Financing Capacity Investment Under Demand Uncertainty: An Optimal Contracting Approach

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Abstract. We study the capacity choice problem of a firm whose access to capital is hampered by financial frictions (i.e., moral hazard). The firm optimizes both its capacity investment under demand uncertainty and its sourcing of funds from a competitive investor. Ours is the first study of this problem to adopt an optimal contracting approach: feasible sources of funds are derived endogenously from fundamentals and include standard financial claims (debt, equity, convertible debt, etc.). Thus, in contrast to most of the literature on financing capacity investments, our results are robust to a change of financial contract. We characterize the optimal capacity level under optimal financing. First, we find conditions under which a feasible financial contract exists that leads to the first-best capacity. When no such contract exists, we find that under optimal financing, the choice of capacity sometimes exceeds strictly the efficient level. Furthermore, the firm invests more when its cash is low, and in some cases less when the project’s unit revenue is high. These results run counter to the newsvendor logic and standard finance arguments. We also show that our main results hold in the case of a strategic monopolist investor, and such an investor may invest more then a competitive one.

1. Introduction

Capacity investment decisions for new products or markets are often made under considerable demand uncertainty. When calibrating capacity, management must navigate between the Scylla of excess demand’s opportunity cost and the Charybdis of excess capacity’s investment cost. While firms investing in capacity need to mobilize human and physical capital, they must also avail of funds to acquire those. When cash is short, they must raise outside funds from any of a variety of sources: bank loans, trade credit, bonds, private equity, stock markets, etc. Hence, firms must optimize not only their capacity investment, but also their sourcing of funds.

This paper takes an optimal contracting approach to study the interplay between the operational and financial facets of capacity investment. We consider the capacity choice problem of a firm with limited cash and whose access to external funds is hampered by financial frictions, that is, moral hazard. When calibrating its capacity investment, the firm must consider the sources of funds available. These are derived endogenously and include standard financial claims (debt, equity, convertible debt, warrants, etc.) and combinations thereof.

After determining the optimal source of funds for a given capacity level, we characterize the optimal capacity and its dependence on important model parameters. This yields new insights into the problem of financing capacity under demand uncertainty. We find that the optimal capacity level can exceed strictly the efficient level, increase with the moral hazard problem’s severity, and decrease with the firm’s cash and, in some cases, with the per-unit revenue. We also show that our main results hold in the case of a strategic monopolist investor, and that such an investor may invest more than a competitive one. Crucially, our approach ensures that no change of financial contract can mitigate these deviations from first best.

These results run counter to the newsvendor logic and existing findings on financing capacity investments. More generally, they challenge the intuition in finance that by raising the cost of external funds and hence the unit capacity cost, financial frictions lower investment in capacity. In our setup, however, financial frictions may actually decrease as capacity increases.

Specifically, we consider a firm with limited cash making a capacity investment choice under demand uncertainty, which we formalize as a newsvendor model with two features.

First, the firm’s owner has limited cash but can raise funds from a competitive investor. This feature alone causes no tangible departure from the standard newsvendor model. Indeed, absent financial frictions,
the Modigliani–Miller theorem implies that the firm’s problem is unaffected by its need for funds and that its choice of a source of funds is a matter of irrelevance.

Second, by exerting a costly effort, the firm can improve the distribution of demand in the sense of the monotone likelihood ratio property (MLRP). Effort can stand for conducting a sales campaign, improving a product’s design, etc. Loosely speaking, the MLRP means that higher demand realizations are more indicative of high effort. Effort is noncontractible, that is, cannot be set by contract. This constitutes a financial friction that renders the firm’s financing relevant.

Given this, the firm optimizes both its capacity and its funding, that is, the financial contract it offers the investor. We adopt an optimal contracting approach, which is now standard in finance: agents optimize over feasible contracts derived endogenously from fundamentals, that is, preferences, physical constraints (e.g., production technology), and contractibility (i.e., which variables can be set in an enforceable contract). This contrasts with existing operations-and-finance studies that assume an exogenous set of feasible claims, typically debt with a loan-size-independent interest rate. That approach runs the risk of concluding that distortions of operations are needed when they could be avoided by a simple contract change (see Section 2).

