Financing Capacity with Stealing and Shirking

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We study a firm's capacity choice under demand uncertainty given it must finance this investment externally. Sharing profits with investors causes governance problems affecting both capacity and demand: the firm may "steal" capital, which reduces effective capacity, and "shirk" on market-development, which reduces demand. We adopt an optimal contracting approach whereby the firm optimizes among feasible financial claims derived endogenously. We characterize its optimal financing and capacity choices. First, debt financing is optimal: it minimizes the incentives to both divert *and* shirk. Second, the firm underinvests (overinvests) if the effort problem is mild (severe) enough relative to the diversion problem. Thus, a worsening of the *same* governance problem can lead to over- or underinvestment depending on circumstances. Third, we find that the diversion and shirking problems interact in their impact on capacity investment. In particular, if the shirking problem is mild enough, the more severe the diversion problem, the *less* the firm invests; However, if the shirking problem is severe enough, the effect of diversion is reversed: the more severe the diversion problem, the *more* the firm invests.

Key words: Capacity Investment, Optimal Contracts, Capital Diversion, Financial Constraints, Newsvendor Model, Moral Hazard.

1. Introduction

In new and fast-changing markets, businesses invest in capacity under considerable demand uncertainty. The operational challenges involved are well established: as firms match capacity and demand, they must trade-off overage vs. underage costs (Arrow et al. 1951, Cachon and Terwiesch 2012). The financing of capacity investments involves challenges too. Indeed, having to share profits with investors raises governance issues (Shleifer and Vishny 1997).[1] These can impact both capacity and demand: firm insiders may divert capital away from its intended use, thus reducing effective capacity and increasing underage risk; They may also fail to allocate suitable resources to market development, thus deflating demand and increasing overage risk. How should firms match capacity and demand when governance issues affect both? How should they account for the impact of capacity choices on funding needs, and thus on governance problems?

This paper studies how capacity investment under demand uncertainty should adjust to finance-implied governance issues affecting both capacity and demand. Responses may be contractual, with incentives set through a financing package, or operational, with deviations from optimal capacity. To consider both, we frame the problem as one of optimal contracting in which a firm facing uncertain demand optimizes both capacity and the financial claim issued to fund it, given two governance problems: diversion ("stealing") of capital which reduces effective capacity, and "shirking" on market development which deflates demand. We characterize the jointly optimal financing and capacity choices.

This analysis yields new insights about capacity financing and investment when capacity and demand are both subject to moral hazard. First, debt financing is optimal: it minimizes the incentives to both divert *and* shirk. Second, the firm underinvests (resp. overinvests) if the diversion problem is relatively more (resp. less) severe than the shirking problem. Hence, a worsening of the *same* governance problem can lead to over- or underinvestment depending on circumstances. Third, and more importantly, the diversion and shirking problems interact to impact capacity investment: if the shirking problem is mild enough,

[1] In their classic survey, Shleifer and Vishny (1997) summarize governance problems as follows: "How do the suppliers of finance get the managers to return some of the profits to them? How do they make sure that managers do not steal the capital they supply or invest in bad projects?"

the more severe the diversion problem, the *less* the firm invests; However, if the shirking problem is severe enough, the effect of diversion is reversed: the more severe the diversion problem, the *more* the firm invests.

Specifically, we frame a firm's problem of matching capacity and demand as a newsvendor model. To finance capacity, the firm's sole owner ("the firm") must seek funds from a competitive investor. Both enter a financial contract whereby the investor provides funds against a financial claim, i.e., a promise of cashflow-contingent repayments. The contract sets values for other variables provided they are contractible, i.e., verifiable by a court.

The firm then takes two actions. First, it can divert funds for its own private benefit. This results in lower effective capacity, increasing underage risk.[2] Second, the firm can shirk on market development (e.g., launching a sales drive), deflating demand and increasing overage risk, which we model as follows. By exerting a costly effort, the firm can improve the distribution of demand in the sense of the Monotone Likelihood Ratio Property (MLRP).

In our model, the first-best outcome involves effort and no diversion. We thus assume diversion to be inefficient: \$1 diverted yields less than \$1 in private benefit for the firm.[3] We also assume effort to be efficient at the first-best capacity level, i.e., to yield an increase in expected revenues exceeding effort's cost. The degree of inefficiency of diversion and the effort cost are key parameters in our analysis: they measure governance problems' severity.

If diversion and effort are contractible, the financial claim is irrelevant (Modigliani-Miller Theorem) and funding needs do not affect operations as the contract sets capacity, diversion and effort at their first-best levels. Instead, we assume them to be non-contractible. In this case, having to share profits with the investor can induce the firm to divert funds or shirk. However, before concluding that it does, we must consider the extent to which the contract between the firm and the investor can mitigate both moral hazards.

To this end we adopt an optimal contracting approach whereby the firm optimizes over a set of feasible financial contracts derived from fundamentals: preferences, technology, and

[2] Evidence abunds that corporate control can entail private benefits (Barclay and Holderness 1989, Dyck and Zingales 2004) that can stem from the diversion of corporate resources (Bertrand et al. 2002). Here, we equate diversion to the firm absconding with funds but other interpretations are possible (see Section 8).

[3] This may reflect dissimulation costs or expected penalties (Shleifer and Wolfenzon 2002) which can be sector-specific (e.g., easy-to-monitor assets) or country-specific (e.g., legal investor protection).

information. Thus deviations from operational efficiency are considered only once contractual solutions are exhausted and hence are robust to simple contract changes. Diversion, effort, and unmet demand are the only non-contractible variables; all others are set by contract. Given the mapping between satisfied demand and cashflow, this amounts to assuming that (only) the cashflow is contractible. This is plausible as cashflows can be audited but unmet demand is usually hard to observe. We make standard assumptions so that financial claims respect limited liability and are monotonic, i.e., repayments cannot exceed the cashflow and must increase weakly with it (Tirole 2006). Feasible financial claims so-defined include debt, equity, convertible debt, warrants, combinations of claims, etc.

Our first set of results relates to optimal financing as we establish that the firm is better off financing the project by issuing debt rather than any other claim.

We start by establishing debt's optimality for each moral hazard alone. Under debt financing, the firm receives nothing when cashflows are low and a high fraction of the cashflows when the latter are large. In fact, debt is the financial claim that maximizes the firm's payoff in high cash flow states (by minimizing it in low cashflow states). This property underpins the optimality of debt for dealing with shirking incentives. Indeed under the MLRP, higher cashflow states are more indicative of effort and maximizing what the firm receives in those states discourages shirking. This pattern also underpins the optimality of debt for dealing with the stealing problem: since stealing diminishes capacity and debt makes the firm a residual claimant of the cashflows, stealing reduces the fraction of the cashflow the firm receives before reducing the fraction the investor receives.

However, as we show, debt minimizes not only the incentives to divert *or* shirk but also those to both divert *and* shirk. This result is new and, in particular, not implied by debt's optimality for each problem *alone*. It relies on a form of complementarity between capacity and demand inherent to the problem of matching capacity and uncertain demand: as diversion decreases, higher demand can be met, encouraging effort; conversely, effort boosts demand, discouraging diversion; hence mitigating one moral hazard with debt mitigates the other as well. Showing debt's optimality under both moral hazards is a contribution in itself but more importantly, constitutes a key methodological step: it allows us to proceed with the analysis assuming optimal financial contracting.

In a second set of results, we characterize the conditions under which, given optimal financing, the firm underinvests or overinvests in capacity relative to the first-best level.

For the diversion problem *alone*, capacity is set at or strictly below the first-best level: the firm underinvests. The result is new and its intuition subtle: it relies on debt financing and decreasing returns to scale. Indeed, under debt financing, by cutting units below the first best level, the firm loses the revenue from these units. But lowering capacity also lower the funding need and thus the debt's face value. This frees up other units that were dedicated to repaying the investor first. Given decreasing returns to scale to capacity investment, the units gained have higher expected revenues than those lost, which discourages diversion.

For the effort problem *alone*, capacity is set at or strictly above the first-best level: the firm overinvests. Indeed, higher capacity widens the set of feasible contracts, which improves the firm's incentives (de Véricourt and Gromb 2018).[4] By extension, the result also holds if the diversion problem is mild enough (relative to the effort problem).

For both moral hazard problems together, we show that the firm underinvests if the diversion problem dominates (in a sense we make precise) and overinvests if the shirking problem dominates. This result is new and no mere juxtaposition of those obtained for each moral hazard alone. This is particularly evident in examples where the firm invests at the first best level absent a diversion problem but *above* that level once a diversion problem is introduced. Conversely, it can be that the firm invests at the first best level absent a shirking problem but *below* (not above) that level once a shirking problem is introduced.

