Financing Capacity with Stealing and Shirking

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We study the problem of a firm making a capacity investment choice under demand uncertainty, which it must finance externally by issuing a financial claim. However, having to share profits with external investors creates governance problems: the firm may divert capital, which reduces the capacity effectively set up, or shirk on market-development effort, which reduces demand. We adopt an optimal financial contracting approach whereby the firm optimizes over a set of feasible financial claims that is derived endogenously. We fully characterize the optimal financial claim and capacity. First, we find that debt financing is optimal, i.e., minimizes the firm's incentive to divert capital or shirk. Second, we find that the diversion and effort problems interact to cause the firm to under- or over-invest. If the diversion problem is severe enough, the firm may need to under-invest in capacity, in which case it under-invests more as the effort problem worsens. Otherwise, the firm over-invests, and more so as the effort problem worsens. In other words, the diversion problem's severity reverses the effort's problem effect on capacity investment.

Key words: Capacity Investment, Optimal Contracts, Capital Diversion, Financial Constraints, Newsvendor Model, Moral Hazard.

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1. Introduction

In new and fast-changing markets, businesses invest in capacity under considerable demand uncertainty, which raises both operational and financial challenges. Firms must not only manage risk, which they typically do by adjusting their scale of operation and taking demand-bolstering actions. They may also need to secure external financing as the funding needs capacity investments create can easily outstrip internal resources.

External financing can be especially difficult as investors must believe their funds will be put to good use and returned to them in fine. Indeed, governance issues arise which financial contracting can mitigate but rarely eliminate: "How do the suppliers of finance get the managers to return some of the profits to them? How do they make sure that managers do not steal the capital they supply or invest in bad projects?" (Shleifer and Vishny 1997).

This paper studies how governance problems such as these affect corporate financing and operating decisions, notably capacity investment under demand uncertainty. Specifically, a firm may "steal", i.e. divert the capital investors supply away from the intended capacity investment. Further, having to share profits with investors, the firm may "shirk", i.e. underdeliver on demand-bolstering actions, thus deflating demand and lowering the return on capacity investment. We study the extent to which financial contracting can mitigate these governance issues, and whether the residual "stealing" and "shirking" problems imply that capacity choices are distorted towards under- or over-investment.

We frame the question as an optimal contracting problem, in which a newsvendor jointly optimizes capacity and the financial claim issued to fund it, given two governance problems: he can divert funds and shirk on demand-boosting effort. In this set-up, we fully characterize the optimal financial contract and capacity choice.

This analysis yields new insights on the problem of financing capacity when both capacity and effort are not contractible. First, debt financing is optimal, i.e., minimizes the incentive to divert funds or shirk. Second, we find the diversion and effort problems interact to cause the firm to either under- or over-invest. If the diversion problem is severe enough, the firm under-invests in capacity, and under-invests more as the effort problem worsens. Else, the firm over-invests, and more so as the effort problem worsens. In other words, the diversion problem's intensity *reverses* the effect of the effort problem on capacity investment. Conversely, the effort problem's severity can reverse the sign of the diversion problem's impact on the optimal capacity choice. In particular, the firm overinvests when effort is contractible but capacity is not, but can underinvest when both are non-contractible.

More specifically, we study a newsvendor model in which the firm's sole owner ("the firm") has limited internal resources ("cash") and seeks funds from a competitive investor. Both enter a financial contract whereby the investor transfers a set amount of funds to the firm against a financial claim, i.e., profit-contingent repayments. The contract also sets other items provided they are contractible.

The firm must then make two choices. First, it can divert funds away from capacity investment and towards private benefits, i.e., excluding the investor.[1] Thus it chooses the effective capacity level set up to serve demand, which can be below the declared capacity level intended in the contract. The remaining funds are diverted to the firm's sole benefit. We assume diversion to be inefficient, i.e., \$1 diverted yields less than \$1 in private benefit. This may reflect dissimulation costs or expected penalities (Shleifer and Wolfenzon 2002). The degree of inefficiency of diversion is a key parameter in our analysis.[2]

Second, the firm must decide how much market development effort to engage in. This may mean tuning product design, launching a marketing or sales campaign, etc. which we model as follows. By exerting a costly effort, the firm can improve the distribution of demand in the sense of the Monotone Likelihood Ratio Property (MLRP).[3] We assume effort to be efficient, i.e., to generate an increase in expected sales that exceeds the cost of effort. The cost of effort is another key parameter in our analysis.

If capacity (and hence diversion) and effort were contractible, i.e., if the firm and investor could set their levels credibly in the contract, the financial claim would be irrelevant

[1] We focus on the interpretation that the firm can abscond with the funds only for brevity. However, other exist as discussed later. For instance, the firm cannot divert funds but can divert capacity towards other private projects. Alternatively, it can make capacity units productive at a private cost.

[2] It can be activity-specific (e.g., the ease with which investors can monitor the firm's capacity investments which may vary with the nature of its assets, the firm's governance mechanisms, etc.) or country-specific and related to, e.g., legal investor protection, enforcement, takeover laws, etc.

[3] In essence, under MLRP, higher demand realizations are more indicative of effort.

(Modigliani-Miller Theorem) and external funding needs would not affect operations as the contract would set capacity and effort at their first-best levels.

Instead, we assume capacity and effort to be non-contractible, which creates two dimensions of moral hazard in the investor-firm relationship. In this case, the need to share profits with the investor might induce the firm to divert funds or shirk on demand-bolstering effort, which reduces profit at the investor's expense. However, before concluding it does, we must consider the extent to which these governance problems can be mitigated by the financial contract the firm and the investor enter.

Thus we follow an optimal contracting approach, whereby the firm optimizes over a set of feasible financial contracts derived endogenously from assumptions about preferences, technology, and information/contractibility. We make standard assumptions implying that financial claims respect limited liability and are monotonic, i.e., repayments do not exceed and increase weakly with profits (Tirole 2006). We also state explicitly which variables are non-contractible, and allow contracts to specify all others. The feasible set so-defined includes debt, equity, convertible debt, warrants, etc. For instance, in a debt claim, repayments equal realized profits if below the debt's face value, and the debt's face value otherwise. In an equity stake, repayments equal a fixed fraction of profits. Both repayment functions are non-decreasing and satisfy limited liability.

We first establish that the firm will optimally use internal cash first and if this proves insufficient, finance the shortfall by issuing a debt claim to the investor. This result constitutes a key methodological step in our analysis: optimal financial contracting ensures that our findings about operations are robust to a simple contract change.

We then show that, given this optimal financing policy, the firm will optimally distort the effective capacity level towards either under- or over-investment relative to the first-best level. We fully characterize the conditions under which it does one or the other.

Specifically, when the diversion's problem is nild enough, the firm sets capacity at or strictly above the first-best level, i.e., there is over-investment. Moreover, in this case, the degree of over-investment increases (i.e., capacity increases further) with the effort problem's severity, i.e., as the cost of effort increases. If the effort problem is too severe, the firm cannot finance capacity and abandons the project altogether.

By contrast, when the diversion problem is severe enough, the firm sets capacity at or strictly below the first-best level, i.e., the distortion is towards under-investment. Further, the degree of under-investment increases (i.e., capacity decreases further) with the effort problem's severity. If the effort problem is too severe, the firm abandons the project.

The paper relates to the vast finance literature on corporate governance problems and their implications for corporate financing and investment (see Shleifer and Vishny (1997) and Becht et al. (2007)'s surveys). That corporate control can give rise to private benefits excluding outside investors has been widely documented empirically (Barclay and Holderness 1989, Nenova 2003, Dyck and Zingales 2004) as has the diversion of corporate resources for private benefits (Bertrand et al. 2002). Diversion appears in corporate governance theory (Burkart et al. 1998), where its inefficiency arises from dissimulation costs or expected punishment (Shleifer and Wolfenzon 2002). Our paper connects the corporate governance and operations literatures and derives original results specific to the newsvendor model, a framework the finance literature has not considered. In particular, this literature has not studied how diversion may interact with financial frictions stemming from the non-contractibility of unmet demands.

Our paper also belongs to the nascent literature linking capacity choices and financial decisions, which has derived implications for how a firm's funding needs affect its capacity (e.g., Alan and Gaur 2015, Li et al. 2013) or technology choices (e.g., Boyabatli and Toktay 2011) and for how these in turn impact the firm's financial policy. Our paper contributes to this literature by studying the impact of diversion and demand-bolstering effort on the choice and financing of capacity. It also makes a methodological contribution to the literature by adopting an optimal contracting approach.

In this approach, agents optimize over a feasible set of financial contracts derived endogenously from assumptions about fundamentals (i.e., preferences, technology, and information/contractibility). We explore the conditions for deviations from efficient outcomes, once contractual solutions are exhausted. To our knowledge, the only operations-and-finance paper this approach is de Véricourt and Gromb (2016)'s study of a similar model but with the effort problem only which shows that over-investment can be optimal. Instead, our paper studies the diversion problem and its interaction with the effort problem, and implications for the direction of optimal capacity distorsions.

Jensen and Meckling (1976) first showed the superiority of debt over equity financing in an “effort problem” model. Innes (1990) extended their analysis to an optimal contracting framework, showing that debt is optimal among all financial claims satisfying the same feasibility conditions as in our model, an analysis de Véricourt and Gromb (2016) extend and specialize to a newsvendor model. Our paper characterizes an optimal contract with two moral hazard problems, where one problem (diversion) scales with the capacity level. The result that debt financing is optimal is thus no simple application of Innes (1990).

Last, our paper builds on the principal-agent literature with risk-neutrality and limited liability (Oyer 2000, Gromb and Martimort 2007, Poblete and Spulber 2012) and more specifically that on agency in capacity choice models. In particular, Dai and Jerath (2013, 2016) and Chu and Lai (2013) study capacity choice models in which a sales agent must be induced by her wage contract to take a demand-enhancing action. All three show that capacity above the first best may be optimal. The salesforce compensation problem they study is quite different from our financing problem. In particular, wage contracts are not constrained by the requirements of financial contracts (notably the monotonicity condition). Moreover, moral hazard makes their sales agent better off whereas it makes our firm worse off. Last, fund diversion is absent from these salesforce studies.

The paper proceeds as follows. Section 2 presents the model. Section 3 lays out the firm’s problem. Section 4 establishes the optimality of debt financing. Section 5 studies the optimal capacity level. Section 6 studies the impact of governance problems on capacity investment. Section 7 concludes. The Appendix contains all proofs.

2. The Model

We study a firm making a capacity investment choice under demand uncertainty, which it must fund externally. It can manage risk by adjusting capacity and by boosting demand via market-development actions. However, having to share profits with investors creates governance problems: the firm may divert capital, which reduces the capacity effectively set up, or shirk on market-development effort, which reduces demand.

We frame the situation in a newsvendor model with risk-neutrality and no discounting. A firm with a sole owner ("the firm") has a project with stochastic demand D_1 with strictly positive distribution $f_1(\cdot)$, cumulative distribution $F_1(\cdot)$, complementary cumulative distribution $\bar{F_1}(\cdot) \equiv 1 - F_1(\cdot)$, and hazard rate $h_1(\cdot) \equiv f_1(\cdot)/\bar{F_1}(\cdot)$.[4] To satisfy this demand, the firm needs to install capacity at unit cost $c > 0$, before demand is realized, which eventually yields revenue $r > c$ per sold unit and salvage value $s < c$ per unsold unit.

To fund capacity, the firm can use internal resources, i.e., cash $W \geq 0$, and if needed, raise additional funds $I \geq 0$ from a competitive investor via a financial contract. In that case, the contract specifies, among other items, the capacity $q \in \mathbb{R}_+$ that the firm agrees to set up, to which we refer as *declared capacity*. If the firm sets $q = 0$, it abandons the project and no other decision is taken. Otherwise, if $q > 0$, the firm receives I from the investor and at that point, it must take two decisions: a diversion choice and an effort choice.