Simple assumptions about fundamentals ensure claims satisfy three conditions. First, effort apart, all variables (e.g., capacity, funding, revenues, etc.) are contractible bar one: demand. With revenues being contractible, and given they map one-to-one with demand when it is below capacity, this amounts to assuming that only unmet demand is not contractible. Hence, financial contracts specify a capacity, an amount of funding by the investor, and a financial claim, that is, revenue-contingent repayments to the investor. Second, we assume the firm to be protected against funds falling below zero. Hence, claims must satisfy limited liability; that is, repayments cannot exceed revenues. Third, the firm can report artificially inflated revenues, and will do so if that leads to lower repayments. To avoid such manipulation, claims must be monotonic; that is, repayments must increase with revenues.

The feasible set so defined includes claims such as debt, equity, convertible debt, or warrants or mixes thereof (e.g., debt plus equity, etc.). For instance, a debt claim defines repayments equal to the realized revenue if below its face value, and otherwise equal to its face value. Similarly, an equity stake defines repayments equal to a fraction of the realized revenue. Both claims are feasible: they respect monotonicity and limited liability, and repayments depend only on revenues.

We start by characterizing the optimal source of funds for a given capacity. This is a key methodological step. Indeed, this paper’s main point is to characterize the firm’s optimal capacity investment given that investment is financed optimally. A priori, different sources of funds have their appeal. For instance, equity may be suitable to control excessive risk taking, a temptation debt might instead exacerbate. However, extending Innes (1990), we show that using cash should be a priority and that if external funds are needed, debt financing dominates.

Given the optimal source of funds, we then characterize the optimal capacity given optimal financing. Absent financial frictions, that is, if effort is contractible, the optimal capacity is the standard newsvendor quantity. This also holds for mild frictions. For intermediate frictions, no feasible contract can achieve first best, and the optimal capacity exceeds strictly the efficient level. Last, if frictions are too strong, financing is impossible and the project is abandoned; that is, capacity is set at zero, hence, below first best.

Overinvestment commits the firm to exerting effort as it increases the firm’s expected payoff by a lump sum (akin to a bonus) for demand realizations above the first best. This bonus effect boosts incentives. In addition, the payoff increase that the extra capacity implies is higher for higher demand realizations. This demand differentiation effect also boosts incentives as under the MLRP, higher demand realizations are more indicative of effort. We show that neither effect can be replicated by a financial claim, as such a claim would have to violate monotonicity to get the bonus effect and the noncontractibility of unmet demand to get the demand differentiation effect.

Several implications arise from these findings. First, under optimal contracting, as long as frictions are mild, financing needs per se do not imply deviating from the first-best capacity investment. Thus, before resorting to doing so, managers should favor changes in financing. Furthermore, for more severe frictions, firms can self-commit by overinvesting, and the more severe the frictions, the larger the overinvestment. This suggest that overinvestment should arise and be more important where incentive provision is more problematic, for example, when human capital is an important input, where legal protection of investors is laxer, etc. In addition, optimal capacity decreases with the firm’s cash, a result that runs counter to the standard finance intuition, and increases with the per unit revenue, a result running counter to the standard newsvendor logic.

Finally, we show that our main results also hold in the case of a monopolist investor, the setup studied by much of the operations-and-finance literature. Solving this Stackelberg game, we find that debt financing is optimal, first-best investment is achieved for mild frictions, but overinvestment occurs for stronger ones. This implies that, under optimal contracting, as long as frictions are mild, monopoly power per se does not
implies a deviation from the first-best capacity. Furthermore, when combined with frictions, monopoly power makes the investor less prone to funding the project, but if so, at larger capacity levels than in the competitive case.

This paper proceeds as follows. Section 2 reviews the literature. Section 3 presents the model. Section 4 studies the frictionless benchmark. Section 5 shows the optimality of debt financing. Section 6 studies the optimal capacity level, optimal overinvestment, and implications. Section 7 considers a monopolist investor model. Section 8 concludes. All proofs are in the online appendix.