The key here is the complementarity between diversion and shirking. Due to complementarity, it is most tempting for the firm not to divert *or* shirk, but to divert *and* shirk. To deter such behavior, the firm must lower the associated gains which equal the sum of *i*) shirking gains absent diversion and *ii*) diversion gains given shirking. Clearly, but importantly, an increase in the diversion problem's severity raises the latter gains but leave the former unchanged. When the diversion problem dominates, decreasing capacity

[4] This occurs through two channels: higher capacity reveals otherwise unmet demand that under the MLRP is particularly indicative of effort and increases what the firm receives in higher demand states.

below the first best level is optimal. Indeed, lower capacity increases shirking gains absent diversion (i) and reduces diversion gains given shirking (ii), but the diversion problem's dominance means the latter effect is large. Conversely, when the shirking problem dominates, increasing capacity above the first best level is optimal. Indeed, higher capacity reduces shirking gains absent diversion and increases diversion gains given shirking but the diversion problem's non-dominance means the latter effect is small.

In our last and main set of results, we characterize the impact of the diversion and shirking problems on capacity investment.

First, consider how the diversion problem's severity impacts optimal capacity. We show that if the project is optimally undertaken at a non-first best capacity, an increase in the diversion problem's severity implies a decrease in capacity when the shirking problem is mild enough but that this effect is reversed when the shirking problem is more severe: the more severe the diversion problem, the *more* the firm invests in capacity.

To understand why, consider first the diversion problem *alone* in which case the firm underinvests. In that case the more severe the diversion problem, the *less* the firm invests. This result aligns with common intuition: the more tempting diversion is, the more capacity must be reduced to discourage it. Yet recall that the intuition must invoke (optimal) debt financing and decreasing returns to scale inherent to the newsvendor problem. We show that the result extends to cases where the shirking problem is mild enough. Importantly, this extension is not "by continuity": it relies on complementarity, as explained below.

Consider the effort problem *alone* in which case the firm over-invests. Now, introduce the possibility of diversion. If the effects of both problems simply added up, a more severe diversion problem would still lead the firm to invest *less*. But in fact, the effect of diversion is reversed: the more severe the diversion problem, the *more* the firm invests. This counterintuitive result is also driven by complementarity.

Indeed, suppose the project is undertaken on a non-first-best scale. In response to a rise in the diversion problem's severity, the firm must deter diversion *and* shirking by raising or cutting capacity. These have opposite effects on the two components (i and ii) of deviation gains. Which route is best depends on the shirking problem's severity. For capacity to deviate from its first-best level when the shirking problem is mild, the diversion problem must be severe enough. If so, cutting capacity is optimal because the impact of a change in capacity on the gains from diversion given shirking (*ii*) is large. Conversely, for a deviation to occur when the shirking problem is severe, the diversion problem must be mild enough (or else the project is abandoned). If so, increasing capacity is optimal because the impact of a change in capacity on the gains from diversion given shirking (*ii*) is small.

Second, we show a similar reversal for the impact of the shirking problem's severity on capacity: a more severe shirking problem leads to the firm investing more (resp. less) in capacity if the diversion problem is mild (resp. severe).

The intuition for this result is along similar lines. Following an increase in the shirking problem's severity, the firm must either raise or cut capacity. When the diversion problem is mild enough, increasing capacity is optimal because the impact of a change in capacity on the gains from diversion given shirking is small. Conversely, when the diversion problem is severe enough, cutting capacity is optimal because the impact of a change in capacity on the gains from diversion given shirking is large.

The paper proceeds as follows. Section 2 reviews the literature. Sections 3 and 4 present the model and the firm's problem. Section 5 proves debt's optimality. Section 6 studies under vs. overinvestment and Section 7 stealing and shirking's impact on capacity choices. Section 8 discusses implications. Section 9 concludes. Proofs are in the online appendix available in the e-companion.

2. Literature Review

Our paper studies a new problem and contributes new results to the corporate governance, operations management, financial contracting, and optimal compensation literatures.

Our paper relates to research on corporate governance (Shleifer and Vishny 1997, Becht et al. 2007) where shirking and diversion also appear (Tirole 2006, Burkart et al. 1998). First, the problem of matching capacity and demand we study here raises specific issues that are new to the corporate finance literature where production is usually blackboxed. For shirking, non-contractible unmet demand, which is natural here but has no equivalent in finance, implies that higher capacity makes higher demand contractible. In turn, this means that capacity affects the power of incentives to exert effort, leading to optimal overinvestment. For diversion, non-contractible capacity and decreasing returns to scale are natural here but absent in finance where diversion usually affects cashflows, not investment.[5] That diversion affects capacity, not the cashflow, is key to our debt optimality result, and decreasing returns to scale are key to our optimal underinvestment result. Second, and more importantly, we study two governance problems' interaction which is new. Moreover, the specificities of the problem of matching capacity and demand yield new predictions. For instance, while more severe shirking or diversion problems lead to less investment in the governance literature, we show that either can lead to more investment.

Our paper fits in the operations literature on capacity and financial choices, with implications for funding's effect on capacity, R&D or technology choices and for how these impact financing (e.g., Boyabatli and Toktay 2011, Li et al. 2013, Alan and Gaur 2017, Ning and Babich 2018). In particular, our work relates to Chod and Zhou (2014), Iancu et al. (2017) and Ni et al. (2017), which study the capacity choice by a firm whose access to debt financing is hampered by one moral hazard (risk-shifting).

Our paper contributes to this literature by studying two moral hazards affecting capacity and demand. It also makes a methodological contribution to this field by adopting an optimal contracting approach, ensuring that results are robust to simple contract changes. This approach, however, has largely been ignored by the OM literature. In particular, the use of optimal financial contracts would eliminate many of the capacity deviations that this literature has uncovered.[6] One exception is de Véricourt and Gromb (2018)'s study of a similar model with shirking only, showing that overinvestment can be optimal. To our

[5] In Jensen (1986)'s free cash flow theory, managers divert the firm's cash to overinvest, not to underinvest as here. Some papers assume diversion of cashflows, not investment (e.g., Bolton and Scharfstein 1990). There, claims are very coarse (and debt is not optimal) as they cannot condition on cashflows (see footnote 14). In other models, shirking can be interpreted as asset diversion (e.g., Tirole 2006). They assume constant returns to scale in investment whereas decreasing returns are key here.

[6] For instance, Dada and Hu (2008) show that a cash-poor newsvendor will underinvest. However, this result holds only assuming that the investor sets a loan-size-independent interest rate and that the firm can choose any loan size at that rate. Since there are no frictions in the model (e.g. no moral hazard), absent this exogenous restriction on financial contracts, the newsvendor would be able invest at the first-best level: by choosing a loan size, the newsvendor commits to making proportional interest payments, which requires the loan size to be contractible (a natural assumption). But if so, loan-size dependent interest debt is feasible and eliminates the underinvestment. Iancu et al. (2017) and Ni et al. (2017)'s models involve a friction (moral hazard) but assume debt financing, whereas equity financing would eliminate the risk-shifting incentives they consider. Chod and Zhou (2014) assume debt and equity financing whereas, for instance, convertible debt can mitigate risk-shifting.

knowledge, the only other is Tang et al. (2017)'s study of optimal financial contracting between a cash-poor supplier and a manufacturer with shirking. However, it considers neither a security design problem (as in its two cashflow states model, all claims are equivalent) nor, more importantly, capacity choices. Instead our paper is the first to study diversion and how its interaction with shirking affects funding and capacity choices.

Our paper relates to the financial contracting literature. Jensen and Meckling (1976) show debt to dominate equity for shirking problems (and vice versa for "risk-shifting" problems). Innes (1990) extends the analysis of shirking problems to optimal contracts: under the MLRP, debt dominates all other claims. We make two contributions. First, we show debt to be optimal for the shirking problem *alone* (as in de Véricourt and Gromb (2018)'s extension of Innes (1990) to a newsvendor model) and the diversion problem *alone*. The latter result is new: it is not implied by Innes (1990) as the MLRP fails to hold for diversion. Second, and more importantly, we show debt's optimality for *both* problems. This result is new and, in particular, not implied by debt's optimality for *each* problem (as Appendix D of the e-companion illustrates). The few papers with multiple moral hazards assume a shirking and a risk-shifting problem (Biais and Casamatta 1999, Hellwig 2009). The issues and results are very different from ours: they show a mix of debt, equity and perhaps other claims to be optimal, not debt alone as here, that underinvestment is optimal and that investment decreases with the moral hazards' severity when we show that overinvestment can be optimal and investment can increase with the severity of either moral hazard.