Diversion. First, the firm can opt to divert part of the funds cq meant to finance declared capacity q as per the contract. That is, the firm can choose to only invest in *effective capacity* $x \in [0, q]$ at cost cx, so that at most demand for x units can be satisfied. This potential violation of the contract is possible as long as variable x is non-contractible, i.e., under no circumstances can a court verify that $x \neq q$. The firm diverts the rest of the funds, $c(q - x)$, which yields benefits $(s + \lambda c)(q - x)$ with $\lambda \in [0, 1]$. Of these, the firm can secure $\lambda c(q - x)$ as a private benefit excluding other parties. It cannot secure the remaining benefits, $s(q - x)$, as this would render x verifiable.[5] We assume diversion to be inefficient, i.e., a unit's cost exceeds its diversion's payoff, i.e., $c > \lambda c + s$ or

$$\lambda < \bar{\lambda} \equiv \frac{c - s}{c} \tag{1}$$

Diversion appears in corporate governance models (e.g., Burkart et al. 1998). It captures instances where the firm can abscond with funds or assets. Inefficiency arises from

[4] The risk-neutrality assumption is standard and important for the result that debt financing is optimal. No-discounting is only for simplicity, as is the strict positivity of f_1.

[5] If the firm secured private benefits above $\lambda c(q - x)$, it would have to repay the investor from diverted funds strictly below $s(q - x)$ and sales revenues as low as sx, in which case the firm would only be able to make a repayment strictly below sq which would prove that $x < q$.

dissimulation costs or expected punishment (e.g., Shleifer and Wolfenzon 2002). It can also reflect less extreme cases of assets being used for pet projects, to favor related parties, etc. ensuring firm insiders exclusive gains not commensurate with the implied profit reduction. Diverting a unit can also be seen as the owner not taking a costly action needed to make it operational, e.g., adequate quality control for the unit to function properly. In this set-up, λc is the cost of taking that action (see Section 7 for further discussion).

Effort. Second, if the firm undertakes the project (i.e., $q > 0$), it chooses the level of its market development effort: high effort ($e = 1$) causes the firm a non-monetary cost $\kappa_1 > 0$ and low effort ($e = 0$) a smaller one $\kappa_0 > 0$, i.e., $\Delta\kappa \equiv \kappa_1 - \kappa_0 \geq 0$.[6] We assume no project adjusment to effort e and effective capacity x (i.e., unit price, cost and salvage value remain r, c and s) but that low effort affects demand.[7] Specifically, demand shifts to D_0 with a distribution f_0 less favorable than f_1 in the sense of the Monotone Likelihood Ratio Property (MLRP), i.e., f_1/f_0 is strictly increasing over $\mathbb{R}_+$.

Effort captures the firm's market development activities: running a marketing campaign, sales actions, tuning product design, etc. Its cost can be a labor cost for the owner, the opportunity cost of shifting attention away from other projects, etc.

For declared and effective capacities q and x, and effort e, the project's monetary payoff is randomly distributed over $[sq, rx]$ as per density $g_{e,x}$ (implied by f_e) and equal to

$$P_{e,x,q} \equiv sq + (r - s)(D_e \wedge x) \tag{2}$$

and the firm also receives private benefit $\lambda c(q - x)$. The firm's expected profit is then

$$\mathbb{E}[P_{e,x,q}] + \lambda c(q - x) - cq. \tag{3}$$

When the firm does not divert capacity ($x = q$), expected profit is as in the standard newsvendor model: $\pi_e(q) \equiv \mathbb{E}[P_{e,q,q}] - cq$.

[6] The assumption that no cost is incurred if $q = 0$ is for simplicity. It amounts to assuming that κ_0 is a fixed cost which will allow us to focus on cases where the firm exerts $e = 1$ or sets $q = 0$ (see Proposition 2).

[7] In fact, while the project could be adjusted to e or x (e.g., with cheaper capacity, lower prices, etc.) which might affect demand, we will show that this would contradict the non-contractibility assumption of$e$ and x.

First-Best. We now characterize the first-best optimum to serve as a benchmark. Ignore diversion, i.e., set $x=q$, and effort costs κ_0 and κ_1. In that case, for a given effort level e, the problem is a standard newsvendor model with demand c.d.f. F_e and, as per standard arguments, the optimal capacity, $\arg\max_{q\in\mathbb{R}_+}\pi_e(q)$, equals $F_e^{-1}\left(\frac{r-c}{r-s}\right)$. Now consider effort costs but not diversion. The cost being fixed, the optimal capacity for effort e remains $F_e^{-1}\left(\frac{r-c}{r-s}\right)$, provided the firm undertakes the project, i.e., sets $q>0$. We assume that in that case, the project is viable if $e=1$ but best abandoned ($q=0$) if $e=0$, i.e.,

$$\max_{q\in\mathbb{R}_+}\pi_1(q)-\kappa_1>0>\max_{q\in\mathbb{R}_+}\pi_0(q)-\kappa_0 \tag{4}$$

Note that condition (4) implies $\underline{q}_1<F_1^{-1}\left(\frac{r-c}{r-s}\right)<\bar{q}_1$ with

$$\bar{q}_1\equiv\max\left\{q\in\mathbb{R}^+ \text{ s.t. } \pi_1(q)\geq\kappa_1\right\} \text{ and } \underline{q}_1\equiv\min\left\{q\in\mathbb{R}^+ \text{ s.t. } \pi_1(q)\geq\kappa_1\right\}. \tag{5}$$

We can now characterize the first-best optimum.

Proposition 1 *The first-best outcome is for the firm to exert effort ($e^{FB}=1$), and set and effectively install the optimal standard newsvendor capacity, i.e., $x^{FB}=q^{FB}=F_1^{-1}\left(\frac{r-c}{r-s}\right)$.*

Indeed, diversion being inefficient (condition (1)), diverting capacity is never first-best optimal. Given this, the first-best outcome is as if diversion were impossible. And given condition (4), this means exerting effort and setting up the standard newsvendor's capacity.

Financial contracts. The firm enters a financial contract with the investor whereby the latter transfers funds $I\geq 0$ to the firm against a financial claim stipulating state-contingent repayments. The contract can also specify any other item provided it is contractible.

In the optimal contracting approach, feasible contracts stem endogenously from assumptions about fundamentals, i.e., preferences, technology and information/contractibility. As a matter of presentation practicality however, we first define the set of feasible contracts and only then lay out the assumptions about fundamentals from which it is derived.

Definition 1 *A financial contract is feasible if it specifies investment I, declared capacity q and a feasible financial claim. A financial claim is feasible if repayments (i) are contingent on profit (only), i.e., $\forall p\in[sq,rq]$, $R(p)$, (ii) satisfy the firm's limited liability, i.e., $\forall p\in[sq,rq]$, $R(p)\leq p$, and (iii) are monotonic, i.e., $R(\cdot)$ is non-decreasing over $[sq,rq]$.*

Denote $\mathcal{C}$ the set of feasible claims, which we represent as repayment functions $R(\cdot)$. Note that a feasible repayment function $R(\cdot) \in \mathcal{C}$ can take negative values, i.e., it may call the investor to pay the firm for some profit realizations. Debt and equity are feasible: a loan with face value K maps into $R(p) = p \wedge K$ and a fraction α of equity into $R(p) = \alpha p$, both of which satisfy Definition 1. Other claims (e.g., convertible debt, call warrants, etc.) are also feasible, as are some combinations of claims (e.g., debt plus levered equity, etc.).

We now turn to the assumptions about fundamentals that underly Definition 1. Parameters are assumed common knowledge so contracting on them is not needed. This standard assumption is plausible, e.g., costs and prices can usually be retrieved from accounts. The model's variables are: investment I, declared and effective capacities q and x, effort e and the realization of demand D_e. Specifying which are contractible or not is critical.

We assume that investment I and declared capacity q are contractible, but that effort e and diversion $(q - x)$ are not. This creates two moral hazard problems. Regarding demand D_e, we assume that (only) unmet demand is non-contractible. Given the mapping from demand to profits $P_{e,x,q}$ is one-to-one below installed capacity x and flat above x, this amounts to assuming that (only) profits are contractible. This is plausible as profits can be audited while unmet demands are usually hard to observe.[8]

Finally, both limited liability and monotonicity are standard assumptions in corporate finance. Limited liability may arise from the firm's need for a minimum subsistence level of wealth. Monotonicity is required when the firm can report artificially inflated revenues (e.g., by borrowing secretly from third parties). Indeed, non-monotonic claims would invite the firm to do so. Similarly, non-monotonic financial contracts may incentivize the investor to engage in sabotage and reduce the payoff. (See Innes 1990, Tirole 2006)

3. The Firm's Problem

The firm's problem is to choose declared capacity $q \in \mathbb{R}_+$, funds $I \in \mathbb{R}_+$ to be raised from the investor and feasible financial claim $R(\cdot) \in \mathcal{C}$ to maximize its expected payoff, i.e.,

$$\max_{(q,I,R) \in \mathbb{R}^+ \times \mathbb{R}^+ \times \mathcal{C}} \bar{r}_e(q,x) + W + I - cq - \kappa_e \tag{6}$$

[8] Note that in our model, if revenues were non-contractible, financial claims would not be feasible at all.

where $\bar{r}_e(q,x) \equiv \mathbb{E}\left[P_{e,x,q} - R(P_{e,x,q})\right] + \lambda c(q-x)$ denotes the firm's expected payoff from all capacity units when x units are effectively installed and $(q-x)$ are diverted.

This choice is made under constraints. First, the investor must at least break even, i.e.,

$$\mathbb{E}\left[R\left(P_{e,x,q}\right)\right] \geq I \tag{7}$$

The firm too should accept the contract, hence a participation constraint written as

$$\text{if } q > 0, \qquad \bar{r}_e(q,x) + W + I - cq - \kappa_e \geq W \tag{8}$$

Second, the firm's total budget should cover the cost of capacity q, i.e.,

$$W + I \geq cq \tag{9}$$

Last, the firm must prefer effort e and effective capacity x to any alternative effort ϵ and effective capacity y, i.e., if $q > 0$, for all $y \in [0,q]$ and $\epsilon \in \{0,1\}$,

$$\bar{r}_e(q,x) + W + I - cq - \kappa_e \geq \bar{r}_\epsilon(q,y) + W + I - cq - \kappa_\epsilon \tag{10}$$

The following result states that when the project can be funded, an optimal contract exists that deters the firm from diverting and shirking. This also narrows down the contract set over which the firm optimizes and thus simplifies the problem.

Proposition 2 *Any contract is (weakly) dominated by a contract* (q,I,R) *in which the firm chooses optimally effort* e *and effective capacity* x *such that:*

(i) The investor breaks even, i.e., $\mathbb{E}\left[R\left(P_{e,x,q}\right)\right] = I$

(ii) Effort is high or the project is abandoned, i.e., if $q > 0$ *then* $e = 1$

(iii) There is no diversion, i.e., $x = q$

Point (i) reflects the assumption that the investor is competitive. Point (ii) stems from the assumption that the project is not viable with low effort. Point (iii) derives from the assumed inefficiency of diversion. The intuition is that if for contract (q,I,R) the firm installs effective capacity $x < q$, it would install the same effective capacity for contract (x,I,R), which avoids the inefficient diversion induced by (q,I,R).

Given this, we denote contracts simply as (q, R), and the problem can be written as:

$$\max_{(q,R)\in\mathbb{R}_+^*\times\mathcal{C}} \pi_1(q) \tag{11}$$

$$\underline{q}_1 \leq q \leq \bar{q}_1 \tag{12}$$

$$\mathbb{E}[R(P_{1,q,q})] \geq (cq - W)^+ \tag{13}$$

$$\forall y \in [0, q], \forall \epsilon \in \{0, 1\}, \quad \bar{r}_1(q, q) - \kappa_1 \geq \bar{r}_\epsilon(q, y) - \kappa_\epsilon \tag{14}$$

and if the previous problem is not feasible then the firm abandons the project $(q = 0)$.

4. The Optimality of Debt Financing

We now establish that debt claims are optimal. A debt claim D is defined by its face value $K^D \in \mathbb{R}_+$ such that the repayment conditional on revenue $p \in \mathbb{R}^+$ is $D(p) = p \wedge K^D$. We denote $\mathcal{D}$ the set of debt claims. Note that $\mathcal{D}$ is a strict subset of $\mathcal{C}$.

Theorem 1 *If problem (11) has a solution, an optimal contract (q, D) exists, where $D \in \mathcal{D}$ is the unique debt claim with face value K such that $E[D(P_{1,q,q})] = (cq - W)^+$, i.e.,*

$$K - (r - s)\int_0^{\hat{d}(q)} F_1(u)\, du = (cq - W)^+ \tag{15}$$

where $\hat{d}(q) \equiv (K - sq)^+ / (r - s)$ is the demand level below which the firm defaults.