2. Related Scholarship

2.1. Corporate Finance

Our overinvestment result contrasts with standard finance theories (Tirole 2006). Those generally assume financial frictions like information asymmetry (Myers and Majluf 1984) or moral hazard (Jensen and Meckling 1976) causing outside funds to be costlier than cash. Hence, firms needing more outside funds incur a higher cost of capital and invest less, not more, than the first-best level. Our overinvestment result stems from the fact that in our model, higher capacity can reduce, not increase, financial frictions.

Overinvestment arises in one finance theory: Jensen’s (1986) free cash flow theory positing that investment-prone managers employ cash for excessive investment. Given managers’ bias, investors will not provide external funds exceeding the needs for optimal investment. Hence, only firms with excess cash overinvest. Our result is the opposite: only firms with insufficient cash overinvest.

Optimal contracting is now the standard methodology in corporate finance (Tirole 2006): agents optimize over feasible contracts derived endogenously from assumptions about fundamentals, that is, preferences, technology, and contractibility. Thus, deviations from efficiency are considered only once all feasible contractual solutions are exhausted so results are robust to a simple contract change. The approach implies that absent financial frictions, efficient outcomes obtain.

Our analysis of optimal financial contracts builds closely on the work of Innes (1990). In our model, as in that of Innes, the MLRP and properties of feasible claims imply debt is optimal. We extend his analysis to the newsvendor model, which is easy, but also to the case of a monopoly investor, which is more involved. In any case, deriving the optimal contract is only a methodological means to and end: characterizing the firm’s optimal capacity investment given that investment is financed optimally. As discussed in the following sections, ours is the first operations–finance paper to follow this approach.

Finally, finance largely overlooks specificities of operations models. For instance, in Innes (1990), there is no discussion of optimal capacity, no unmet demand, etc. Our work demonstrates that operations do matter in finance problems and can even reverse standard results, as is the case for our overinvestment result and the result that investment level decreases with the firm’s cash.

2.2. Operations–Finance Interface

Our paper belongs to the nascent literature on the interplay between capacity choices and financing, which has derived a rich set of results on how a firm’s funding needs affect its capacity or technology choices. In a study by Alan and Gaur (2017), a newsvendor seeking a bank loan sets capacity below the efficient level. In a dynamic inventory model by Li et al. (2013), optimal inventory and financial decisions are found to be myopic and increasing in inventory level and retained earnings. Boyabatli and Toktay (2011) study a multiproduct firm making capacity and (flexible or dedicated) technology choices and show that the firm’s need for funds affects demand uncertainty’s impact on those choices.

None of this literature follows an optimal contracting approach. Instead, an investor sets a fixed (i.e., loan-size independent) interest rate in a first stage and, in a second stage, the newsvendor chooses the loan size given that interest rate. In Section 2 of the paper by Dada and Hu (2008), the interest rate is set by a monopolist bank. This is the case in the works by Buzacott and Zhang (2004) and Alan and Gaur (2017), too, but in their studies the loan size is capped. In the study by Boyabatli and Toktay (2011), the rate is set by a competitive bank for each technology and loans may be secured or not. Li et al. (2013) assume an exogenous interest rate.

A downside relative to optimal contracting is that one cannot tell how much of a result (usually, a distortion of operations away from first best) is only due to the restriction to specific financial contracts, and how much would resist a simple contract change. In fact, some papers assume no financial frictions, in which case optimal contracting ensures efficiency and so distortion results go away. Others assume only bankruptcy costs, a friction specific to debt, and hence eliminated by a simple switch to another financial contract (e.g., equity).

In contrast, ours is the first paper in the operations- and finance literature to adopt an optimal contracting approach. This ensures that the deviations from first best we characterize are robust to a simple contract change. In particular, we do not assume debt financing a priori, and when we consider debt financing, we do not assume a priori fixed interest rates. We also assume a financial friction (i.e., moral hazard), which ensures financing choices are not irrelevant in the first place.
2.3. Salesforce Compensation

Technically, our paper is linked to the principal–agent literature with risk neutrality and limited liability (e.g., Oyer 2000, Grömb and Martimort 2007, Poblete and Spulber 2012). In operations, such models have been used to analyze capacity choice in the context of the salesforce compensation problem. Chu and Lai (2013) and Dai and Jerath (2013, 2016) study a newsvendor model in which a wage contract must induce a sales agent to take a demand-enhancing action. Notably, Chu and Lai (2013) and Dai and Jerath (2013) show that capacity above the first best may be optimal and that the optimal compensation contract includes a bonus when demand exceeds capacity.