Our paper also builds on the principal-agent literature with risk-neutrality and limited liability (Oyer 2000, Gromb and Martimort 2007, Poblete and Spulber 2012). In particular, Dai and Jerath (2013, 2016) and Chu and Lai (2013) study capacity choice models in which a sales agent's wages must incite her to boost demand, and show that overinvestment can be optimal. Their salesforce compensation problem is quite different from our financing problem: constraints on wage contracts differ from those on financial contracts (notably monotonicity); moral hazard benefits their sales agent but hurts our firm; they focus on one moral hazard (shirking), not two moral hazards (shirking and diversion) and their interaction as we do. Thus our questions, frameworks and results are different.

3. The Model

We study a firm making a capacity investment choice under demand uncertainty, which it must fund externally. Profit-sharing with investors creates governance problems affecting both capacity and demand: the firm may divert capital, which reduces effective capacity, or shirk on market-development, which deflates demand.

We frame the situation in a newsvendor model with risk-neutrality and no discounting. A firm with a sole owner ("the firm") has a project with stochastic demand D_1 with strictly positive distribution $f_1(\cdot)$, cumulative distribution $F_1(\cdot)$, and complementary cumulative distribution $\bar{F}_1(\cdot) \equiv 1 - F_1(\cdot)$.[7] (As we clarify later, the index 1 indicates a high effort level.) The firm sets up capacity at unit cost $c > 0$ and receives revenue $r > c$ per unit sold. For simplicity, we assume no salvage value for unsold units.[8]

The firm has no cash to finance capacity, but it can raise funds $I \geq 0$ from a competitive investor.[9] If so, the firm and the investor enter a financial contract specifying, among other variables, the capacity $q \in \mathbb{R}_+$ that the firm agrees to set up, to which we refer as *declared capacity*. If the firm sets $q = 0$, it abandons the project and no other decision is taken. Otherwise, if $q > 0$, the firm receives I from the investor. At that point, the firm must take two decisions: a diversion choice and an effort choice.

Diversion. The firm can divert part of the amount cq meant to fund declared capacity q. That is, the firm can choose to only invest in *effective capacity* $x \in [0, q]$ at cost cx, so that at most demand for x units can be served. This is possible provided x is non-contractible, i.e., courts cannot verify that $x \neq q$. The firm diverts the rest of the funds, $c(q - x)$, which yields private benefit $\lambda c(q - x)$ with $\lambda \in [0, 1)$. Note that $\lambda < 1$ ensures that diversion is inefficient, i.e., a unit's cost exceeds its diversion's payoff.

Diversion captures instances where the firm's insiders can abscond with funds or assets, its inefficiency arising from dissimulation costs or expected punishment. It can also capture

[7] The risk-neutrality assumption is standard and important for the result that debt financing is optimal. No-discounting is only for simplicity, as is the strict positivity of f_1.

[8] A positive salvage value raises contracting issues that are immaterial to our point but obfuscate the exposition. All our results hold for any unit salvage value less than c (de Véricourt and Gromb 2017).

[9] The assumption of a cashless firm is for simplicity. Our analysis easily extends to the case where the firm has cash is can use to fund part of its capacity.

asset use for pet projects, to favor related parties, etc. giving insiders private gains less than the implied profit loss. Diversion can also reflect the owner not taking an action to make units operational, e.g., maintenance, λc being the action's cost (see Section 8).

Effort. Second, if the firm undertakes the project ($q > 0$), it chooses a non-contractible market development effort: high effort ($e = 1$) implies a non-monetary cost $\kappa_1 > 0$ for the firm and low effort ($e = 0$) a smaller one $\kappa_0 > 0$, i.e., $\Delta\kappa \equiv \kappa_1 - \kappa_0 \geq 0$.[10] Demand shifts to D_0 with a distribution f_0 less favorable than f_1 in the sense of the Monotone Likelihood Ratio Property (MLRP), i.e., f_1/f_0 is strictly increasing over $\mathbb{R}_+$.

Effort captures the firm's market development activities: running a marketing campaign, sales actions, tuning product design, etc. Its cost can be a labor cost for the owner, the opportunity cost of shifting attention away from other projects, etc.

For declared and effective capacities q and x, and effort e, the project generates a unique cashflow that is randomly distributed over $[0, rx]$ and equal to

$$P_{e,x,q} \equiv r\,(D_e \wedge x) \tag{1}$$

and the firm also receives private benefit $\lambda c\,(q - x)$, so the project's value is[11]

$$\mathbb{E}\,[P_{e,x,q}] + \lambda c\,(q - x) - cq - \kappa_e. \tag{2}$$

First-Best. We now characterize the first-best optimum benchmark, when effort and diversion are contractible. For effort e, ignoring diversion ($x = q$) and effort cost κ_e, the project's value is the profit in a standard newsvendor model with demand c.d.f. F_e, i.e.,

$$\pi_e\,(q) \equiv \mathbb{E}\,[P_{e,q,q}] - cq.$$

As per standard arguments, the capacity maximizing $\pi_e\,(q)$ is $\bar{F}_e^{-1}\,(c/r)$. Now consider effort costs but not diversion. The cost being fixed, the optimal capacity for effort e remains

[10] The assumption that no cost is incurred if $q = 0$ is for simplicity. It amounts to assuming that κ_0 is a fixed cost which will allow us to focus on cases where the firm exerts $e = 1$ or sets $q = 0$ (see Lemma 2).

[11] Note that we assume no project adjustment to effort e and effective capacity x (i.e., unit price and cost remain r and c). In fact, while the project could be adjusted to e or x (e.g., with cheaper capacity, lower prices, etc.), this would violate the assumed non-contractibility of e and x.

$\bar{F}_e^{-1}(c/r)$, provided the firm undertakes the project ($q>0$). In that case, we assume that the project is viable if $e=1$ but best abandoned ($q=0$) if $e=0$, i.e.,[12]

$$\max_{q\in\mathbb{R}_+}\pi_1(q)-\kappa_1>0>\max_{q\in\mathbb{R}^+}\pi_0(q)-\kappa_0 \tag{3}$$

Note that condition (3) implies $\underline{q}_1<\bar{F}_1^{-1}(c/r)<\bar{q}_1$ where

$$\bar{q}_1\equiv\max\left\{q\in\mathbb{R}^+\text{ s.t. }\pi_1(q)\geq\kappa_1\right\}\quad\text{and}\quad\underline{q}_1\equiv\min\left\{q\in\mathbb{R}^+\text{ s.t. }\pi_1(q)\geq\kappa_1\right\}. \tag{4}$$

Define q^{FB}, x^{FB} and e^{FB} as the first-best declared capacity, effective capacity and effort.

Lemma 1 *At the first-best, the firm exerts effort, i.e.,* $e^{FB}=1$*, and declares and effectively installs the optimal standard newsvendor capacity, i.e.,* $x^{FB}=q^{FB}=\bar{F}_1^{-1}(c/r)$*.*

Indeed, diversion being inefficient (i.e., $\lambda<1$), diverting capacity is never first-best optimal. Given this, the first-best outcome is as if diversion were impossible. And given condition (3), this means exerting effort and setting up the standard newsvendor's capacity.

Financial contracts. The firm enters a financial contract with the investor whereby the latter transfers funds $I\geq 0$ to the firm against a financial claim stipulating state-contingent repayments. The contract can also specify any other item provided it is contractible.

In the optimal contracting approach, feasible contracts stem endogenously from assumptions about fundamentals. For presentation practicality however, we first define the set of feasible contracts and then lay out the standard assumptions from which it derives.

Definition 1 *A financial contract is feasible if it specifies investment* I*, declared capacity* q *and a feasible financial claim. A financial claim is feasible if repayments (i) are contingent only on cashflow, (ii) satisfy the firm's limited liability, and (iii) are monotonic, i.e., for cashflow realization* $p\in[0,rq]$*, the firm must make a repayment* $R(p)\leq p$ *with* $R(\cdot)$ *non-decreasing over* $[0,rq]$*.*

[12] Assuming two effort levels, one for which the project is not viable, is a common simplification. It avoids running and comparing multiple optimization problems for the firm: one for each possible value of e.

Let $\mathcal{C}$ be the set of feasible claims, represented as repayment functions $R(\cdot)$.[13] Debt and equity are feasible: a loan with face value K maps into $R(p) = p \wedge K$ and a fraction α of equity into $R(p) = \alpha p$, which both satisfy Definition 1. (We ignore the distinction between interest and principal which is immaterial here.) Other claims such as convertible debt, call warrants, etc. are also feasible, as are some combinations of claims (e.g., debt plus equity, etc.). Hence a feasible financial contract is a triplet $(q, I, R) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathcal{C}$.