The theorem establishes that the firm finds it (weakly) optimal to raise no more than the funds it needs, i.e., $(cq - W)^+$, and to do so by borrowing. It extends Innes (1990)'s debt optimality result obtained in an "effort problem" model, which de Véricourt and Gromb (2016) extend and specialize to a newsvendor model. Our result extends the analysis to two moral hazard problems. Notably, while the effort problem is independent of the investment's scale, the diversion problem scales with the investment. The result relies on a form of complementarity between the diversion and effort problems: as diversion increases, effective capacity reduces, which makes reaching higher demand levels less useful and thus discourages effort. Our debt optimality result is thus no simple application of Innes (1990).

More importantly, the result is a key methodological step: the optimal financial contracting approach ensures that our results about operations are robust to simple contract changes. This contrasts with most the operations-and-finance literature.

The analysis implies that internal funds should be used in priority and if this is insufficient, debt financing, safe or risky, dominates all other external sources of funds.

Corollary 1 *If problem (11) has a solution, the optimal contract of Theorem 1 implies:*

- *If* $W \geq (c-s)\,q$*, the firm finances the project with a combination of cash and riskfree debt:* $K \in [0, sq]$ *and* $\hat{d}(q) = 0$*.*
- *If* $(c-s)\,q > W$*, the firm finances the project by using all of its cash* W *and issuing risky debt to fund the shortfall:* $K \in (sq, rq]$ *and* $\hat{d}(q) > 0$*.*

5. Optimal Capacity Investment

Having characterized the optimal financing for a given capacity, we now study the firm's optimal capacity choice accounting for the fact that the firm optimizes its financing too.

First assume the firm's cash covers enough of the investment cost of the first-best level of capacity that the short-fall, if any, is small enough to be financed with riskfree debt.

Proposition 3 *If* $W \geq (c-s)q_1^{FB}$*, the firm exerts high effort (*$e^* = 1$*) and effectively sets up the first-best capacity level (*$x^* = q^* = q_1^{FB}$*) which it finances with cash and riskfree debt.*

Indeed, when the firm can self-finance, it internalizes the project's value and makes first-best choices. With riskfree debt, the investor's payment being fixed, the firm bears the full consequences of its effort and diversion choices, Thus, it makes first-best choices.

We now consider the case where the firm's cash is so low as to make it impossible to finance the first-best level of capacity with cash and riskfree debt. The following result, our main result, characterizes the firm's optimal decisions in that case.

Theorem 2 *Assume* $W < (c-s)q_1^{FB}$*. Two thresholds* $\hat{\lambda} < \tilde{\lambda}$ *exist such that for a given* λ*,*

- *If* $\lambda \leq \tilde{\lambda}$*, two thresholds* $0 \leq \Delta\underline{\kappa} \leq \Delta\bar{\kappa}$ *exist such that* $\Delta\underline{\kappa} = \Delta\bar{\kappa}$ *if* $\lambda = \hat{\lambda}$ *and,*
  - *— If* $\Delta\kappa \leq \Delta\underline{\kappa}$*, the first-best investment obtains (*$q^* = q_1^{FB}$*);*
  - *— If* $\Delta\underline{\kappa} < \Delta\kappa < \Delta\bar{\kappa}$ *(and thus* $\lambda \neq \hat{\lambda}$*):*
    - $*$ *If* $\lambda < \hat{\lambda}$*, over-investment obtains (*$q^* > q_1^{FB}$*) and* q^* *increases with* $\Delta\kappa$*;*
    - $*$ *If* $\lambda > \hat{\lambda}$*, under-investment obtains (*$q^* < q_1^{FB}$*) and* q^* *decreases with* $\Delta\kappa$*;*
  - *— If* $\Delta\kappa \geq \Delta\bar{\kappa}$*, the project is abandoned (*$q^* = 0$*).*

- *If $\lambda > \tilde{\lambda}$, a threshold $\Delta\bar{\kappa} \geq 0$ exists such that:*
  - *— If $\Delta\kappa < \Delta\bar{\kappa}$, under-investment obtains ($q^* < q_1^{FB}$) and q^* decreases with $\Delta\kappa$;*
  - *— If $\Delta\kappa \geq \Delta\bar{\kappa}$, the project is abandoned ($q^* = 0$).*

Further, if the firm invests ($q^* > 0$), it does not shirk ($e^* = 1$) or divert ($x^* = q^*$).

The cost difference $\Delta\kappa$ captures the effort problem's severity: a higher $\Delta\kappa$ makes shirking more tempting. Similarly λ captures the diversion problem's severity: a higher λ means that diversion is less wasteful and therefore more tempting.

Consider first a mild enough diversion problem ($\lambda < \hat{\lambda}$). In this case, if the effort problem is also mild enough ($\Delta\kappa \leq \Delta\underline{\kappa}$), the first-best outcome obtains. At the other extreme, if the effort problem is too severe ($\Delta\kappa \geq \Delta\bar{\kappa}$), the project is abandoned ($q^* = 0$). The case of an effort problem of intermediate severity is richer ($\Delta\kappa \in (\Delta\underline{\kappa}, \Delta\bar{\kappa})$). There, the optimal capacity can strictly exceed the first-best level. Moreover, as the effort problem's severity increases, so does the optimal capacity, i.e., q^* increases with $\Delta\kappa$.

Consider now a more severe diversion problem ($\lambda \in (\hat{\lambda}, \tilde{\lambda})$). Again, the first-best obtains if the effort problem is mild enough ($\Delta\kappa \leq \Delta\underline{\kappa}$) and the project is abandoned if it is too severe ($\Delta\kappa \geq \Delta\bar{\kappa}$). However, the case of an effort problem of intermediate severity is different, reversed in fact. Indeed, the optimal capacity can be strictly below the first-best level. Moreover, the effect of the effort problem's severity is also reversed: as the effort problem worsens, the firm under-invests more, i.e., q^* decreases with $\Delta\kappa$.

Finally, if the diversion problem is too severe ($\lambda > \tilde{\lambda}$), the first-best outcome never arises, the firm always strictly under-invest, and under-invests more as the effort problem worsens.

The result is driven to a large extent by a form of complementarity between the diversion and effort problems. Indeed, as diversion increases, effective capacity decreases. As a result, is becomes more likely that effective capacity will not suffice to absorb demand, which has the effect of discouraging demand-boosting effort.

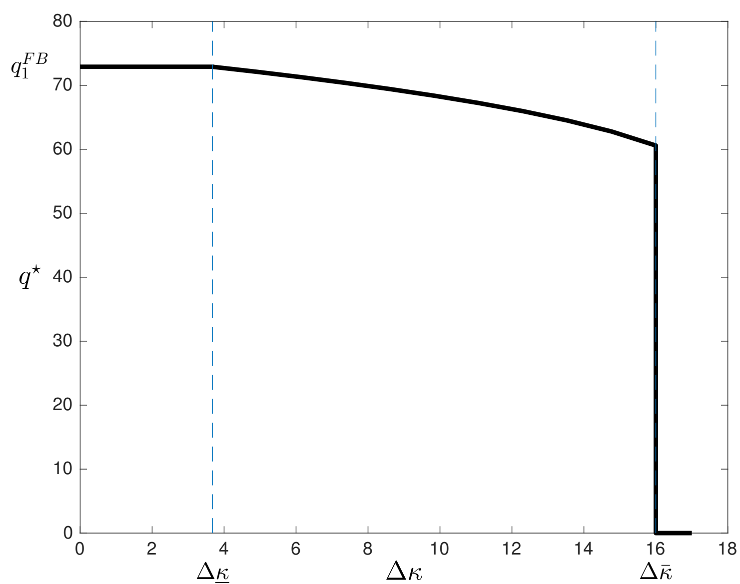
6. Diversion and Shirking's Impact on Capacity Investment

The previous result can be interpreted in terms of governance problems' severity ($\Delta\kappa$ for the effort problem and λ for the diversion problem), which may be related to the firm's activity and its institutional environment. For instance, fixed assets may be more easily monitored by investors (low λ) and thus harder to divert than intangible assets or human capital (high λ). Similarly, investors may be better able to assess the firm's operations in well traveled activities than in new business territories. Governance mechanisms (e.g., board composition, ownership structure, etc.) and their effectiveness also vary across firms. Further, countries vary in the legal protection of external investors, which has been shown to affect governance and outcomes (Shleifer and Vishny 1997).

The previous result reveals that both governance problems interact with each other and that this interaction can cause the firm to under- or over-invest.

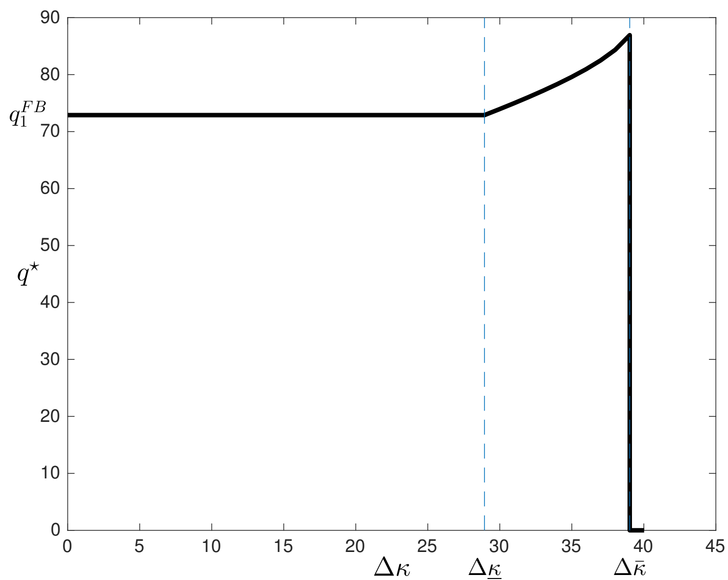
First, depending on the diversion problem's severity, the direction in which capacity is distorted can change: the firm over-invests if the diversion problem is mild, but under-invests otherwise. Moreover, the effort problem's impact on capacity does itself vary with the severity of the diversion problem.

Figure 1 **Optimal capacity** q^* **as a function of** $\Delta\kappa$ **when** $\lambda > \hat{\lambda}$**.** **(**$\lambda = 0.2, \hat{\lambda} = 0.157, W = 0$**,** $r = 10$**,** $c = 8$**,** $s = 0$**,** $D_1 \sim Gamma(10,10)$ **and** $D_0 \sim Gamma(8,10)$**.)**



Corollary 2 *Assume* $W < (c - s)q$*. If the firm undertakes the projects (*$q^* > 0$*),*

Figure 2 **Optimal Capacity q^* as a function of $\Delta\kappa$ when $\lambda < \hat{\lambda}$. ($\lambda = 0.1, \hat{\lambda} = 0.157, W = 0$, $r = 10$, $c = 8$, $s = 0$, $D_1 \sim Gamma(10, 10)$ and $D_0 \sim Gamma(8, 10)$.)**



- *The firm under-invests ($q \leq q_1^{FB}$) if the diversion problem is severe enough ($\lambda \geq \hat{\lambda}$) and over-invests ($q^* \geq q_1^{FB}$) otherwise.*
- *The effort problem's severity amplifies the capacity distorsion: it leads to greater under-investment if the firm under-invests and greater over-investment if it over-invests (q^* decreases with $\Delta\kappa$ if $q \leq q_1^{FB}$ and increases with $\Delta\kappa$ otherwise).*

Figures 1 and 2 illustrate this point. Both consider the same example in which $\hat{\lambda} = 0.157$ and $q_1^{FB} = 72.89$ and depict q^* as a function of $\Delta\kappa$. Figure 1 assumes $\lambda = 0.2$, in which case $\lambda > \hat{\lambda}$. When $\Delta\kappa$ is low ($\Delta\kappa \leq \Delta\underline{\kappa} = 3.68$), the optimal capacity is at first best. When $\Delta\underline{\kappa} < \Delta\kappa < \Delta\bar{\kappa}$ (where $\Delta\bar{\kappa} = 16$), the optimal capacity drops below the first-best. When $\Delta\kappa \geq \Delta\bar{\kappa}$, the project is abandoned. Figure 2 assumes $\lambda = 0.1$, in which case $\lambda < \hat{\lambda}$. Now when $\Delta\underline{\kappa} < \Delta\kappa < \Delta\bar{\kappa}$, the distortion in capacity is reversed and the optimal capacity increases above first best.[9]

[9] Note that for $\lambda = \hat{\lambda} = 0.157$, the corresponding figure - not reported here - depicts a straight horizontal line at $q^* = q_1^{FB}$ for $0 \leq \Delta\kappa < \Delta\bar{\kappa}$, which drops to $q^* = 0$, when $\Delta\kappa \geq \Delta\bar{\kappa}$.