However, the salesforce compensation problem is connected to but in fact quite different from our financing problem, both technically and in terms of implications.

The main technical differences are two. First, in the salesforce problem, the newsvendor tries to control her sales agent, whereas in our model, she tries to self-control, that is, to commit to work once the project is underway. Thus, the optimization problem’s objective is different. Second, wage contracts are not subject to the same constraints as financial contracts, notably the monotonicity condition, a must in finance. Thus, the optimization problem’s constraints are different.

These technical differences imply that the results, and the reasons why they hold, are also different. For instance, the different objective implies that more severe moral hazard makes the sales agent better off but our firm worse off. The different constraints imply that the reasons for the overinvestment result are not the same. Notably, our bonus effect stems from financial contracts’ monotonicity condition and thus cannot possibly arise in salesforce models (only an effect akin to the demand differentiation effect does). Thus, situations exist in which overinvestment arises in our model but not in salesforce models. (See Section 6.3 for a detailed discussion.)

Finally, the implications of our analysis cannot be compared with those of a salesforce model. For instance, we study the role of the newsvendor’s cash, which is irrelevant in the salesforce model. We further show that higher unit revenue decreases overinvestment (by relaxing financing constraints), whereas it would increase overinvestment in a salesforce model.

We also analyze the interaction between financial frictions and investor market power, an issue absent from the salesforce literature.

3. A Model of Capacity Investment Financing

3.1. The Newsvendor Model with Financing Under Moral Hazard

We study the problem of a firm with limited cash making a capacity investment choice under demand uncertainty. We formalize this situation as a newsvendor model, with two features: the firm’s owner has limited wealth but can raise funds externally, and he can affect the distribution of demand by taking a noncontractible action. It is worth noting that limited wealth per se (see Section 4) or the noncontractibility of the action per se (see Section 5) causes no meaningful departure from the standard newsvendor model. Yet, their combination does.

3.1.1. Newsvendor Model. Assume universal risk neutrality and no discounting. Consider a firm whose sole owner (henceforth, “the firm”) has an investment project. The firm faces stochastic demand $D_t$ with distribution $f_t(\cdot)$, cumulative distribution $F_t(\cdot)$, and complementary cumulative distribution $\bar{F}_t(\cdot) \equiv 1 - F_t(\cdot)$. To simplify, we assume that $f_t$ is strictly positive over $\mathbb{R}_+$. The firm chooses a capacity $q \in \mathbb{R}_+$ at unit cost $c > 0$ before demand is realized, and eventually receives a revenue $r > c$ per unit sold and salvage value $s < c$ per unsold unit.

3.1.2. Effort Choice. If the firm does not set up any capacity (i.e., if $q = 0$), we say it abandons the project. Otherwise, once capacity $q > 0$ is installed but before demand is realized, the firm can opt to run the project diligently (e.g., conduct a suitable sales campaign, improve product design, etc.) or not, which we model as an effort choice. If the firm works (effort $e = 1$), it incurs a nonmonetary cost $\kappa_1 > 0$. This can be a labor cost for the owner, the opportunity cost of allocating attention away from other projects, etc. If instead the firm shirks (effort $e = 0$), it incurs a smaller nonmonetary cost $\kappa_0 > 0$, that is, with $\Delta \kappa \equiv \kappa_1 - \kappa_0 > 0$.

A priori, the firm could adjust the project to the lower effort level (e.g., cut the product price, etc.), which might affect demand. To simplify, we assume that irrespective of such adjustments, the project’s value is negative if the firm shirks, so that abandoning it ($q = 0$) is preferable.