We now discuss the assumptions underlying Definition 1. Parameters are assumed common knowledge so contracting on them is not needed. This standard assumption is plausible, e.g., costs and prices can usually be retrieved from accounts. Thus the model's variables are investment I, declared and effective capacities q and x, effort e and demand D_e.

Specifying which variables are contractible is key. Investment I and declared capacity q are assumed contractible, but not effort e and diversion $(q - x)$. This creates two moral hazards. Regarding demand D_e, we assume that (only) unmet demand is non-contractible. The mapping from demand to cashflow $P_{e,x,q}$ being one-to-one below installed capacity x and flat above x, this amounts to assuming that (only) the cashflow is contractible. This is plausible as cashflows can be audited while unmet demand is usually hard to observe.[14]

Last, limited liability and monotonicity are standard assumptions in corporate finance. Limited liability may arise from the firm's need for a minimum subsistence level of wealth. Monotonicity is required when the firm can report artificially inflated revenues (e.g., by borrowing secretly from third parties), which non-monotonic claims would invite. Similarly, non-monotonic claims may induce sabotage by the investor to reduce cashflows (Tirole 2006).

[13] $R(\cdot)$ can take negative values, i.e., the investor may have to pay the firm for some cashflow realizations.

[14] With non-contractible cashflows, financial claims would be unfeasible, and thus financing impossible. Note also that the non-contractibility of diversion requires that of (unmet) demand. With contractible demand, realizations above effective capacity would reveal diversion. Taking limited liability to mean that the firm's payoff must be nonnegative *in equilibrium*, arbitrarily harsh punishments for revealed diversion would be feasible (they would deter diversion and thus not be used in equilibrium) and implement the first best.

4. The Firm's Problem

The firm's problem is to choose declared capacity $q \in \mathbb{R}_+$, funds $I \in \mathbb{R}_+$ raised from the investor and feasible financial claim $R(\cdot) \in \mathcal{C}$ to solve

$$\max_{(q,I,R)\in\mathbb{R}^+\times\mathbb{R}^+\times\mathcal{C}} \mathbb{E}\left[P_{e,x,q} - R(P_{e,x,q})\right] + \lambda c(q-x) + I - cq - \kappa_e \tag{5}$$

Indeed, on average, the firm receives the project's expected cashflow $\mathbb{E}[P_{e,x,q}]$ net of the expected repayment to the investor $\mathbb{E}[R(P_{e,x,q})]$ as well as private benefit $\lambda c(q-x)$ if x units are effectively installed and $(q-x)$ diverted. Further, the firm receives funds I from the investor, sets aside the investment cost cq for capacity q and incurs effort cost κ_e.

This choice is made under constraints. First, the investor must at least break even, i.e.,

$$\mathbb{E}\left[R\left(P_{e,x,q}\right)\right] \geq I \tag{6}$$

Second, the firm too should accept the contract, hence the participation constraint

$$\text{if } q>0, \qquad \mathbb{E}\left[P_{e,x,q} - R(P_{e,x,q})\right] + \lambda c(q-x) + I - cq - \kappa_e \geq 0 \tag{7}$$

Third, the funds raised should cover the cost of capacity q, i.e.,

$$I \geq cq \tag{8}$$

Last, the firm must prefer effort e and effective capacity x to any alternative effort ϵ and effective capacity y, i.e., if $q>0$, for all $\epsilon \in \{0,1\}$ and $y \in [0,q]$,

$$\mathbb{E}\left[P_{e,x,q} - R(P_{e,x,q})\right] + \lambda c(q-x) + I - cq - \kappa_e \geq \mathbb{E}\left[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})\right] + \lambda c(q-y) + I - cq - \kappa_\epsilon \tag{9}$$

We show that if the project can be funded, an optimal contract exists that deters diversion and shirking. This narrows down the contract set over which the firm optimizes.

Lemma 2 *Any contract is (weakly) dominated by a contract (q,I,R) under which the firm chooses optimally effort e and effective capacity x such that: (i) the investor breaks even, i.e., $\mathbb{E}\left[R\left(P_{e,x,q}\right)\right] = I$; (ii) effort is high or the project is abandoned, i.e., if $q>0$ then $e=1$; (iii) there is no diversion, i.e., $x=q$.*

Point (i) reflects the competitive investor assumption. Point (ii) stems from the assumption that the project is not viable with low effort. Point (iii) derives from diversion's assumed inefficiency. The intuition is that if for contract (q, I, R) the firm installs effective capacity $x < q$, it would do so too for contract (x, I, R), which avoids inefficient diversion.

Given this, we denote contracts simply by (q, R) and the problem simplifies to:

$$\max_{(q,R)\in\mathbb{R}_+^*\times\mathcal{C}} \quad \pi_1(q) \tag{10}$$

$$\underline{q}_1 \leq \; q \; \leq \bar{q}_1 \tag{11}$$

$$\mathbb{E}[R(P_{1,q,q})] \geq cq^+ \tag{12}$$

$$\forall y \in [0, q], \forall \epsilon \in \{0, 1\},$$

$$\mathbb{E}\left[P_{1,q,q} - R(P_{1,q,q})\right] - \kappa_1 \geq \mathbb{E}\left[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})\right] + \lambda c(q-y) - \kappa_\epsilon \tag{13}$$

and if the previous problem is not feasible then the firm abandons the project $(q = 0)$.

5. The Optimality of Debt Financing

In this section, we establish the optimality of debt. A debt claim, i.e., a loan contract L is defined by its face value $K^L \in \mathbb{R}_+$ such that the repayment conditional on cashflow $p \in \mathbb{R}^+$ is $L(p) = p \wedge K^L$. Denote by $\mathcal{L} \subset \mathcal{C}$ the set of debt claims.

Theorem 1 *If problem (10)-(13) has a solution, an optimal contract (q, L) exists, where $L \in \mathcal{L}$ is the unique debt claim with face value K such that $E\left[L\left(P_{1,q,q}\right)\right] = cq$, i.e.,*

$$K - r\int_0^{K/r} F_1(u)\,du = cq \tag{14}$$

where K/r is the demand level below which the firm defaults. The firm finances the project by issuing risky debt ($K \in (0, rq]$).

To prove the theorem, we show that debt minimizes not only the incentive to divert *or* shirk but also that to both divert *and* shirk. To understand the result, discussing the case of a single moral hazard is useful.

Corollary 1 *Assume problem (10)-(13) has a solution. Debt is optimal for (i) the effort problem alone ($\lambda = 0$) and (ii) the diversion problem alone ($\Delta\kappa = 0$).*

First, it is not the case that debt is optimal for any moral hazard (Jensen and Meckling 1976). Instead, the optimal claim depends on the moral hazard's impact on the cashflow distribution, hence the MLRP's relevance. As in de Véricourt and Gromb (2018), point (i) extends to the newsvendor model Innes (1990)'s result that debt is optimal for an effort problem. The idea is as follows. With debt, the firm receives nothing when cashflows are low and a high fraction of the cashflow in high cashflow states. In fact, among all feasible claims, debt is the one that maximizes what the firm receives in high cashflow states (by minimizing what it receives in low cashflow states). This property underpins the optimality of debt for dealing with shirking incentives. Indeed under the MLRP, higher cashflow states are more indicative of effort and maximizing what the firm receives in those states encourages it to exert effort.

Point (ii) stems from a different idea: while diversion does not satisfy the MLRP, maximizing what the firm receives in high cashflow states remains optimal.[15] Indeed, under debt financing, the firm must first use capacity to repay the face value before receiving any part of the cashflow the remaining capacity generates. Hence if the firm diverts capacity, it must first incur the opportunity cost of diverting all units it would otherwise benefit from, before gaining from diverting units that would otherwise benefit the investor. This minimizes diversion incentives.

The theorem states that debt remains optimal for both moral hazards. This is in no way a direct implication of points (i) and (ii): debt's optimality for each problem does *not* imply its optimality for both problems (see Appendix D of the e-companion for a simple counterexample).[16] Instead, the result relies on a form of complementarity: as diversion decreases, higher demand can be served, encouraging effort; conversely, higher effort improves the distribution of demand, which increases the opportunity cost of diversion, discouraging it. Hence mitigating one moral hazard with debt can only mitigate, not worsen,

[15] To see that diversion fails to satisfy the MLRP, hold declared capacity q and effort e constant and let g_x denote the distribution of cash flow $p = r(D_e \wedge x)$ for effective capacity $x \in [0, q]$. (Note that g_x is loosely defined as the cashflows are not differentiable at rx). Diversion satisfying the MLRP would mean that for all x and y with $q \geq x > y \geq 0$, the ratio $g_x(p)/g_y(p)$ increases with p. To see that this is not the case, notice that the ratio equals 1 for $p < ry$, 0 for $p = ry$ and $+\infty$ for $p \in (ry, rx)$.