Second and conversely, the effort problem's severity can also reverse the sign of the diversion problem's impact on capacity. To see this, assume the case $\Delta\kappa = 0$. This corresponds to effort being contractible: since the firm is indifferent between both effort levels, we can assume it chooses $e = 1$.[10] In that case, the firm always underinvests, and underinvests more as the capacity diversion problem becomes more severe , as stated below.

Corollary 3 *Assume $W < (c-s)q_1^{FB}$. If $\Delta\kappa = 0$ (i.e., effort is contractible), the firm under-invests ($q^* \le q_1^{FB}$) and capacity investment decreases with the severity of the diversion problem, i.e., q^* decreases with λ.*

By contrast, when effort is not contractible, the effect of the diversion problem on capacity investment can be reversed, as stated in the following result.

Proposition 4 *Assume $W < (c-s)q_1^{FB}$. If $\Delta\kappa > 0$ (i.e. effort is non-contractible), two thresholds $\Delta\underline{\kappa}_0 \le \Delta\bar{\kappa}_0$ exist such that for $\Delta\underline{\kappa}_0 < \Delta\kappa < \Delta\bar{\kappa}_0$, the firm over-invests ($q^* \ge q_1^{FB}$) and capacity investment increases with the severity of the diversion problem, i.e., q^* increases with λ.*

7. Concluding Remarks

This paper studies capacity investment under demand uncertainty when the firm may divert capital or shirk on market-development effort. We study how financial contracting can mitigate these issues, and whether the implied capacity choice is distorted towards under- or over-investment. We find that debt financing is optimal and that the diversion and effort problems interact to determine whether the firm under- or over-invests.

We adopt an optimal financial contracting approach whereby agents optimize over a feasible set of financial contracts derived endogenously from assumptions about preferences, technology, and information/contractibility. This ensures that possible effects of a firm's financing needs on its operational decisions are considered only once feasible contractual solutions are exhausted. This is important because operational changes away from first-best are costly, while contractual changes are not.

[10] We have assumed $\Delta\kappa > 0$ so to be perfectly consistent with our model, we should here consider $\Delta\kappa$ arbitrarily close to zero, which corresponds to effort being near-contractible.

We have emphasized one interpretation of the diversion problem, i.e., the firm can divert part of the funds, but other potentially fruitful interpretations are possible.

First, it may be that the level of capacity investment is contractible (and so the firm cannot abscond with the funds) but that capacity usage is not. For instance, the firm may be able to allocate capacity to other projects than that intended by the investor. In our model, these uses simply generate private benefits for the firm but this is one in a broader class of problems. In particular, this points to the issue of priority rules, a fundamental decision in operations. A research question an extension of our model could address is how to finance and control the operations of a firm whose priority rules are not contractible.

Second, it may be that diversion is not possible but that the firm must exert an effort to improve the capacity's yield. In this case, our model's private benefits of diversion map into (the opposite of) the effort cost, and our framework could be extended to study questions related to random capacity yield problems, another important theme in operations (Yano and Lee 1995, Okyay et al. 2014).

Our work identifies situations where financial contracts cannot eliminate capacity distortions. The firm may thus adopts governance measures be it on the asset side, e.g., hire reputable auditors, tilt investment towards tangible assets, or the liabilities side, e.g., favour relationship banking over arm's length financing, etc. (Shleifer and Vishny 1997, Becht et al. 2007). In our context, the firm could also commit to measuring unmet demand. In fact, making unmet demand contractible eliminates stealing and mitigates shirking.

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Appendix A: Proof of Proposition 1

For all $q \geq x$, objective (3)'s first-order derivative with respect to q is $s - c + \lambda c$, which is negative from condition (1) so that $x = q$ at an optimum. For $x = q$, expected profit $\pi_e(\cdot)$ is as in the standard newsvendor model and the optimal capacity is $F^{-1}((r-c)/(r-s))$. Condition (4) implies $e = 1$ is optimal. ■

Appendix B: Proof of Proposition 2

Point (i): Objective (6) is strictly increasing in I. The only upper bound on I is set by condition (7), which must thus be binding. This proves point (i).

Point (ii): The firm's profit is zero if the project is abandoned ($q = 0$) and must thus be non-negative at an optimum. To prove that $e = 0$ cannot be optimal, we show that if $e = 0$, the firm's profit is strictly negative. For $e = 0$, profit is, with $P_{0,x,q} = sq + (r-s)(D_0 \wedge x)$,

$$\mathbb{E}\left[P_{0,x,q} - R(P_{0,x,q})\right] + \lambda c(q-x) + I - cq - \kappa_0, \quad \text{which is equivalent to}$$

$$(\mathbb{E}\left[P_{0,x,q}\right] - \mathbb{E}\left[P_{0,x,x}\right]) + \mathbb{E}\left[P_{0,x,x}\right] - \mathbb{E}\left[R(P_{0,x,q})\right] + \lambda c(q-x) + I - (cq - cx) - cx - \kappa_0$$

or, noting further that $P_{0,x,q} = P_{0,x,x} + s(q-x)$,

$$(\mathbb{E}\left[P_{0,x,x}\right] - cx - \kappa_0) + (I - \mathbb{E}\left[R(P_{0,x,q})\right]) + (q-x)(\lambda c + s - c)$$

The first term is $\pi_0(x) - \kappa_0$ which is strictly negative from condition (4). The second term is negative due to investor participation constraint (7). The third term is negative condition (1). Hence profit is strictly negative, which implies that $q = 0$ dominates $q > 0$ and $e = 0$.

Point (iii): Suppose contract (q, I, R) satisfies the problem's constraints, (i.e. (7) to (10)). We show that if this contract implies diversion ($x < q$), it is strictly dominated by an alternative contract with no diversion. Consider alternative contract (x, I', R') with investment $I' = I - c(q-x)$ and claim $R'(\cdot)$ defined as for all p, $R'(p) = R(p + s(q-x)) - c(q-x)$. Note that for a given realization of demand D_e the repayment implied by R' is exactly $c(q-x)$ below that implied by $R(\cdot)$:

$$R'(P_{0,x,x}) = R(P_{0,x,x} + s(q-x)) - c(q-x) = R(P_{0,x,q}) - c(q-x)$$

First, we check contract (x, I', R') is feasible, i.e. claim $R'(\cdot)$ is feasible. $R'(\cdot)$ inherits $R(\cdot)$'s monotonicity and, since $P_{0,x,x} = P_{0,x,q} - s(q-x)$, it also inherits its satisfying limited liability:

$$R(P_{0,x,q}) \leq P_{0,x,q} \implies R(P_{0,x,q}) - c(q-x) \leq P_{0,x,q} - s(q-x) - (c-s)(q-x)$$

$$\implies R'(P_{0,x,x}) \leq P_{0,x,x} - (c-s)(q-x) \implies R'(P_{0,x,x}) < P_{0,x,x}$$

the last implication being due to our assumption that $c > s$.

We now check that contract (x, I', R') satisfies all of the problem's constraints, i.e., conditions (7) to (10), given that contract (q, I, R) does. Condition (7) holds as

$$\mathbb{E}[R(P_{e,x,q})] \geq I \implies \mathbb{E}[R(P_{e,x,x}) - c(q-x)] \geq I - c(q-x) \implies \mathbb{E}[R'(P_{e,x,x})] \geq I'$$

Condition (8) is satisfied because

$$\text{if } q > 0, \quad \mathbb{E}[P_{e,x,q} - R(P_{e,x,q})] + \lambda c(q-x) + W + I - cq - \kappa_e \geq W$$

$$\implies \text{if } q > 0, \quad \mathbb{E}[P_{e,x,q} - s(q-x) - (R(P_{e,x,q}) - c(q-x)) - (c-s)(q-x)]$$

$$+\lambda c(q-x) + W + I - c(q-x) + c(q-x) - cq - \kappa_e \geq W$$

$$\implies \text{if } q > 0, \quad \mathbb{E}[P_{e,x,x} - R'(P_{e,x,x})] + W + [I - c(q-x)] - cx - \kappa_e \geq W + (c - s - \lambda c)(q-x)$$

$$\implies \text{if } x > 0, \quad \mathbb{E}[P_{e,x,x} - R'(P_{e,x,x})] + W + I' - cx - \kappa_e > W$$

the last implication being due to our assumption that $c > s + \lambda c$ (condition (1)).

The firm's budget constraint, condition (9), is satisfied because

$$W + I \geq cq \implies W + (I - c(q-x)) \geq cq - c(q-x) \implies W + I' \geq cx$$

The firm's incentive compatibility constraint, condition (10), holds (i.e., there is no diversion) as

$$\forall y \in [0,q], \ \forall \epsilon \in \{0,1\}, \quad \mathbb{E}[P_{e,x,q} - R(P_{e,x,q})] + \lambda c(q-x) + W + I - cq - \kappa_e$$

$$\geq \mathbb{E}[P_{\epsilon,y,q} - R(P_{\epsilon,y,q})] + \lambda c(q-y) + W + I - cq - \kappa_\epsilon$$

$$\implies \forall y \in [0,q], \ \forall \epsilon \in \{0,1\}, \quad \mathbb{E}[P_{e,x,q} - s(q-x) - (R(P_{e,x,q}) - c(q-x)) - (c-s)(q-x)]$$

$$+\lambda c(q-x) + W + [I - c(q-x)] + c(q-x) - cq - \kappa_e$$

$$\geq \mathbb{E}[P_{\epsilon,y,q} - s(q-x) - R(P_{\epsilon,y,q}) + c(q-x) - (c-s)(q-x)] + \lambda c(q-y) + W + [I - c(q-x)] + c(q-x) - cq - \kappa_\epsilon$$

$$\implies \forall y \in [0,x], \ \forall \epsilon \in \{0,1\}, \quad \mathbb{E}[P_{e,x,x} - R'(P_{e,x,x})] + \lambda c(x-x) + W + I' - cx - \kappa_e$$

$$\geq \mathbb{E}[P_{\epsilon,y,x} - R'(P_{\epsilon,y,x})] + \lambda c(x-y) + W + I' - cx - \kappa_\epsilon$$

Last, we show that objective (6) is strictly larger for contract (x, I', R') than for contract (q, I, R):

$$\mathbb{E}[P_{e,x,q} - R(P_{e,x,q})] + \lambda c(q-x) + W + I - cq - \kappa_e$$

$$= \mathbb{E}[P_{e,x,q} - s(q-x) - (R(P_{e,x,q}) - c(q-x)) - (c-s)(q-x)] + \lambda c(q-x) + W + [I - c(q-x)] + c(q-x) - cq - \kappa_e$$

$$= \mathbb{E}[P_{e,x,x} - R'(P_{e,x,x})] + W + I' - cx - \kappa_e - (c - s - \lambda c)(q-x)$$

$$< \mathbb{E}[P_{e,x,x} - R'(P_{e,x,x})]\lambda c(x-x) + W + I' - cx - \kappa_e$$

the last implication is due to (1). Thus the optimal contract must induce no diversion, i.e., $x = q$. ■

Appendix C: Proof of Theorem 1

For contract $(q,R)\in\mathbb{R}^+\times\mathcal{C}$, denote the firm's optimal decisions as $e^*_{q,R}$ and $x^*_{q,R}$. For a given $x\in[0,q]$ and $e=0,1$ define $e^*_{q,R}(x)=\arg\max_{e=0,1}\bar{r}_e(q,x)$ and $x^*_{q,R}(e)=\arg\max_{x\in[0,q]}\bar{r}_e(q,x)$. Further, for all $x\le q$, denote the payoff implied by demand realization $u\ge 0$ as $p_{x,q}(u)\equiv sq+(r-s)(u\wedge x)$.

We prove the theorem by showing that if optimal contracts with $q>0$ exist, they include one with a debt claim (Lemma 3) raising no more than the funds it needs, i.e., $(cq-W)^+$ (Lemma 4).