A case of interest is when the firm shirks ($e = 0$) but does not adjust the project relative to when it works ($e = 1$); that is, unit price, cost, and salvage value remain equal to $r$, $c$, and $s$. In this case, shirking shifts demand to $D_0$ with a distribution $f_0$ less favorable than $f_1$ in the sense of the MLRP, that is, $f_1/f_0$ is strictly increasing over $\mathbb{R}_+$. Loosely speaking, MLRP means that higher demand realizations are more indicative of high effort.

3.1.3. First Best. For given capacity $q \geq 0$ and effort $e \in \{0, 1\}$, the project’s monetary payoff is randomly distributed over $[sq, rq]$ as per density $g_{e,q}$ (implied by density $f_e$) and is

$$P_{e,q} \equiv sq + (r - s)(D_t \land q).$$

Denote the expected profit in the standard newsvendor model as

$$\pi_e(q) \equiv E[P_{e,q}] - cq.$$
As per standard arguments, \( \pi_c(\cdot) \) is strictly concave over \( \mathbb{R}_+ \), and the first-best optimal capacity is
\[
q_e^{FB} = \arg \max_{q \in \mathbb{R}_+} \pi_c(q) = F_e^{-1} \left( \frac{r - c}{r - s} \right).
\]
(3)

We assume that given \( r, c \) and \( s \), the project is viable if \( e = 1 \) but not if \( e = 0 \), that is,
\[
\max_{q \in \mathbb{R}_+} \pi_1(q) - \kappa_1 > 0 > \max_{q \in \mathbb{R}_+} \pi_0(q) - \kappa_0,
\]
(4)
so that the first-best optimal outcome is \( e = 1 \) and \( q_e^{FB} \).

3.1.4. Financial Contracts. The firm has cash \( W \geq 0 \). If this is not enough to fund its desired capacity, the firm can raise funds from a competitive investor via a financial contract. Such a contract specifies a capacity \( q \geq 0 \) and an investment \( I \geq 0 \) by the investor against a financial claim, that is, the promise of a repayment contingent on variables that are contractible, in the sense that they can be set in an enforceable contract. Assumptions about fundamentals ensure that claims satisfy three conditions.

First, we assume that all variables (e.g., capacity \( q \), funding \( I \), cost \( c \), revenue \( P_c,q \), etc.) are contractible except for effort \( e \) and demand \( D \). For instance, costs and revenues can typically be audited. Effort represents softer inputs, like whether the firm’s best people are allocated to the project, or the intensity of a sales effort. As for demand, with revenues being contractible and given they map one-to-one with demand when demand is below capacity, demand’s noncontractibility boils down to only unmet demand being noncontractible. This assumption seems reasonable as unfilled demands are usually difficult to observe. Here, this implies that financial contracts cannot distinguish between realizations of demand \( D \) above capacity \( q \) as they all yield the same revenue \( r q \). Hence, financial contracts specify a capacity, an amount of funds contributed by the investor, and a repayment function \( R(\cdot) \): \( [sq, r q] \mapsto \mathbb{R} \) such that given payoff \( p \in [sq, r q] \), the repayment to the investor is \( R(p) \), leaving the firm with net payoff \( p - R(p) \).

Second, we assume the firm to be protected against funds falling below zero. Hence, claims must satisfy limited liability; that is, repayments cannot exceed revenues: for all \( p \in [sq, r q] \), \( R(p) \leq p \).

Last, we assume the firm to be able to report artificially inflated revenues by secretly raising funds from a third party. The firm may opportunistically do so if the financial claim specifies a lower repayment for higher (reported) revenues. To avoid such manipulation, financial claims must be monotonic, that is, \( R(\cdot) \) nondecreasing over \( [sq, r q] \). This assumption is standard in finance.\(^6\)

All claims satisfying these conditions are deemed feasible. Note that all repayments \( R(p) \) need not be positive; that is, a claim can call the investor to pay the firm for some payoff realizations. Note also that borrowing is feasible, as a debt contract with face value \( K \) maps into \( R(p) = p \land K \), which satisfies limited liability and monotonicity.\(^7\) Equity is feasible too, as a fraction \( \alpha \) of equity maps into \( R(p) = \alpha p \). Other claims (e.g., convertible debt, put and call warrants, etc.) are also feasible, as are some combinations of claims (e.g., debt plus equity, etc.).