[16] The counterexample has two moral hazards each satisfying the MLRP, implying that debt is optimal for each problem alone, and yet, debt is not optimal for both problems together.

the other. This ensures that debt is also optimal for both moral hazards. The property underlying the argument is not generally true, but holds here due to capacity and demand's natural complementarity.

The theorem contributes to the optimal financial contracting literature but more importantly it is a key methodological step in our analysis: the optimal financial contracting approach ensures that our results about operations are robust to simple contract changes.

6. Optimal Under- vs. Overinvestment

Our second set of results characterize the conditions under which, given optimal financing, the firm's optimal capacity lies above or below its first-best level. We link this choice to the severity of the shirking and diversion problems, as measured by $\Delta\kappa$ and λ.

Theorem 2 *There exist a unique threshold $\hat{\lambda} \in (0,1)$ and for all $\lambda \in [0,1]$, two unique thresholds $(\Delta\underline{\kappa}, \Delta\bar{\kappa}) \in \mathbb{R}^2$, with $\Delta\underline{\kappa} \leq \Delta\bar{\kappa}$, such that there is no diversion ($x^* = q^*$) and*
(i) If $\Delta\kappa \leq \Delta\underline{\kappa}$, the firm invests at the first-best level ($q^* = q^{FB}$);
(ii) If $\Delta\kappa \in (\Delta\underline{\kappa}, \Delta\bar{\kappa})$, the firm overinvests ($q^* > q^{FB}$) if $\lambda < \hat{\lambda}$ but underinvests ($q^* < q^{FB}$) if $\lambda > \hat{\lambda}$;
(iii) If $\Delta\kappa > \Delta\bar{\kappa}$, the firm abandons the project ($q^* = 0$).
Moreover, $\Delta\underline{\kappa}$ and $\Delta\bar{\kappa}$ are continuous non-increasing in λ and equal iff $\lambda = \hat{\lambda}$. Further, unique thresholds $(\underline{\lambda}, \bar{\lambda}) \in (0,1]$ exist such that $\Delta\underline{\kappa} < 0$ iff $\lambda > \underline{\lambda}$, and $\Delta\bar{\kappa} < 0$ iff $\lambda > \bar{\lambda}$.

The theorem partitions the $(\lambda, \Delta\kappa)$ parameter space in four regions (Figure 1). If the mix of governance problems is mild enough, i.e. $(\lambda, \Delta\kappa)$ with $\Delta\kappa \leq \Delta\underline{\kappa}(\lambda)$, the firm can invest at the first best level. Instead, if that mix is severe enough, i.e. $(\lambda, \Delta\kappa)$ with $\Delta\kappa > \Delta\bar{\kappa}(\lambda)$, it must abandon the project. In the middle region, where the mix is neither too mild nor too severe, i.e. $(\lambda, \Delta\kappa)$ with $\Delta\kappa \in (\Delta\underline{\kappa}(\lambda), \Delta\bar{\kappa}(\lambda))$, the firm undertakes the project but not at its first-best scale. That region itself is split into two. Denote $\Delta\hat{\kappa} \equiv \Delta\underline{\kappa}(\hat{\lambda}) = \Delta\bar{\kappa}(\hat{\lambda})$. If the diversion problem is severe enough and thus the shirking problem mild enough, with $\lambda > \hat{\lambda}$ and $\Delta\kappa < \Delta\hat{\kappa}$, the firm underinvests, but if the shirking problem is severe enough and thus the diversion problem mild enough, with $\lambda < \hat{\lambda}$ and $\Delta\kappa > \Delta\hat{\kappa}$, it overinvests.

To understand the result, discussing the case of a single moral hazard is useful.

Corollary 2 *Assume that the project is undertaken ($q^* > 0$), which requires $\Delta\kappa < \Delta\bar{\kappa}$.*
(i) For the effort problem alone ($\lambda = 0$), the firm never underinvests ($q^* \geq q^{FB}$) and it overinvests ($q^* > q^{FB}$) if and only if the effort problem is severe enough ($\Delta\kappa > \Delta\underline{\kappa}$).
(ii) For the diversion problem alone ($\Delta\kappa = 0$), the firm never overinvests ($q^* \leq q^{FB}$) and it underinvests ($q^* < q^{FB}$) if and only if the diversion problem is severe enough ($\lambda > \underline{\lambda}$).

Point (i), optimal over-investment (absent diversion), is counter-intuitive. The idea is in de Véricourt and Gromb (2018). As capacity increases, the set of feasible claims widens, mitigating the shirking problem in two ways. First, higher capacity reveals higher demand which would otherwise be unmet and that has stronger incentive effects under the MLRP. Second, higher capacity implies higher scale and thus higher payoffs for high demand, which also increases incentives.

Point (ii) is new. Underinvestment absent shirking seems natural but the intuition is in fact subtle: it relies on optimal debt financing and decreasing returns to scale. Under debt financing, the firm will divert all units or none ($x \in \{0, q\}$). Indeed, as we saw, debt forces the firm to bear the cost of diverting all units it would otherwise benefit from, before it can gain from diverting units that would otherwise benefit the investor. But once that opportunity cost incurred, the firm finds it optimal to divert all of the investor's units. Given this "all or nothing" choice, condition (13) amounts to the firm's expected payoff per unit exceeding diversion's benefit per unit ($\pi_1(q)/q \geq \lambda c$). Given the newsvendor's decreasing returns to scale, lowering capacity makes this condition more likely to hold.[17]

Theorem 2 extends these results to two moral hazards: the firm underinvests if diversion is the dominant problem ($\lambda > \hat{\lambda}$ and $\Delta\kappa \in (\Delta\underline{\kappa}, \Delta\hat{\kappa})$), and overinvests if shirking dominates ($\lambda < \hat{\lambda}$ and $\Delta\kappa \in (\Delta\hat{\kappa}, \Delta\bar{\kappa})$). This is no mere juxtaposition of the single moral hazard results (Corollary 2). Instead, the key is the complementarity between diversion and shirking.

[17] For $\Delta\kappa = 0$ and $x \in \{0, q\}$, condition (13) writes $\mathbb{E}[P_{1,q,q}] - \mathbb{E}[R(P_{1,q,q})] \geq \lambda cq$. Given $\mathbb{E}[R(P_{1,q,q})] = I = cq$ (Lemma 2(i), Theorem 1), it writes $\pi_1(q)/q \geq \lambda c$. The intuition for why lower capacity discourages diversion is less obvious than may seem. First, it relies on debt financing: under non-debt financing, partial diversion ($x \in (0, q)$) can be optimal in which case condition (13) does not write as above. It also relies on decreasing returns to scale: with a fixed cost C, condition (13) writes $(\pi_1(q) - C)/q \geq \lambda c$ and for C large enough, the LHS increases with q in which case lower capacity *encourages* diversion.

Due to complementarity, the most tempting for the firm is not to divert *or* shirk, but to divert *and* shirk. To deter such a "deviation", financial and operational choices must lower the associated gains. These equal the sum of the gains from (a) shirking absent diversion and of (b) those from diversion given shirking. The former are independent of the diversion problem's severity while the latter obviously do increase with the diversion problem's severity.

When the moral hazards are mild, the deviation is not profitable at the first best capacity and the firm can invest at that level. When the moral hazards worsen to the point that the deviation is profitable, the firm has two ways to deter it. One is to increase capacity above the first best level: this decreases the gains from shirking absent diversion (a) but increases those of diversion given shirking (b). Another is to cut capacity below the first best level, with the opposite effects. When the diversion problem is mild enough (λ small), the impact of a change in capacity on the gains from diversion given shirking (b) is smaller than that on the gains from shirking absent diversion (a). In that case, increasing capacity is optimal. The reverse is true when the diversion problem is more severe.

Some configurations highlight that the two moral hazard case is no mere juxtaposition of both single moral hazard cases. For instance, a shirking problem can never lead to underinvestment absent a diversion problem, but can do so if a diversion problem exists. Similarly, in the presence of a shirking problem, introducing a diversion problem can cause overinvestment, which it can never do absent a shirking problem (see Figure 1).

7. Moral Hazards' Impact on Capacity Investment

In our last and main set of results, we characterize the impact of the diversion and shirking problems on capacity investment.