Lemma 3 *If an optimal contract $(q,R)\in\mathbb{R}^*_+\times\mathcal{C}$ exists, then (i) a debt claim $D\in\mathcal{D}$ exists such that $\mathbb{E}[D(P_{1,q,q})]=\mathbb{E}[R(P_{1,q,q})]$, and (ii) it is such that contract (q,D) is also optimal.*

Proof. Consider an optimal contract (q,R) with $q>0$. We have $e^*_{q,R}=1$ and $x^*_{q,R}=q$ (Lemma 2).

Point (i): For $D_K\in\mathcal{D}$ with face value $K\in[0,rq]$ and given q, define $\hat{d}(K)\equiv(K-sq)^+/(r-s)$ as the lowest demand level such that the firm can repay K, i.e., solution to $p_{q,q}(\hat{d}(K))=K$. We have

$$\mathbb{E}[D_K(P_{1,q,q})]=\int_0^{\hat{d}(K)}p_{q,q}(u)f_1(u)du+K\cdot\bar{F}_1(\hat{d}(K))$$

For $K=0$, $\hat{d}(0)=(0-sq)^+/(r-s)=0$ and $\mathbb{E}[D_0(P_{1,q,q})]=\int_0^0 p_{q,q}(u)f_1(u)du+0\cdot\bar{F}_1(\hat{d}(K))=0$. This is, debt with zero face value has zero value. For $K=rq$, $\hat{d}(rq)=(rq-sq)^+/(r-s)=q$ and

$$\mathbb{E}[D_{rq}(P_{1,q,q})]=\int_0^q(p_{q,q}(u)\wedge rq)f_1(u)du=\int_0^q p_{q,q}(u)f_1(u)du=\mathbb{E}[P_{1,q,q}].$$

That is, if debt's face value rq exceeds the highest profit realization possible, the investor receives all profits. Moreover, the expression is continuous and strictly increasing in K over $[0,rq]$, taking values from 0 to $\mathbb{E}[P_{1,q,q}]$ since

$$\frac{\partial\mathbb{E}[D_K(P_{1,q,q})]}{\partial K}=\frac{\partial\hat{d}(K)}{\partial K}p_{q,q}(\hat{d}(K))f_1(\hat{d}(K))+K\cdot\frac{\partial\hat{d}(K)}{\partial K}(-f_1(\hat{d}(K)))+\bar{F}_1(\hat{d}(K)).$$

Given that $p_{q,q}(\hat{d}(K))=K$ by definition of $\hat{d}(K)$, we have $\partial\mathbb{E}[D_K(P_{1,q,q})]\partial K=\bar{F}_1(\hat{d}(K))>0$.

Finally, we have $\mathbb{E}[R(P_{1,q,q})]\in[0,rq]$. Indeed, constraint (13) implies $\mathbb{E}[R(P_{1,q,q})]\ge(cq-W)^+\ge 0$, limited liability implies $R(P_{1,q,q})\le P_{1,q,q}$ and thus $\mathbb{E}[R(P_{1,q,q})]\le\mathbb{E}[P_{1,q,q}]$.

Point (ii): To show that contract (q,D) is optimal, it suffices to show that $e^*_{q,D}=1$ and $x^*_{q,D}=q$. To do so, we show that the fact that optimal contract (q,R) satisfies condition (14) implies that contract (q,D) satisfies it as well, i.e., $\forall\epsilon\in\{0,1\},\ \forall y\in[0,q]$

$$\mathbb{E}[P_{1,q,q}-R(P_{1,q,q})]-\lambda c(q-x)-\kappa_1\ge\mathbb{E}[P_{\epsilon,y,q}-R(P_{\epsilon,y,q})]-\lambda c(q-x)-\kappa_\epsilon$$

$$\Longrightarrow\ \mathbb{E}[P_{1,q,q}-D(P_{1,q,q})]-\lambda c(q-x)-\kappa_1\ge\mathbb{E}[P_{\epsilon,y,q}-D(P_{\epsilon,y,q})]-\lambda c(q-x)-\kappa_\epsilon$$

After simplification, this amounts to showing that

$$\mathbb{E}[P_{1,q,q}-R(P_{1,q,q})]-\mathbb{E}[P_{\epsilon,y,q}-R(P_{\epsilon,y,q})]\geq\kappa_1-\kappa_\epsilon \implies \mathbb{E}[P_{1,q,q}-D(P_{1,q,q})]-\mathbb{E}[P_{\epsilon,y,q}-D(P_{\epsilon,y,q})]\geq\kappa_1-\kappa_\epsilon$$

which, both inequalities' right-hand sides being identical, amounts to showing

$$\mathbb{E}[P_{1,q,q}-D(P_{1,q,q})]-\mathbb{E}[P_{\epsilon,y,q}-D(P_{\epsilon,y,q})]\geq\mathbb{E}[P_{1,q,q}-R(P_{1,q,q})]-\mathbb{E}[P_{\epsilon,y,q}-R(P_{\epsilon,y,q})]$$

Denoting $\Delta(\cdot)\equiv D(\cdot)-R(\cdot)$ and given that $\mathbb{E}[D(P_{1,q,q})]=\mathbb{E}[R(P_{1,q,q})]$, the condition amounts to

$$\forall\epsilon\in\{0,1\}\,,\forall y\in[0,q]\,,\quad \mathbb{E}[\Delta(P_{\epsilon,y,q})]\geq 0 \tag{16}$$

We complete the proof in two steps, first showing that for all $y\in[0,q]$, condition (16) holds for $\epsilon=1$ and then showing that it holds for $\epsilon=0$.

We start with the case $\epsilon=1$. We have for all $u\in\mathbb{R}^+$,

$$\begin{aligned} p_{y,q}(u) &= sq+(r-s)(u\wedge y)=sq+(r-s)(u\wedge y\wedge q) \quad \text{(because } y\leq q) \\ &= sq+(r-s)[(u\wedge q)\wedge(y\wedge q)]=[sq+(r-s)(u\wedge q)]\wedge[sq+(r-s)(y\wedge q)]=p_{q,q}(u)\wedge p_{q,q}(y) \end{aligned}$$

Thus, denoting debt claim D's face value as K, we have

$$D\left(p_{y,q}\left(u\right)\right)=p_{y,q}\left(u\right)\wedge K=p_{q,q}(u)\wedge p_{q,q}(y)\wedge K.$$

We consider two cases in turn.

Case 1: $y\geq\hat{d}(K)$ which amounts to $p_{q,q}(y)\geq K$. In that case, we have:

$$\forall u\in\mathbb{R}^+,\quad D(p_{y,q}(u))=p_{q,q}(u)\wedge p_{q,q}(y)\wedge K=p_{q,q}(u)\wedge K=D(p_{q,q}(u))$$

Hence $\mathbb{E}[\Delta(P_{\epsilon,y,q})]\equiv\mathbb{E}[D(P_{1,y,q})]-\mathbb{E}[R(P_{1,y,q})]=\mathbb{E}[D(P_{1,q,q})]-\mathbb{E}[R(P_{1,y,q})]$. Moreover, $R(\cdot)$ being non-decreasing, $p_{y,q}(u)\leq p_{q,q}(u)$ implies $R(p_{y,q}(u))\leq R(p_{q,q}(u))$ which implies, by definition of $D(\cdot)$,

$$\mathbb{E}[\Delta(P_{1,y,q})]\geq\mathbb{E}[D(P_{1,q,q})]-\mathbb{E}[R(P_{1,q,q})]\geq 0.$$

Case 2: $y<\hat{d}(K)$ which amounts to $p_{q,q}(y)<K$. In that case, we have:

$$\forall u\in\mathbb{R}^+,\quad p_{y,q}(u)\leq p_{y,q}(y)\leq p_{q,q}(y)<K \tag{17}$$

$$\begin{aligned} \text{Therefore,}\quad \forall u\in\mathbb{R}^+,\quad D(p_{y,q}(u))-R(p_{y,q}(u)) &= p_{y,q}(u)\wedge K-R(p_{y,q}(u)) \\ &= p_{y,q}(u)R(p_{y,q}(u)) \quad \text{from condition (17)} \\ &\geq 0 \quad \text{due to limited liability} \end{aligned}$$

which implies $\mathbb{E}[\Delta(P_{1,y,q})] \equiv \mathbb{E}[D(P_{1,y,q})] - \mathbb{E}[R(P_{1,y,q})] \geq 0$.

Now we consider the case $\epsilon = 0$. First, there exists $p^* \in [sq, rq]$ such that $\Delta(p) \geq 0$ for all $p \in [sq, p^*)$ and $\Delta(p) \leq 0$ for all $p \in (p^*, rq]$. (This is because $\partial D(p)/\partial p$ is as high as possible for $p < K$ and as low as possible for $p > K$.) For all $y \in [0, q]$, we have $\mathbb{E}[\Delta(P_{0,y,q})] = \int_{sq}^{p_{y,q}(y)} \Delta(p) g_{0,y}(p) dp$. We consider two cases in turn.

Case 1: $p^* \leq p_{y,q}(y)$. In that case, we have:

$$\begin{aligned} \mathbb{E}[\Delta(P_{0,y,q})] &= \int_{sq}^{p^*} \Delta(p) g_{0,y}(p) dp + \int_{p^*}^{p_{y,q}(y)} \Delta(p) g_{0,y}(p) dp \\ &= \int_{sq}^{p^*} \Delta(p) g_{1,y}(p) \frac{g_{0,y}(p)}{g_{1,y}(p)} dp + \int_{p^*}^{p_{y,q}(y)} \Delta(p) g_{1,x}(p) \frac{g_{0,y}(p)}{g_{1,y}(p)} dp \end{aligned}$$

From MLRP, $g_{0,y}(p)/g_{1,y}(p)dp$ increases strictly over $[sq, p_{y,q}(y)]$ (de Véricourt and Gromb 2016).

$$\mathbb{E}[\Delta(P_{0,y,q})] \geq \frac{g_{0,y}(p^*)}{g_{1,y}(p^*)} \int_{sq}^{p^*} \Delta(p) g_{1,y}(p) dp + \frac{g_{0,y}(p^*)}{g_{1,y}(p^*)} \int_{p^*}^{p_{y,q}(y)} \Delta(p) g_{1,y}(p) dp \geq \frac{g_{0,y}(p^*)}{g_{1,y}(p^*)} \cdot \mathbb{E}[\Delta(P_{1,y,q})].$$

Since we have shown that $\mathbb{E}\Delta(P_{1,y,q})] \geq 0$, this implies $\mathbb{E}[\Delta(P_{0,y,q})] \geq 0$.

Case 2: $p^* > p_{y,q}(y)$, which amounts to $\forall p \in [sq, p_{y,q}(y)],\ \Delta(p) > 0$. In that case, we have:

$$\mathbb{E}[\Delta(P_{0,y,q})] = \int_{sq}^{p_{y,q}(y)} \Delta(p) g_{0,y}(p) dp > 0.$$

Last we show (q, D) to be an optimal contract. We have shown that $e^*_{q,D} = 1$ and $x^*_{q,D} = q$. Hence (q, D) is optimal because objective (11) and condition (12) only depend on e, q and x, which are the same as for (q, R), and $\mathbb{E}\left[D\left(P_{1,q^*,q^*}\right)\right] = \mathbb{E}\left[R\left(P_{1,q^*,q^*}\right)\right]$ ensures condition (13) holds. ■

Lemma 4 *If contract $(q, D) \in \mathbb{R}^*_+ \times \mathcal{D}$ is optimal, a unique $D^* \in \mathcal{D}$ exists such that $E\left[D^*\left(P_{1,q,q}\right)\right] = (cq - W)^+$ and it is such that (q, D^*) is also optimal.*

Proof. For all $e \in \{0, 1\}$, $q \in \mathbb{R}^*_+$ and $x \in [0, q]$, the firm's expected payoff given debt with face value K is, using a slight abuse of notations,

$$\Pi\left(K, e, x, q\right) = (r - s)\left(\int_{\hat{d}(K,q)}^{x} \bar{F}_e\left(u\right) du\right)^+ + \lambda c(q - y) - \kappa_e,$$

where $\hat{d}(K, q) = \left(\frac{K - sq}{r - s}\right)^+$. We have,

$$\frac{\partial \Pi}{\partial K}\left(K, e, x, q\right) = -\mathbf{1}_{\left\{x \geq \hat{d}(K,q) > 0\right\}} \cdot \bar{F}_e\left(\hat{d}\left(K, q\right)\right) \leq 0 \tag{18}$$

which also implies that

$$\frac{\partial \Pi}{\partial K}(K,1,q,q) - \frac{\partial \Pi}{\partial K}(K,e,x,q) = -\mathbf{1}_{\left\{q \geq \hat{d}(K)\right\}} \bar{F}_1\left(\hat{d}(K) > 0\right) + \mathbf{1}_{\left\{x \geq \hat{d}(K) > 0\right\}} \cdot \bar{F}_e\left(\hat{d}(K)\right) \leq 0, \tag{19}$$

where the inequality holds because $x \leq q$ and $\bar{F}_1(\cdot) \geq \bar{F}_0(\cdot)$ from MLRP. Now consider contract (q,D) as in the lemma, with D's face value K. By Lemma 2, $e_D^* = 1$, $x_D^* = q$. Since $\mathbb{E}[D(P_{1,q,q})] \geq (cq-W)^+$, $D^* \in \mathcal{D}$ with face value $K^* \leq K$ exists such that $\mathbb{E}[D^*(P_{1,q,q})] = (cq-W)^+$as per previous arguments. By (19), we have $e_{D^*}^* = 1$ and $x_{D^*}^* = q$. Moreover, given (18), the firm's expected profit decreases with K and so (7) holds for K^* as it does for K. Hence, (q,D^*) is also optimal. ■

Appendix D: Proof of Corollary 1

Define function h as

$$h(K) \equiv K - (r-s)\int_0^{\left(\frac{K-sq}{r-s}\right)^+} F_1(x)\,dx - (cq-W)^+ \tag{20}$$

Expression (15) is written as $h(K) = 0$. Note that $h(K)$ is strictly increasing in K as $h'(K) = 1$ for $K \leq sq$ and $h'(K) = 1 - F_1\left(\frac{K-sq}{r-s}\right)$ for $K \geq sq$.