Importantly, the firm optimizes over the entire set of feasible financial contracts derived from assumptions about fundamentals, that is, preferences, physical constraints, and contractibility.

3.2. The Firm’s Problem

The firm’s problem is to choose capacity \( q \geq 0 \), the funds \( I \) raised from the investor, and the repayments function \( R(\cdot) \) to maximize its expected payoff,
\[
\max_{q,I,R(\cdot)} \{ \mathbb{E}[E[P_{c,q} - R(P_{c,q})] + W + I - cq - \kappa] \}.
\]
(5)

These choices are subject to constraints. First, the investor should accept the contract. Given risk neutrality and no discounting, this requires that its expected payoff be no less than its investment. Hence, the following investor participation constraint must hold:
\[
\mathbb{E}[R(P_{c,q})] \geq I.
\]
(6)

Similarly, the firm should also accept the contract, which requires that its expected payoff be no less than its payoff under the status quo when the project is abandoned; that is, the following firm participation constraint must hold:
\[
\text{if } q > 0, \quad \mathbb{E}[P_{c,q} - R(P_{c,q})] + W + I - cq - \kappa_e \geq W.
\]
(7)

Second, the firm’s cash and the funds raised should cover the capacity cost; that is, the following funding constraint must hold:
\[
I + W \geq cq.
\]
(8)

Third, the financial claim must be feasible; that is, the following limited liability and monotonicity constraints must hold:
\[
\forall (p,p') \in [sq, r q]^2 \text{ with } p \geq p', \quad R(p) \leq p, \quad \text{and } R(p') \leq R(p) \text{.}
\]
(9)

Finally, effort being noncontractible, it cannot be set by contract. Instead, the firm chooses its level based on postfinancing incentives. Therefore, unless the project is abandoned, the firm must prefer exerting the assumed effort \( e \) rather than the alternative effort \( 1 - e \); that is, the following incentive compatibility constraint must hold:
\[
\text{if } q > 0, \quad \mathbb{E}[P_{c,q} - R(P_{c,q})] + W + I - cq - \kappa_e
\]
\[
\geq \mathbb{E}[P_{(1-e),q} - R(P_{(1-e),q})] + W + I - cq - \kappa_{(1-e)}. \]
(10)
The previous problem can be simplified. First, note that $I$, the amount raised from the investor, increases the firm’s objective (5) and is bounded from above only by the investor’s participation constraint (6), which must thus be binding, that is,

$$E[R(P_{e,q})] = I,$$

(11)

an expression we can use to eliminate $I$ from the objective and the constraints.

Second, using definition (2) and eliminating constant $W$, objective (3) can be rewritten as

$$\max_{q,R(\cdot)} \pi_e(q) - \kappa_e.$$  

(12)

Hence, the firm maximizes the project’s expected profit net of effort cost. This simply reflects the fact that, the investor being competitive, the firm captures the project’s full value.

After similar simplifications, the firm’s participation constraint (7) can be written as

$$\begin{cases} q > 0, & \pi_e(q) - \kappa_e \geq 0, \quad (13) \\
\end{cases}$$

which, given condition (4), cannot hold for $e = 0$ and holds for $e = 1$ only for $q \in [\bar{q}_1, \bar{q}_1]$ with

$$\bar{q}_1 \equiv \min\{q \text{ s.t. } \pi_e(q) - \kappa_1 \geq 0\} \text{ and } \bar{q}_1 \equiv \max\{q \text{ s.t. } \pi_e(q) - \kappa_1 \geq 0\}. \quad (14)$$

Note that $q_{e FB} \in (\bar{q}_1, \bar{q}_1)$. This implies that optimization can be restricted to the following: either the firm sets up capacity $q \in [q_1, \bar{q}_1]$ and exerts $e = 1$ or the firm abandons the project ($q = 0$). In other words, incentive compatibility condition (10) should in fact state that the firm should exert effort $e = 1$ if $q > 0$.