Theorem 3 *For all $\lambda \in [0,1)$, $\Delta\tilde{\kappa} \in [\Delta\underline{\kappa}, \Delta\bar{\kappa}]$ exists s.t., for $\Delta\kappa \in (\Delta\underline{\kappa}, \Delta\bar{\kappa})$,*
(i) If $\lambda < \hat{\lambda}$ and $\Delta\kappa \leq \Delta\tilde{\kappa}$, $q^* > q^{FB}$ is constant in λ and strictly increasing in $\Delta\kappa$;
(ii) If $\lambda < \hat{\lambda}$ and $\Delta\kappa > \Delta\tilde{\kappa}$, $q^* > q^{FB}$ is strictly increasing in both λ and $\Delta\kappa$;
(iii) If $\lambda \geq \hat{\lambda}$, $q^* < q^{FB}$ is strictly decreasing in both λ and $\Delta\kappa$.
Moreover, α and β with $0 < \alpha < \beta < \hat{\lambda}$ exist such that $\Delta\tilde{\kappa} = \Delta\bar{\kappa}$ for $\lambda \in [0,\alpha]$, $\Delta\tilde{\kappa}$ is strictly decreasing for $\lambda \in [\alpha,\beta]$, and $\Delta\tilde{\kappa} = \Delta\underline{\kappa}$ for $\lambda \in [\beta,1)$. (Recall there is no diversion: $x^* = q^*$).

Figure 1 Shirking and diversion problems' impact on optimal capacity q^* in the $(\lambda, \Delta\kappa)$ parameter space. ($r = 2.1$, $c = 1$, $D_1 \sim Gamma(100/4, 4)$ and $D_0 \sim Gamma(75/4, 4)$.)

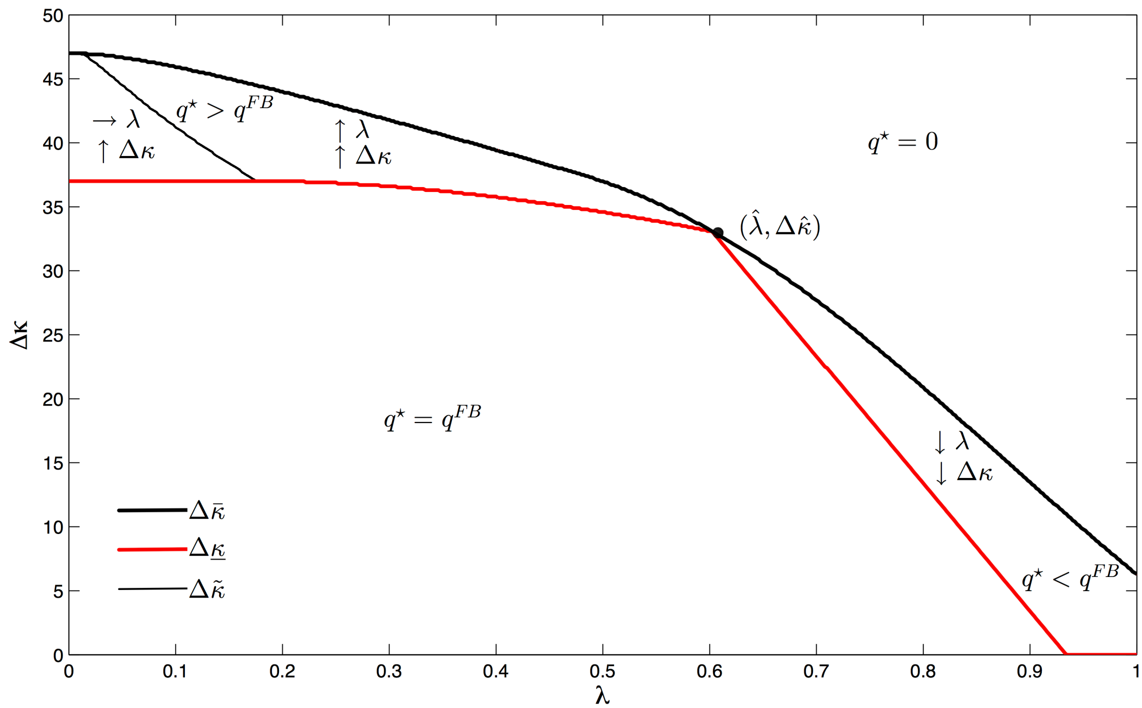

Figure 1 illustrates Theorems 2 and 3. It shows the space of moral hazard parameters $(\lambda, \Delta\kappa)$ to be partitioned in four regions (Theorem 2). In the bottom-left region, the moral hazards are mild enough so the first best obtains ($q^\star = q^{FB}$). In the top-right region, both moral hazards are so severe as to preclude investment altogether ($q^\star = 0$). In between these regions, the moral hazards are mild enough so the project is undertaken ($q^\star > 0$) but severe enough so capacity is not at its first-best level ($q^\star \neq q^{FB}$). That region, between the two curbs, is itself split in two areas. In the right/bottom-most area, where diversion is severe enough and thus shirking mild enough, the firm underinvests ($q^\star < q^{FB}$ as per Theorem 2) and capacity decreases with both λ and $\Delta\kappa$, as indicated by $\downarrow$ (Theorem 3). In the left/upper-most area, where diversion is mild enough and thus shirking severe enough, the firm overinvests ($q^\star > q^{FB}$ as per Theorem 2) and capacity increases with $\Delta\kappa$, as indicated by $\uparrow$, and is either increasing or constant in λ, as indicated by $\downarrow$ and $\rightarrow$ (Theorem 3).

The figure also illustrates how the over- and underinvestment results with both moral hazards (Theorems 2) are no mere juxtaposition of the results obtained for each moral hazard alone (Corollary 2). For instance, consider $\lambda = 0.8$. Note that absent the shirking problem ($\Delta\kappa = 0$), the optimal capacity is at the first best level. Now introduce the shirking problem (e.g., $\Delta\kappa = 20$). If the two effects in Corollary 2 simply added up, the firm would now overinvest (weakly). In fact, it underinvests. A similar point can be seen by considering $\Delta\kappa = 36$: the firm invests at the first best level absent the diversion problem ($\lambda = 0$) but overinvests when the diversion is severe enough (e.g., $\lambda = 0.55$).

Diversion problem's impact

As per Theorem 3, the impact of each moral hazard on optimal capacity depends on the severity of the other moral hazard. We begin with the impact of the diversion problem.

Corollary 3 *Assume the project is undertaken ($q^* > 0$) at a non-first-best capacity ($q^* \neq q^{FB}$) and consider an increase in λ, the diversion problem's severity. (i) If the shirking problem is mild enough ($\Delta\kappa < \Delta\tilde{\kappa}$), capacity decreases weakly, but (ii) if it is severe enough ($\Delta\kappa > \Delta\tilde{\kappa}$), the effect is reversed: capacity increases.*

To understand point (i), first consider the diversion problem *alone*, in which case the firm under-invests. That the more severe the diversion problem, the *less* the firm invests seems intuitive. Yet recall that the intuition must invoke (optimal) debt financing, under which condition (13) amounts to the expected payoff per unit exceeding diversion's benefit per unit ($\pi_1(q)/q \geq \lambda c$). Due to cutting returns to scale, lowering capacity increases the profit per unit. Since cutting capacity is inefficient given under-investment, the firm will lower capacity as little as needed for condition (13) to hold. As diversion becomes more severe (λ increases), the firm must lower capacity further. Point (i) extends this result to an effort problem that is not absent but mild enough. As explained below, this extension is not "by continuity": it relies on the complementarity between diversion and shirking.

Point (ii) is more counter-intuititive. Again, to start with, consider the effort problem *alone* in which case the firm over-invests. Now, introduce the possibility of diversion. If the effects of both moral hazards simply "added up" a more severe diversion problem would still lead the firm to invest *less*. However, the effect of diversion is in fact reversed: the more severe the diversion problem, the *more* the firm invests.

The intuition for points (i) and (ii) relies again on complementarity. Say the project is undertaken on a non-first-best scale. This requires the moral hazard mix to be neither too mild not too severe. Now consider an increase in the diversion problem's severity. The firm must react to deter diversion *and* shirking, the relevant deviaiton under complementarity, by either raising or cutting capacity. These two routes have opposite effects on deviation gains' two components (a) and (b).

The corollary states that which route is optimal depends on the shirking problem's severity. For capacity to deviate from its first-best level when the shirking problem is mild, the diversion problem must be severe enough. If so, cutting capacity is optimal because the negative impact on the gains from diversion given shirking (b) is large. Conversely, for a deviation to occur when the shirking problem is severe, the diversion problem must be mild enough (or else the project is abandoned). If so, increasing capacity is optimal because the positive impact on the gains from diversion given shirking is small.