If $W \geq (c-s)q$ then $(cq-W)^+ \leq sq$. Thus, taking $K = (cq-W)^+$ we have $\hat{d} = 0$ and $h\left((cq-W)^+\right) = (cq-W)^+ - 0 - (cq-W)^+ = 0$. Therefore $K(q) = (cq-W)^+ \in [0,sq]$. Debt is riskfree because $K(q) \leq sq$ while $P_{1,q} \geq sq$.

Conversely, if $W < (c-s)q$ then $(cq-W)^+ > sq$. Thus, taking $K = sq$, we have $\hat{d} = 0$ and $h(sq) = sq - 0 - (cq-W)^+ < 0$. Moreover, when $K = rq$,

$$h(rq) = rq - (r-s)\int_0^q F_1(x)dx - (cq-W)^+ = [\pi_1(q) - \kappa_1] + [\kappa_1 + cq - (cq-W)^+] > 0$$

Indeed, the first bracket term non-negative over $\left[\underline{q}_1, \bar{q}_1\right]$ and the second one is strictly positive because $\kappa_1 > 0$. Therefore $K(q) \in (sq, rq)$. Moreover, debt is risky because $K(q) > sq$ and $P_{1,q} \geq sq$.

Appendix E: Proof of Proposition 3

To show the result, we establish that constraint (14) is satisfied at q_1^{FB} (all other constraints clearly hold). Note first that $\hat{d}(q_1^{FB}) = 0$ from Corollary 1. It follows that

$$\bar{r}_e\left(q_1^{FB}, x\right) = (r-s)\int_0^x \bar{F}_e(u)\,du + \lambda c\left(q_1^{FB} - x\right) = \mathbb{E}\left[P_{e,x,q_1^{FB}}\right] - sq_1^{FB} + \lambda c\left(q_1^{FB} - x\right).$$

Hence, we have $\bar{r}_1(q_1^{FB}, q_1^{FB}) - (c-s)q_1^{FB} = \pi_1(q_1^{FB})$ and for $e \in \{0,1\}$,

$$\bar{r}_e\left(q_1^{FB}, x\right) - (c-s)q_1^{FB} = \mathbb{E}\left[P_{e,x,q_1^{FB}}\right] + \lambda c\left(q_1^{FB} - x\right) - cq_1^{FB} \leq \pi_e\left(q_e^{FB}\right)$$

from Proposition 1. Constraint (14) holds at q_1^{FB} since for $x \leq q_1^{FB}$ and $e \in \{0,1\}$,

$$\bar{r}_1\left(q_1^{FB}, q_1^{FB}\right) - \kappa_1 = \pi_1\left(q_1^{FB}\right) + (c-s)q_1^{FB} - \kappa_1 \geq \pi_e\left(q_e^{FB}\right) + (c-s)q_1^{FB} - \kappa_e \geq \bar{r}_e\left(q_1^{FB}, x\right) - \kappa_e,$$

where the first inequality holds for $e = 0$ from (4).

Appendix F: Proof of Theorem 2 and its Corollaries

For capacity q and effort e, the firm's expected revenues under the debt claim of Theorem 1 is,

$$\bar{r}_e(q,x) = (r-s)\left(\int_{\hat{d}(q)}^{x} \bar{F}_e(u)\,du\right)^{+} + \lambda c(q-x). \tag{21}$$

F.1. Problem reformulation

We first characterize the optimal effective capacity, given capacity q and effort e.

Lemma 5 *Define* $q_e^{\lambda\star} = \bar{F}_e^{-1}\left(\frac{\lambda c}{r-s}\right)$. *We have* $\arg\max_{y\in[0,q]} \bar{r}_e(q,y) \in \{0, q\wedge q_e^{\lambda\star}\}$ *with* $r_e(q,q) \le r_e(q, q\wedge q_e^{\lambda\star})$, *where* $q_e^{\lambda\star}$ *is strictly decreasing in* λ *over* $[0,\bar{\lambda}]$ *from* $+\infty$ *to* q_e^{FB}.

Proof. Fix capacity q and consider $\bar{r}_e(q,x)$ as a function of x. For $x \le \hat{d}(q)$ from (21) we have $\bar{r}_e(q,x) = -\lambda c x$, which is decreasing in x. For $x > \hat{d}(q)$, $d\bar{r}_e(q,x)/dx = (r-s)\bar{F}_e(x) - \lambda c$ which is decreasing and equal to zero at $x = q_e^{\lambda\star} = \bar{F}_e^{-1}\left(\frac{\lambda c}{r-s}\right)$.

It follows that the maximand of $\bar{r}_e(q,\cdot)$ over $[\hat{d}(q), q]$ is $q\wedge q_e^{\lambda\star}$, while $x_e^*(q) = 0$ over $[0,\hat{d}(q)]$. Thus for all $x\in[\hat{d}(q),q]$, $\bar{r}_e(q,x) \le \bar{r}_e(q, q\wedge q_e^{\lambda\star})$ and for all $x\in[0,q]$, $\bar{r}_e(q,x) \le \max[\bar{r}_e(q,q\wedge q_e^{\lambda\star}), \bar{r}_e(q,0)]$, where $\bar{r}_e(q,0) = \lambda c q$.

Finally, $q_e^{\lambda\star}$ decreases in λ with $q_e^0 = +\infty$ because $F_e(\cdot)$ is a cumulative probability function. Further, note that $\bar{F}_e(q_e^{FB}) = (c-s)/(r-s) \ge \lambda c/(r-s) = \bar{F}_e(q_e^{\lambda\star})$, and thus $q_e^{FB} \le q_e^{\lambda\star}$, where the equality holds for $\lambda = \bar{\lambda}$. ■

Lemma 5 allows us to rewrite incentive constraints (14) as

$$\bar{r}_1(q,q) \ge \lambda c q, \tag{22}$$

$$\bar{r}_1(q,q) \ge \bar{r}_1(q,\, q\wedge q_1^{\lambda\star}), \tag{23}$$

$$\bar{r}_1(q,q) \ge \lambda c q + \Delta\kappa \tag{24}$$

$$\bar{r}_1(q,q) \ge \bar{r}_0(q,\, q\wedge q_0^{\lambda\star}) + \Delta\kappa, \tag{25}$$

(24) implies (22), i.e., if the firm diverts the entire capacity ($x=0$), it has no incentive to exert effort. Further from Lemma 5, if $q > q_1^{\lambda\star}$ then $\arg\max_{y\in[0,q]} \bar{r}_e(q,y) < q$ and so (22) or (23) is violated. Hence, it must be that $q \le q_1^{\lambda\star}$, in which case (23) is satisfied. Thus the firm's problem rewrites,

$$\max_q \; \pi_1(q) \tag{26}$$

$$s.t.$$

$$\underline{q}_1 \le q \le q_1^{\lambda\star} \wedge \bar{q}_1,$$

$$K(q)-(r-s)\int_0^{\hat{d}(q)} F_1(u)\,du=(cq-W)^+$$
$$L_1^\lambda(q,\Delta\kappa)\equiv \bar{r}_1(q,q)-\lambda cq-\Delta\kappa\ \geq 0$$
$$L_2^\lambda(q,\Delta\kappa)\equiv \bar{r}_1(q,q)-\bar{r}_0(q,\,q\wedge q_0^{\lambda\star})-\Delta\kappa\ \geq 0$$

Define now $L^\lambda(q,\Delta\kappa)$ as $L^\lambda(q,\Delta\kappa)=L_1^\lambda(q,\Delta\kappa)\wedge L_2^\lambda(q,\Delta\kappa)$. In particular, (14) is equivalent to $L^\lambda(q,\Delta\kappa)\geq 0$ with $q\leq q_1^{\lambda\star}$. Define further $q^{\lambda\max}$ as the maximand of $L^\lambda(\cdot,\Delta\kappa)$ for $q\leq \bar{q}_1\wedge q_1^{\lambda\star}$, i.e., $q^{\lambda\max}\equiv argmax_{q\leq\bar{q}_1\wedge q_1^{\lambda\star}}L^\lambda(q,\Delta\kappa)$. This capacity provides the strongest incentive for effort.

We next study the monotonicity of L^λ, which helps determine when the constraints in (26) bind.

F.2. Properties of L_1^λ, L_2^λ and L^λ

We start with a technical lemma.

Lemma 6 *If $W<(c-s)q$ then $\partial\hat{d}(q)/\partial q=\bar{F}_1(q_1^{FB})/\bar{F}_1\left(\hat{d}(q)\right)$*

Proof. If $W<(c-s)q$ then equation (15) implies $K(q)>sq$ and $\hat{d}(q)=\left(\frac{K(q)-sq}{r-s}\right)$. Taking the first-order derivative of condition (15) with respect to q yields

$$\frac{\partial K(q)}{\partial q}-(r-s)\frac{\partial\hat{d}(q)}{\partial q}F_1\left(\frac{K-sq}{r-s}\right)=c$$

Since $\partial\hat{d}(q)/\partial q=(\partial K(q)/\partial q-s)/(r-s)$, this expression can be rewritten as

$$s+(r-s)\frac{\partial\hat{d}(q)}{\partial q}-(r-s)\frac{\partial\hat{d}(q)}{\partial q}F_1\left(\hat{d}(q)\right)=c$$

$$\text{or,}\qquad \frac{\partial\hat{d}(q)}{\partial q}=\frac{c-s}{r-s}\frac{1}{1-F_1(\hat{d}(q))}=\left(1-\frac{r-c}{r-s}\right)\frac{1}{1-F_1\left(\hat{d}(q)\right)}=\frac{\bar{F}_1(q_1^{FB})}{\bar{F}_1\left(\hat{d}(q)\right)}.$$

■

We then provide a condition under which L_1^λ dominates L_2^λ at first-best.

Lemma 7 *A unique $\hat{\lambda}_1\in(0,\bar{\lambda}]$ exists such that,*

$$\lambda\in[0,\hat{\lambda}_1]\Leftrightarrow L_1^\lambda(q_1^{FB},\Delta\kappa)\geq L_2^\lambda(q_1^{FB},\Delta\kappa), \tag{27}$$

with equality if and only if $\lambda=\hat{\lambda}_1$ and $\hat{\lambda}_1<\bar{\lambda}$.

Further, if $\lambda\geq\hat{\lambda}_1$ then $L_1^\lambda(q,\Delta\kappa)\leq L_2^\lambda(q,\Delta\kappa),\forall q\geq q_1^{FB}$.