Overall, the firm’s problem can be written as follows:

$$\max_{q,R(\cdot)} \pi_e(q)$$

(15)

s.t. $q \in [\bar{q}_1, \bar{q}_1]$, $E[R(P_{1,q})] \geq (c q - W)^{+}$, $\forall (p, p') \in [sq, rq]^{2}$ with $p \geq p'$, $R(p) \leq p'$, and $R(p') \leq R(p)$,

$$E[P_{1,q} - R(P_{1,q})] - E[P_{0,q} - R(P_{0,q})] - \Delta \kappa \geq 0, \quad (19)$$

and if the previous problem is not feasible, then the firm abandons the project ($q = 0$). (Note that (16) is the firm’s participation constraint, (17) is the funding constraint, (18) is the constraint that the contract be feasible, and (19) is the incentive compatibility constraint).

4. Frictionless Financing

This section studies the frictionless benchmark. In our model, this means effort is contractible. Modigliani and Miller’s (1958) irrelevance theorem holds and implies that the firm’s need to raise funds and the financial claim employed are immaterial.8

Specifically, since the contract can now also specify the effort level, the firm’s problem is as before but without incentive compatibility constraint (19) and with $e$ now an optimization variable; that is, the firm maximizes its objective $\pi_1(q)$ by choosing $q$, $R(\cdot)$, and $e$. Since repayments $R(\cdot)$ do not appear in objective (15), effort $e$ can be set independently from $R(\cdot)$. This implies that the firm is indifferent between all functions $R(\cdot)$ satisfying the other constraints.

**Lemma 1.** If effort is contractible, for a given capacity $q \in [q_1, \bar{q}_1]$, the firm is indifferent between all financial claims satisfying constraints (17) and (18).

Note that for all $q \in [q_1, \bar{q}_1]$, such financial claims exist. Indeed, consider the claim such that for all $p \in [sq, rq]$, $R(p) = p$, which corresponds to selling the project to the investor.9 Clearly, this claim is feasible, that is, satisfies condition (18). It satisfies condition (17) as, by definition of $R(\cdot)$ and $\bar{q}_1$,

$$E[R(P_{1,q})] = E[P_{1,q}] \geq cq. \quad (20)$$

Hence, all capacities $q \in [q_1, \bar{q}_1]$ being fundable by feasible claims, the problem reduces to the standard newsvendor problem

$$\max_{q \in [0, q_1]} \pi_1(q). \quad (21)$$

**Proposition 1.** If effort is contractible, the firm’s optimal decisions are to exert effort $e = 1$ and set capacity at the corresponding first-best level $q_{e FB}$.

In a sense, Proposition 1 justifies the use in the literature of the standard newsvendor model. Indeed, despite the firm’s need for funds, the analysis boils down to the standard model. Absent financial frictions, the need to raise funds has no impact per se. By the same token, absent financial frictions, capacity distortions caused by funding needs are never robust to a contract change. In contrast, our paper specifies a friction (i.e., moral hazard) and optimizes over feasible claims, and so no contract change can alleviate the deviations from first best derived in the following sections.

5. The Optimality of Debt Financing

Effort is now determined by the firm’s incentives *after* financing, as per incentive compatibility constraint (19): effort’s noncontractibility causes a financial friction, and the Modigliani–Miller theorem no
longer holds. This raises the question of the optimal source of funds for a given capacity $q$ and its influence on the firm’s choice of capacity. To address this problem, we adapt Innes’s (1990) optimal contracting approach.

Specifically, we show that in the newsvendor model, the MLRP extends from demands to payoffs (see Lemma 1 in Online Appendix A). MLRP is important for the incentive compatibility constraint: it implies that, holding expected repayments constant, claims with higher repayments for higher payoffs are more detrimental to incentives for effort (see Lemma 2 in Online Appendix A). A debt claim is appealing because it minimizes repayments for higher payoffs by maximizing repayments for lower payoffs. We show next that for a given capacity, the optimal claim is indeed a debt claim, which we characterize.