Figure 2 depicts the impact of the diversion problem's severity λ on optimal capacity $q^\star$ for the same parameters as Figure 1 but holding $\Delta\kappa$ fixed. It illustrates the reversal of the diversion problem's impact: optimal capacity $q^\star$ can increase strictly with λ if the shirking problem is severe enough (Figure 2(a)), but decreases with λ otherwise (Figure 2(b)).

In Figure 2(a), $\Delta\kappa = 38$. For $\lambda = 0$, $\Delta\tilde{\kappa} = \Delta\bar{\kappa} = 47$ and $\Delta\underline{\kappa} = 37$. (Thus $\Delta\kappa > \Delta\underline{\kappa}$ for all $\lambda \in [0, 1)$ since $\Delta\underline{\kappa}$ is decreasing in λ). Hence, $q^\star = 101$ which exceeds the first-best capacity level, $q^{FB} = 99.8$ (Theorem 2(ii)). As λ increases, $\Delta\underline{\kappa}$, $\Delta\tilde{\kappa}$ and $\Delta\bar{\kappa}$ decrease weakly. At first ($\lambda \leq 0.16$), $\Delta\kappa < \Delta\tilde{\kappa}$ and $q^\star$ stays constant (Theorem 3(i)). As λ rises further ($\lambda \in (0.16, 0.46)$), $\Delta\tilde{\kappa} < \Delta\kappa < \Delta\bar{\kappa}$ and $q^\star$ increases strictly (Corollary 3(ii)). For λ large enough ($\lambda \geq 0.46$), $\Delta\kappa \geq \Delta\bar{\kappa}$ and the project is abandoned, i.e., $q^\star = 0$ (Theorem 2(iii)).

In Figure 2(b), $\Delta\kappa = 22$. For $\lambda = 0$, $\Delta\underline{\kappa} = 37$ so $\Delta\kappa < \Delta\underline{\kappa}$ and first-best capacity is optimal, i.e., $q^\star = q^{FB} = 99.8$ (Theorem 2(i)). As λ increases, at first ($\lambda \leq 0.71$), $\Delta\kappa < \Delta\underline{\kappa} \leq \Delta\tilde{\kappa}$ and $q^\star$ stays constant (Theorem 2(i)). As λ rises further ($\lambda \in (0.71, 0.79)$), $\Delta\underline{\kappa} < \Delta\kappa < \Delta\bar{\kappa}$ and $q^\star$ decreases strictly (Theorem 3(iii)). For λ large enough ($\lambda \geq 0.79$), $\Delta\kappa \geq \Delta\bar{\kappa}$ and the project is abandoned, i.e., $q^\star = 0$.

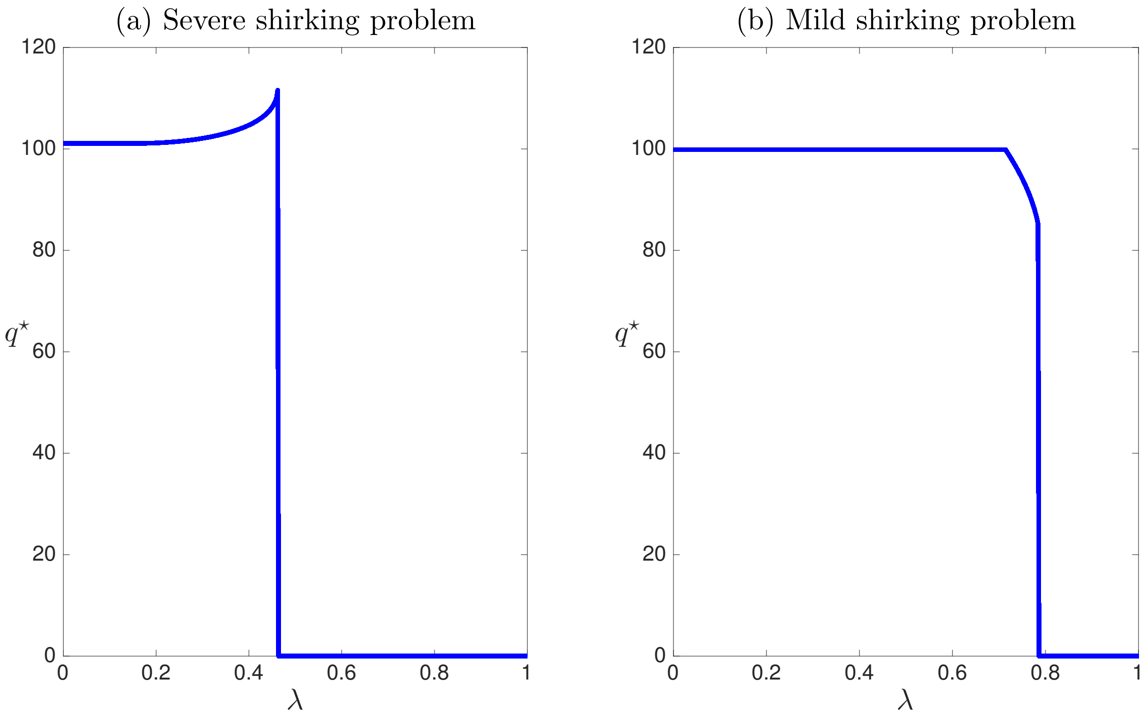

Figure 2 **Impact of the diversion problem's severity λ on optimal capacity q^*, when the shirking problem is severe ($\Delta\kappa = 38$) or mild ($\Delta\kappa = 22$).**
($r = 2.1$, $c = 1$, $D_1 \sim Gamma(100/4, 4)$ and $D_0 \sim Gamma(75/4, 4)$.)

Shirking problem's impact

A similar reversal result holds for the shirking problem's impact on optimal capacity.

Corollary 4 *Assume the project is undertaken ($q^* > 0$) at a non-first-best capacity ($q^* \neq q^{FB}$) and consider an increase in $\Delta\kappa$, the shirking problem's severity. (i) If the diversion problem is mild enough ($\lambda < \hat{\lambda}$), capacity increases, but (ii) if it is severe enough ($\lambda > \hat{\lambda}$), the effect is reversed: capacity decreases.*

To understand point (i), consider first the shirking problem *alone*. In that case, the firm over-invests and the more severe the shirking problem, the *more* the firm invests (de Véricourt and Gromb 2018). Indeed, higher capacity allows for financial contracts with more powerful incentive to exert effort. Since increasing capacity is inefficient given over-investment, the firm will increase capacity as little as needed to deter shirking. As shirking

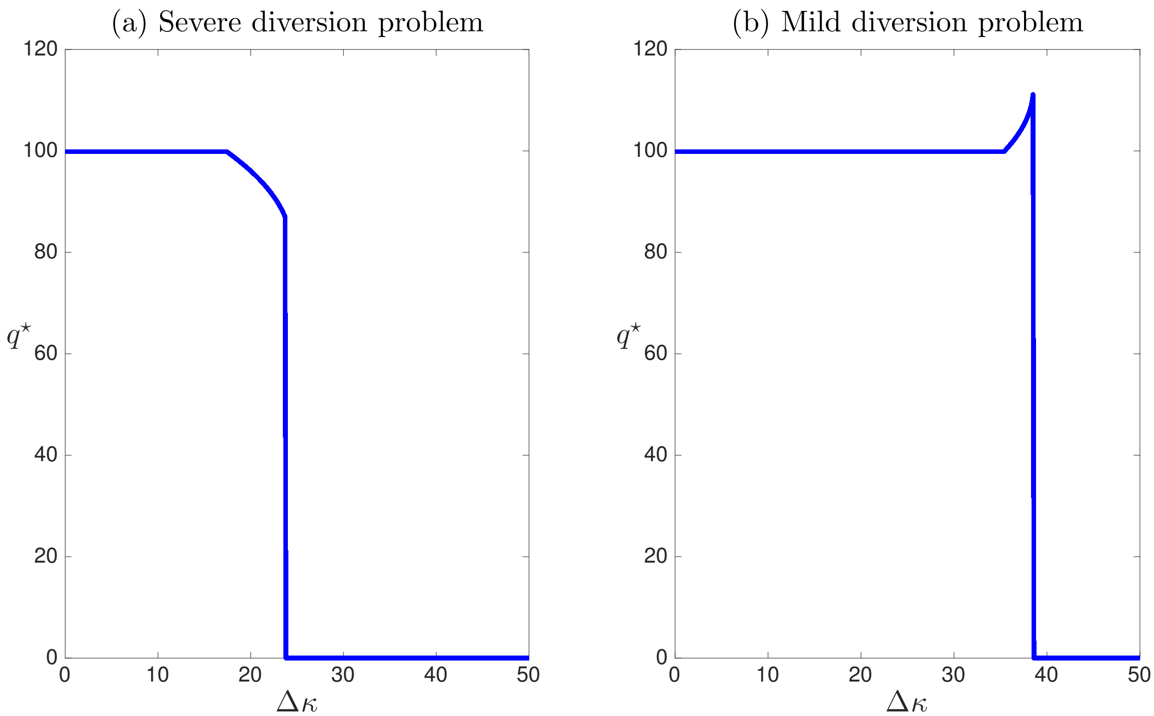

Figure 3 Impact of the shirking problem's severity $\Delta\kappa$ on optimal capacity q^*, when the diversion problem is severe ($\lambda = 0.76$) or mild ($\lambda = 0.44$).
($r = 2.1$, $c = 1$, $D_1 \sim Gamma(100/4, 4)$ and $D_0 \sim Gamma(75/4, 4)$.)

becomes more severe ($\Delta\kappa$ increases), the firm must raise capacity further. Point (i) extends this result to cases where the diversion problem is not necessarily absent but mild enough. Again, the intuition invokes complementarity, not merely continuity.