Proof. Define $\phi(q,\lambda)\equiv L_1^\lambda(q,\Delta\kappa)-L_2^\lambda(q,\Delta\kappa)$. From the definition of $L_i^\lambda(q,\Delta\kappa)$, $i=1,2$, and equation (21), we have

$$\phi(q,\lambda)\,=\,\bar{r}_0(q,\,q\wedge q_0^{\lambda\star})-\lambda cq=(r-s)\left(\int_{\hat{d}(q)}^{q\wedge q_0^{\lambda\star}}\bar{F}_0(u)\,du\right)^+-\lambda c\,\left(q\wedge q_0^{\lambda\star}\right).$$

Take first $q = q_1^{FB}$. Recall that $q_0^{\lambda\star}$ is decreasing from $+\infty$ to 0 as λ goes from 0 to $\bar{\lambda}$. When $q_0^{\lambda\star} \geq q$, $\phi(q,\lambda) = (r-s)\int_{\hat{d}(q)}^{q} \bar{F}_0(u)\,du - \lambda cq$, which is decreasing in λ. When $\hat{d}(q) \leq q_0^{\lambda\star} < q$, the first order derivative of $\phi(q,\lambda)$ with respect to λ yields,

$$\frac{\partial \phi}{\partial \lambda}(q,\lambda) = (r-s)\bar{F}_0(q_0^{\lambda\star})\frac{\partial q_0^{\lambda\star}}{\partial \lambda} - \lambda c \frac{\partial q_0^{\lambda\star}}{\partial \lambda} - cq_0^{\lambda\star} = -cq_0^{\lambda\star} < 0,$$

and $\phi(q,\cdot)$ is still decreasing. When $q_0^{\lambda\star} < \hat{d}(q)$, $\phi(q,\lambda) = -c\lambda q_0^{\lambda\star} < 0$. Hence, since $q_0^{\lambda\star}$ is decreasing in λ, $\phi(\cdot)$ is decreasing for all λ such that $q_0^{\lambda\star} \geq \hat{d}(q)$ and is negative otherwise. Further we have $\phi(0) \geq 0$ and threshold $\hat{\lambda}_1$ exists where $\hat{\lambda}_1 = arg\max_{\lambda\in[0,\bar{\lambda}]}\{\phi(q_1^{FB},\lambda) \geq 0\}$.

Consider now the case where $q \geq q_1^{FB}$. For $q \leq q_0^{\star}$, we have

$$\begin{aligned}
\frac{\partial \phi}{\partial q}(q,\lambda) &= (r-s)\left[\bar{F}_0(q) - \bar{F}_0(\hat{d}(q))\frac{d\hat{d}}{\partial q}(q)\right] - \lambda c \\
&< (r-s)\left[\bar{F}_0(q) - \bar{F}_1\left(q_1^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\right] \\
&< (r-s)\left[\bar{F}_0(q) - \bar{F}_1\left(q_1^{FB}\right)\frac{\bar{F}_0(q)}{\bar{F}_1(q)}\right] < 0
\end{aligned}$$

where the derivative of $\hat{d}(\cdot)$ is given by Lemma 6, the second inequality holds from MLRP and the last one from the fact that $q \geq q_1^{FB}$. For $q \geq q_0^{\lambda\star}$, $\phi(q,\lambda) = (r-s)\int_{\hat{d}(q)}^{q_0^{\lambda\star}} \bar{F}_0(u)\,du - \lambda c q_0^{\lambda\star}$, which is decreasing in q. Hence $\phi(\cdot,\lambda)$ is decreasing for $q \geq q_1^{FB}$. Further, if $\lambda \geq \hat{\lambda}_1$, then $L_1^{\lambda}(q_1^{FB},\Delta\kappa) \leq L_2^{\lambda}(q_1^{FB},\Delta\kappa)$ and thus $\phi(q,\lambda) \leq 0$ for all $q \geq q_1^{FB}$. ■

We now show that L^{λ} is concave.

Lemma 8 $L_1^{\lambda}(q,\Delta\kappa)$ *is strictly concave for* $q \geq 0$ *and is decreasing for* $q \geq q_1^{FB}$.

Proof. From Lemma 6 the first order derivative of L_1^{λ} with respect to q yields,

$$\frac{\partial L_1^{\lambda}}{\partial q}(q,\Delta\kappa) = (r-s)\left[\bar{F}_1(q) - \bar{F}_1(\hat{d}(q))\frac{d\hat{d}}{\partial q}(q)\right] - \lambda c = (r-s)\left[\bar{F}_1(q) - \bar{F}_1(q_1^{FB})\right] - \lambda c,$$

which is strictly decreasing in q and negative for $q \geq q_1^{FB}$. ■

This implies that when $L_1^{\lambda}(q_1^{FB},\Delta\kappa) < 0$, the constraint can be relaxed by decreasing q. Similarly, the next lemma indicates that when constraint $L_2^{\lambda}(q,\Delta\kappa) \geq 0$ is binding at q_1^{FB}, the constraint can be relaxed by decreasing or increasing q depending on whether λ is above or below a threshold.

Lemma 9 *A unique threshold* $\hat{\lambda}_2 \in (0,\bar{\lambda})$ *exists such that,*

- *If* $\lambda < \hat{\lambda}_2$, $L_2^\lambda(q,\Delta\kappa)$ *is strictly increasing in* q *for all* $q \le q_1^{FB}$,
- *If* $\lambda > \hat{\lambda}_2$, $L_2^\lambda(q,\Delta\kappa)$ *is strictly decreasing in* q *for all* $q \ge q_1^{FB}$.
- *If* $\lambda = \hat{\lambda}_2$, $L_2^\lambda(q,\Delta\kappa) \le L_2^\lambda(q_1^{FB},\Delta\kappa)$ *for all* $q \in [0,\bar{q}_1]$.

Proof. To simplify the exposition define $L_2^\lambda(q) \equiv (L_2^\lambda(q,\Delta\kappa)+\Delta\kappa)/(r-s)$. Maximizing this affine transform of $L_2^\lambda(q,\Delta\kappa)$ is equivalent to maximizing $L_2^\lambda(q,\Delta\kappa)$. We have,

$$L_2^\lambda(q) = \int_{\hat{d}(q)}^{q} \bar{F}_1(y)\,dy - \left(\int_{\hat{d}(q)}^{q\wedge q_0^{\lambda\star}} \bar{F}_0(u)\,du\right)^+ - \frac{\lambda c}{r-s}(q-q_0^{\lambda\star})^+,$$

such that, with $L_2^\lambda(q)$ is piece-wise differentiable,

$$\begin{aligned}\frac{\partial L_2^\lambda}{\partial q}(q) &= \bar{F}_1(q) - \bar{F}_1(\hat{d}(q))\frac{\partial \hat{d}}{\partial q}(q) - \bar{F}_0(q)\mathbb{1}_{\{q\le q_0^{\lambda\star}\}} + \bar{F}_0\left(\hat{d}(q)\right)\frac{\partial \hat{d}}{\partial q}(q)\mathbb{1}_{\{q_0^{\lambda\star}>\hat{d}(q)\}} - \frac{\lambda c}{r-s}\mathbb{1}_{\{q>q_0^{\lambda\star}\}}\\ &= \bar{F}_1(q) - \bar{F}_1(q_1^{FB}) - \bar{F}_0(q)\mathbb{1}_{\{q\le q_0^{\lambda\star}\}} - \bar{F}_0\left(q_0^{\lambda\star}\right)\mathbb{1}_{\{q>q_0^{\lambda\star}\}} + \bar{F}_1\left(q_1^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\mathbb{1}_{\{q_0^{\lambda\star}>\hat{d}(q)\}}\\ &= \bar{F}_1(q) - \bar{F}_1(q_1^{FB}) - \bar{F}_0\left(q\wedge q_0^{\lambda\star}\right) + \bar{F}_1\left(q_1^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\mathbb{1}_{\{q_0^{\lambda\star}>\hat{d}(q)\}},\end{aligned} \tag{28}$$

where $\mathbb{1}_{\{\cdot\}}$ is the indicator function and the second equality holds from Lemma 6.

From Lemma 5, $q_0^{\lambda\star}$ is strictly decreasing over $[0,\bar{\lambda}]$ from $+\infty$ to $q_0^{FB} < q_1^{FB}$. Hence, a unique $\hat{\lambda}_0$ exists such that $q_1^{FB} \le q_0^{\lambda\star}$ if and only if $\lambda \le \hat{\lambda}_0$.

Consider then the case $\lambda \le \hat{\lambda}_0$. The first order derivative reduces to

$$\frac{\partial L_2^\lambda}{\partial q}(q) = \bar{F}_1(q) - \bar{F}_0(q) - \bar{F}_1\left(q_1^{FB}\right)\left(1 - \frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\right) \tag{29}$$

For $q \le q_1^{FB}$, we have $\bar{F}_1(q) \ge \bar{F}_1(q_1^{FB})$ and so

$$\frac{dL_2^\lambda}{dq}(q) \ge \bar{F}_1(q) - \bar{F}_0(q) - \bar{F}_1(q)\left(1 - \frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\right) = \bar{F}_1(q)\left[\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)} - \frac{\bar{F}_0(q)}{\bar{F}_1(q)}\right] > 0$$

where the last inequality holds from MLRP. Hence, $L_2^\lambda(\cdot)$ increases for $q \le q_1^{FB}$ when $\lambda \le \hat{\lambda}_0$.

For the case $\lambda > \hat{\lambda}_0$ when $q \le q_0^{\lambda\star}$ and hence $q \le q_1^{FB}$, $dL_2^\lambda(q,\Delta\kappa)/dq$ reduces to (29) which is also positive. When $q > q_0^{\lambda\star}$, $dL_2^\lambda(q,\Delta\kappa)/dq$ reduces to

$$\frac{\partial L_2^\lambda}{\partial q}(q) = \bar{F}_1(q) - \bar{F}_1(q_1^{FB}) - \bar{F}_0\left(q_0^{\lambda\star}\right) + \bar{F}_1\left(q_1^{FB}\right)\frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\mathbb{1}_{\{q_0^{\lambda\star}>\hat{d}(q)\}},$$

which is decreasing in q since $\bar{F}_1(q)$ is decreasing, while $\bar{F}_0(x)/\bar{F}_1(x)$ and $\mathbb{1}_{\{q_0^{\lambda\star}>x\}}$ are both non-negative non-increasing functions in x and $\hat{d}(q)$ is increasing. Thus, $L_2^\lambda(q)$ is concave in q if $q \geq q_0^{\lambda\star}$. It follows that when $\lambda > \hat{\lambda}_0$, $L_2^\lambda(\cdot)$ is unimodal and has a unique maximand $q_2^{\lambda\max} \equiv argmax_{0\leq q\leq \bar{q}_1} L_2^\lambda(q,\Delta\kappa)$, with $q_2^{\lambda\max} > q_0^{\lambda\star}$.

Further, $dL_2^\lambda(q)/\partial q$ in (28) is also decreasing in λ since $q_0^{\lambda\star}$ is decreasing and hence $\bar{F}_0\left(q \wedge q_0^{\lambda\star}\right)$ and $\mathbb{1}_{\{q_0^{\lambda\star}>\hat{d}(q)\}}$ are non-decreasing and non-increasing in λ, respectively. Thus $L_2^\lambda(q)$ is concave and submodular in (q,λ) for $\lambda \geq \hat{\lambda}_0$ and $q_2^{\lambda\max}$ is decreasing in λ.

When $\lambda = \hat{\lambda}_0$, $dL_2^\lambda(q_1^{FB})/\partial q > 0$ from (29) and thus $q_2^{\lambda\max} > q_1^{FB}$. When $\lambda = \bar{\lambda}$, we have $\bar{F}_0(q_0^{\bar{\lambda}\star}) = \bar{\lambda}c/(r-s) = (c-s)/(r-s) = \bar{F}_1(q_1^{FB})$ and

$$\frac{\partial L^{\bar{\lambda}}}{\partial q}(q_1^{FB}) = -\bar{F}_1\left(q_1^{FB}\right)\left(1 - \frac{\bar{F}_0\left(\hat{d}(q)\right)}{\bar{F}_1\left(\hat{d}(q)\right)}\mathbb{1}_{\{q_0^{\lambda\star}>\hat{d}(q)\}}\right) < 0,$$

where the inequality holds with $\bar{F}_0(\cdot) < \bar{F}_1(\cdot)$ from the MLRP assumption. Thus, a unique value $\hat{\lambda}_2 \in (\hat{\lambda}_0, \bar{\lambda})$ exists, such that $q_2^{\lambda\max} \geq q_1^{FB}$ if and only if $\lambda \leq \hat{\lambda}_2$ and where the equality holds only for $\lambda = \hat{\lambda}_2$. The result then holds because $L_2^\lambda(\cdot)$ is unimodal. ■

Finally, the next lemma uses the previous two lemmas to characterize the monotonicity of L^λ.