Given this, point (ii) is counter-intuititive and illustrates that the moral hazards do not simply add up but interact. The effect of shirking is reversed: the more severe the shirking problem, the *less* the firm invests. (This is in contrast to de Véricourt and Gromb (2018).)

The intuition for points (i) and (ii) relies again on complementarity. Say the mix of moral hazards be neither too mild not too severe so the project is undertaken on a non-first-best scale. In response to an increase in the severity of the diversion problem, the firm must either raise or cut capacity. When the diversion problem is mild enough, increasing capacity is optimal because the corresponding increase in the gains from diversion given shirking (b) is small. When the diversion problem is severe enough, decreasing capacity is optimal because the negative impact on these gains (b) is large.

Figure 3 depicts the impact of the shirking problem's severity $\Delta\kappa$ on optimal capacity $q^\star$ holding λ constant. It illustrates the reversal of the shirking problem's impact: optimal capacity $q^\star$ can decrease strictly with $\Delta\kappa$ if the diversion problem is severe enough (Figure 3(a)), but increases with λ otherwise (Figure 3(b)).

8. Discussions

We discuss instances where governance issues map into more or less severe shirking and diversion problems as in our model, and list our main managerial implications.

Shirking problem

The shirking problem stems from unmet demand and effort's non-contractibility.

Unmet demand tends to be non-observable and thus not directly contractible. Instances where reliable indirect measures (e.g., waitlines, consumer feedback, peer firm sales) exist map into mild shirking problems in our model ($\Delta\kappa$ small). Yet unmet demand estimates are often noisy or costly, which maps into more severe shirking problems ($\Delta\kappa$ large). For example, for products with few close substitutes, unmet demand cannot be inferred from close product sales. Similarly, estimating unmet demand may be more challenging for firms with mostly private peers, which are subject to laxer disclosure requirements.

Our model's effort corresponds to hard-to-assess actions affecting demand. Demand may be impacted by sales drives and marketing campaigns but also a product's design, quality and other items affecting its market appeal. Whether these are easily contractible varies. For instance, a product's market fit or an ad campaign's effectiveness may be easier to assess in a well known market or for an existing product than in new ones.

Diversion problem

Diversion, which affects the firm's ability to meet demand, has several interpretations.

First, diversion can stand for misrepresenting assets. Instances in which due diligence is reliable and cheap map into cases of mild diversion problems in our model (λ small). Yet due diligence is often imperfect and costly, which would map into more severe diversion problems (λ large). For example, financiers' expertise may be limited, e.g., for new technologies or assets abroad.[18] Due diligence may also be less effective when transparency is low, e.g., for private firms as they are subject to laxer auditing requirements. Such issues can also arise when the assets are mobile, e.g., human capital or mobile physical assets.[19]

Second, diversion of (the funding of) a capacity unit can stand for the firm not taking an action raising the unit's output, and private benefits for the opposite of the action's cost. For example, productivity may rely on maintenance or well-trained staff. Similarly assets' use may be hard-to-contract, notably for flexible capacity, like human capital, that can be redeployed swiftly. Diversion also maps into not making adequate quality controls that affect product quality or fit and hence sales.

Managerial implications

Several implications arise from our findings.

First, our adopting an optimal financial contracting approach highlights a key point: if frictions are mild, the need for external funding does not justify deviating from the first-best level of investment in capacity. Thus before resorting to distorting operating decisions, managers should explore ajusting the mode of financing of these operations. This insight is very general but has been largely ignored in the managerial literature.

Second, our over- vs. underinvestment result (Theorem 2) has broad implications.

For a start, different conditions require over- or underinvesting relative to the unconstrained efficient level. Overinvestment may be in order when shirking-type problems are severe but diversion-type problems milder. For instance, a private firm entering a new market with a new product whose production involves mostly physical fixed assets may find it optimal to operate on a larger-than-efficient scale. However, underinvestment may

[18] In 2011-2012, fraud scandals wiped out billions of dollars in stock value for many US-listed Chinese firms. Irregularities (some avered, others not) included overstating of scale of operation, in some cases by as much as 40 times, and conflicts of interest in related party transactions. This also pointed to a failure of due diligence and auditing, with U.S. auditors paying hundreds of millions of dollars to settle lawsuits.

[19] Referring to human capital, Brealey and Myers (2000) warn against "paying too much for assets that go down in the elevator and out to the parking lot at the close of each business day. They may drive into the sunset and never return." Bankruptcy law recognizes that the concern extends to mobile physical capital by allowing creditors secured by airplanes or ships to seize them faster than other assets.

be optimal in the opposite configuration, as may hold for a firm in a well travelled activity but operating in a location remote from its financiers.

Moreover, a worsening of the *same* governance problem can lead to over- or underinvestment depending on circumstances. For instance, starting from a situation where capacity is optimally set at its first best level, a new shirking-type problem can lead the firm to over-invest if diversion-type problems are mild but to underinvest if they are more severe. This may be the case for two firms employing mostly fixed physical and human capital respectively. In well known markets, both may operate at their efficient scale, but if entering a new market, the former might overinvest while the latter underinvests.

Third, our results shed light on how investment decisions should be tilted relative to a benchmark (Theorem 3), which is particularly relevant when the optimal capacity level is difficult to establish precisely. For example, compare two firms producing well established ($\Delta\kappa$ small) and new ($\Delta\kappa$ large) goods respectively. A decline in legal investor protection (the higher λ) might lead the former to reduce its investment and the latter to increase it.

These findings also have empirical implications. First, the previous effects provide possible hypotheses for the empirical study of corporate governance on capacity investments. More generally, our results caution against considering different dimensions of governance in isolation. For instance, the staggered adoption of business combination laws in different U.S states has been extensively used to identify effects of corporate governance (Bertrand and Mullainathan 2003). To the extent that these laws reduced the threat of hostile takeovers, they reduced the expected "punishment" associated with diversion-type activities (higher λ). However, our analysis warns that the effect of this regulatory shock may be modulated by other governance problems, and identification strategies ignoring this issue may generate biased estimates.

9. Conclusion

This paper studies capacity investment under demand uncertainty when external financing creates governance problems: the firm may divert capital, which reduces capacity, and shirk on market-development effort, which deflates demand. We adopt an optimal contracting approach whereby the firm optimizes among financial claims derived endogenously. This ensures that financing's effect on operations is considered only once contractual solutions are exhausted. This is important because operational changes away from first-best are costly, while contracting is not. We find that debt financing is optimal, that the firm under-invests or overinvests depending on whether the diversion or the effort problem dominates, and that the problem's intensity can reverse the direction of the other's impact on capacity. All these findings stem from the complementarity between shirking and diversion, which arises in the problem of matching supply and uncertain demand.

Our work can be extended in several directions. First, as discussed in Section 8, capacity may not be contractible because the firm may allocate capacity towards other, unintended uses. This raises the issue of priority rules and an extension of our model could address the question of capacity financing when priority rules are non-contractible. Second, diversion can also amount to the firm not taking actions (e.g., maintenance, quality control) raising the capacity's yield. Another extension could thus account for random capacity yield problems (Yano and Lee 1995, Okyay et al. 2014).

Last, financial contracts cannot always eliminate capacity distortions. The firm may thus adopt governance measures on the asset side, e.g., hire auditors, favor tangible assets, or the liabilities side, e.g., favor relationship banking over arm's length financing, etc. Here, the firm could also commit to better measuring unmet demand to improve contracting.

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