Lemma 10 *Assume that $W < (c-s)q_1^{FB}$. A unique threshold $\hat{\lambda} < \bar{\lambda}$ exists such that,*

- *If $\lambda < \hat{\lambda}$, $q^{\lambda\max} > q_1^{FB}$ and $L^\lambda(q,\Delta\kappa)$ is strictly increasing in q for all $q \leq q_1^{FB}$,*
- *If $\lambda > \hat{\lambda}$, $q^{\lambda\max} < q_1^{FB}$ and $L^\lambda(q,\Delta\kappa)$ is strictly decreasing in q for all $q \geq q_1^{FB}$,*
- *If $\lambda = \hat{\lambda}$, $q^{\lambda\max} = q_1^{FB}$.*

Proof. To simplify the notations, take $L_i^\lambda(q) = L_i^\lambda(q,\Delta\kappa)$, $i = 1,2$ and $L^\lambda(q) = L^\lambda(q,\Delta\kappa)$. Define $\hat{\lambda} \equiv \hat{\lambda}_1 \wedge \hat{\lambda}_2$. We have $\hat{\lambda} \leq \hat{\lambda}_2 < \bar{\lambda}$ from Lemma 9.

First assume the case where $\lambda < \hat{\lambda}$. $L_1^\lambda(\cdot)$ is then strictly concave (Lemma 8) and $L_2^\lambda(q)$ strictly increasing for $q \leq q_1^{FB}$ (Lemma 9 with $\lambda < \hat{\lambda}_2$). Further $L_2^\lambda(q_1^{FB}) < L_1^\lambda(q_1^{FB})$ (Lemma 7 with $\lambda < \hat{\lambda}_1$). Thus, $L^\lambda(q)$ is strictly increasing for $q \leq q_1^{FB}$. Indeed, denote $\tilde{q}$ the unique maximand of $L_1^\lambda(\cdot)$ over $[0, q_1^{FB}]$. When $q < \tilde{q}$, $L_1^\lambda(q)$ and hence $L_1^\lambda(q) \wedge L_2^\lambda(q)$ increase in q. When $\tilde{q} \leq q \leq q_1^{FB}$, if $L_1^\lambda(q) \leq L_2^\lambda(q)$ then $L_2^\lambda(q_1^{FB}) > L_1^\lambda(q_1^{FB})$ since $L_1^\lambda(\cdot)$ and $L_2^\lambda(\cdot)$ are striclty decreasing and increasing, respectively. This yields a contradiction and thus $L^\lambda(q) = L_2^\lambda(q)$ for $q \in [\tilde{q}, q_1^{FB}]$, which is also increasing. Finally, $q_1^{FB} < q_1^{\lambda\star}$ for $\lambda < \hat{\lambda}$ (Lemma 5) and hence $q_1^{FB} < \bar{q}_1 \wedge q_1^{\lambda\star}$. It follows that $q_1^{FB} < q^{\lambda\max}$.

Next, assume $\lambda > \hat{\lambda}$. Note that $L_1^\lambda(q)$ is strictly decreasing for $q \geq q_1^{FB}$ (Lemma 8). If $\lambda > \hat{\lambda}_2$, $L_2^\lambda(q)$ is also decreasing in $q \geq q_1^{FB}$ (Lemma 9), and the result holds. If $\lambda > \hat{\lambda}_1$, then $L^\lambda(q) = L_1^\lambda(q)$ for $q \geq q_1^{FB}$ from Lemma 7 so thats $L^\lambda(q)$ is still strictly decreasing, which yields the result.

The continuity of $L^\lambda(q)$ in λ implies the last part of the result. ■

F.3. Proof of Theorem 2

Note that $L_1^\lambda(q,\Delta\kappa)$ is continuous and decreasing in λ. Define $\tilde{\lambda}$ as the unique value of λ such that $L_1^\lambda(q_1^{FB},0)=0$. From Lemma 5, $L_2^\lambda(q,0)=\bar{r}_1(q,q)-\bar{r}_0(q,\,q\wedge q_0^{\lambda\star})\geq \bar{r}_1(q,q)-\bar{r}_0(q,q)\geq 0$ and since $L_1^{\tilde{\lambda}}(q_1^{FB},0)\leq 0$, we have $\hat{\lambda}\leq\tilde{\lambda}$.

Similarly, $L^\lambda\,(q,\Delta\kappa)$ is continuous and decreasing in $\Delta\kappa$ since $L_i^\lambda\,(q,\cdot)$, $i=1,2$ are continuous decreasing functions. Define $\Delta\bar{\kappa}$ as $\Delta\bar{\kappa}\equiv\max\left[\Delta\kappa,\, s.t.\, \Delta\kappa\geq 0 \text{ and } L^\lambda\,(q^{\lambda\max},\Delta\kappa)\geq 0\right]$. Thus, for all $\Delta\kappa>\Delta\bar{\kappa}$, $L^\lambda\,(q,\Delta\kappa)<0$ for all q and the firm abandons the project ($q^*=0$).

Consider first the case $\lambda\leq\tilde{\lambda}$. We have $L_1^\lambda\,(q,0)\geq 0$ and thus $L^\lambda\,(q,0)\geq 0$ so that a unique non-negative threshold $\Delta\underline{\kappa}\geq 0$ exists such that $\Delta\underline{\kappa}\equiv\min\left[\Delta\kappa\geq 0\; s.t.\; L^\lambda\,(q_1^{FB},\Delta\kappa)\leq 0\right]$. Thus, for all $\Delta\kappa\leq\Delta\underline{\kappa}$, $L^\lambda\,(q_1^{FB},\Delta\kappa)\geq 0$. Further for $\lambda\leq\bar{\lambda}$, we have $q_1^{FB}\leq q_1^{\lambda\star}$, and thus all constraints of problem (26) hold in $q=q_1^{FB}$, which also maximizes the objective. Hence, we have $q^*=q_1^{FB}$. Further when $\lambda=\hat{\lambda}$, $q_1^{FB}=q^{\lambda\max}$ from Lemma 10 and hence $L^\lambda\,(q_1^{FB},\Delta\kappa)=L^\lambda\,(q^{\lambda\max},\Delta\kappa)$, which implies that $\Delta\underline{\kappa}=\Delta\bar{\kappa}$ since $L^\lambda(q,\cdot)$ is a continuous and decreasing function.

Assume further that $\Delta\underline{\kappa}<\Delta\kappa<\Delta\bar{\kappa}$, which implies $\lambda\neq\hat{\lambda}$. Take $\lambda>\hat{\lambda}$. (A similar approach yields the result for $\lambda<\hat{\lambda}$.) From Lemma 10, we have $q^{\lambda\max}<q_1^{FB}$. Because $\Delta\kappa<\Delta\bar{\kappa}$, $L^\lambda\,(q^{\lambda\max},\Delta\kappa)>0$ and the set $F=\{q\geq q^{\lambda\max}\; s.t.\; L^\lambda\,(q,\Delta\kappa)>0\}$ is not empty. For $q\geq q_1^{FB}$, Lemma 10 implies $L(q,\Delta\kappa)\leq L(q_1^{FB},\Delta\kappa)$, which is negative since $\Delta\underline{\kappa}<\Delta\kappa$. Thus, $q_1^{FB}>q$ for $q\in F$ and objective (26) is increasing in q for $q\in F$. It follows that $q^*=\max[q\; s.t.\; q\in F]$, with $q^*<q_1^{FB}$. And q^* is decreasing in $\Delta\kappa$ since $L^\lambda(q,\cdot)$ is decreasing, with $q^*=q^{\lambda\max}$ when $\Delta\kappa=\Delta\bar{\kappa}$.

Finally, consider the case $\lambda>\tilde{\lambda}$, which implies $\lambda>\hat{\lambda}$. We have $L^\lambda(q_1^{FB},0)=L_1^\lambda(q_1^{FB},0)<0$ and since $L^\lambda(q_1^{FB},0)$ decreases in $\Delta\kappa$, $L^\lambda(q_1^{FB},\Delta\kappa)<0$ for all $\Delta\kappa$ so that $\Delta\underline{\kappa}$ does not exist. The previous argument holds for $0\leq\Delta\kappa<\Delta\bar{\kappa}$ and $\lambda>\hat{\lambda}$. ■

F.4. Proof of Proposition 3

Assume $\Delta\kappa=0$. $L_2^\lambda(q,0)\geq 0$ for all q and constraint (14) is always satisfied, which corresponds to the case of contractible effort. The result holds directly from Theorem 2 with $\Delta\kappa=0$. ■

F.5. Proof of Corollary 4

We first show that $L^\lambda(q,\Delta\kappa)$ is decreasing in λ. Recall that $q_0^{\lambda\star}$ is decreasing in λ. Note that $L_1^\lambda(q,\Delta\kappa)$ and $L_2^\lambda(q,\Delta\kappa)$ are linear decreasing for all λ, and for λ such that $q_0^{\lambda\star}\geq q$, respectively. When λ is such that $q_0^{\lambda\star}<q$, $dL_2^\lambda(q,\Delta\kappa)/d\lambda=-c(q-q_0^{\lambda\star})-\left[(r-s)\bar{F}_0(q_0^{\lambda\star})-\lambda c\right]dq_0^{\lambda\star}/d\lambda=-c(q-q_0^{\lambda\star})<0$, where the second equality holds from the definition of $q_0^{\lambda\star}$. It follows that $L_i^\lambda(q,\Delta\kappa)$, $i=1,2$ and hence $L^\lambda(q,\Delta\kappa)$ are decreasing in λ.

From Theorem 2, when $\lambda = 0 < \hat{\lambda}$, two thresholds $\Delta\underline{\kappa}_0$ and $\Delta\bar{\kappa}_0$ exist such that $L^0(q_1^{FB}, \Delta\kappa) < 0$ and $q^* > q_1^{FB}$ when $\Delta\underline{\kappa}_0 < \Delta\kappa < \Delta\bar{\kappa}_0$. Define then $\bar{\lambda}_0$ as $\bar{\lambda}_0 \equiv \max_{0 \leq \lambda \leq \bar{\lambda}} \left[\lambda \; s.t. \; L^{\lambda}\left(q^{\lambda \max}, \Delta\kappa\right) \geq 0\right]$.

When $\hat{\lambda} \leq \lambda$, then $L^{\lambda}(q^{\lambda \max}, \Delta\kappa) \leq L^0(q^{\lambda \max}, \Delta\kappa) \leq L^0(q_1^{FB}, \Delta\kappa) < 0$, where the first inequality holds since $L^{\lambda}(q, \Delta\kappa)$ is non-increasing in λ and the second one since $q^{\lambda \max} < q_1^{FB}$ and $L^0(\cdot, \Delta\kappa)$ is increasing in $q \leq q_1^{FB}$ from Lemma 10. It follows that $\bar{\lambda}_0 < \hat{\lambda}$.

When $\lambda < \bar{\lambda}_0$ define the set $F = \{q \leq q^{\lambda \max} \; s.t. \; L^{\lambda}\left(q, \Delta\kappa\right) > 0\}$. For $q \leq q_1^{FB}$, we have $L^{\lambda}(q, \Delta\kappa) < L^{\lambda}(q_1^{FB}, \Delta\kappa) < L^0(q_1^{FB}, \Delta\kappa) < 0$. Thus, $q_1^{FB} < q$ for $q \in F$ where the first inequality holds from Lemma 10 and the second one from the monotonicity of $L^{\lambda}(q_1^{FB}, \Delta\kappa)$ in λ. The objective of (26) is then decreasing in q for $q \in F$ and thus $q^* = \min[q \; s.t. \; q \in F]$, with $q^* > q_1^{FB}$. It follows that q^* is increasing in $\lambda < \bar{\lambda}_0$ since $L^{\lambda}(q, \Delta\kappa)$ is decreasing in λ.

When $\lambda \geq \bar{\lambda}_0$, $L^{\lambda}\left(q^{\lambda \max}, \Delta\kappa\right) < 0$ and $q^* = 0$, which yields the result. ■