**Proposition 2.** For a given $q \in [\bar{q}, \hat{q}]$, if claims satisfying conditions (17)–(19) exist, the firm exerts effort $e = 1$, and an optimal financing of capacity cost $cq$ is to use cash $W$ and fund any shortfall $I(q) = (cq - W)^+$ with debt with face value $K(q)$, the unique solution to

$$K - (r - s) \int_{0}^{(K - sq)/(r - s)^{+}} F_{r}(x) dx = (cq - W)^{+}. \quad (22)$$

- If $W \geq cq$, then $K = 0$ and the firm does not raise funds.
- If $cq > W \geq (c - s)q$, then $K = I(q) \in (0, sq]$, and debt is risk free.
- If $(c - s)q > W$, then $K \in (sq, rq]$, and debt involves default risk.

Hence, $R(p) = p \wedge K(q)$ is an optimal repayment function. Equation (22) corresponds to funding condition (17) and investor’s binding participation constraint (11), that is, $\mathbb{E}[R(P_{i, q})] = (cq - W)^{+}$.

Note that for $q \in [\bar{q}, \hat{q}]$, Equation (22) has a unique solution. Indeed, its left-hand side, equal to $\mathbb{E}[P_{i, q} \wedge K]$, is continuous and strictly increasing in $K$ over $[sq, rq]$. For $K = 0$, $\mathbb{E}[P_{i, q} \wedge K] = 0$, which is less than $(cq - W)^{+}$. For $K = rq$, $\mathbb{E}[P_{i, q} \wedge K] = \mathbb{E}[P_{i, q}]$, which, from condition (4), exceeds $(cq + \kappa_{e})$ for $q \in (0, \hat{q})$. This ensures existence and uniqueness of a solution. The firm pays its debt with face value $K(q)$ in full if demand exceeds

$$d(q) \equiv \left( \frac{K(q) - sq}{r - s} \right)^{+}. \quad (23)$$

Finally, when the firm’s cash suffices to fund the first-best capacity, that is, $W \geq cq_{1}^{FB}$, the claim $R(\cdot) \equiv 0$ (or, as per Proposition 2, $K = 0$) is optimal. Indeed, all constraints hold, and the objective is maximized. With the results of Section 4, this illustrates that limited wealth per se or the noncontractibility of effort per se causes no meaningful departure from the standard newsvendor model. Instead, their combination does, as we now show.

### 6. Optimal Investment Decisions

#### 6.1. Optimal Capacity

We now determine the optimal capacity choice $q^*$ solving problem (15). For a given capacity $q$, it may be that no contract satisfies (17)–(19), which is equivalent to incentive compatibility condition (19) being violated if $R$ is the debt claim with face value $K(q)$. Noting that for $e = 0, 1$,

$$\mathbb{E}[P_{c, q} - P_{c, q} \wedge K(q)] - \kappa_{e},$$

$$= (rq - K(q)) - (r - s) \int_{d(q)}^{q} F_{r}(x) dx - \kappa_{e}, \quad (24)$$

can we rewrite incentive compatibility constraint (19) as

$$L(q, \Delta \kappa) \equiv \int_{d(q)}^{q} (F_{0}(x) - F_{1}(x)) dx - \frac{\Delta \kappa}{r - s} \geq 0. \quad (25)$$

Recall that face value $K(q)$ affects the choice of effort via the default threshold, $d(q)$. Note that $L(q, \Delta \kappa)$ is decreasing in $\Delta \kappa$: the larger the extra effort cost is, the more tempting shirking is. However, it is not clear how $L(q, \Delta \kappa)$ varies with $q$. For $q_{1}^{\text{max}}$, the capacity $q \in [\bar{q}, \hat{q}]$ for which the firm’s incentive to work is strongest, the following holds.

**Lemma 2.** Defining $q_{1}^{\text{max}} \equiv \arg \max_{q \in [\bar{q}, \hat{q}]} L(\cdot, \Delta \kappa)$, we have $q_{1}^{\text{max}} > q_{1}^{FB}$.

This key result means that increasing capacity beyond the first-best level can boost incentives, that is, relax incentive compatibility constraint (25). (In Section 6.3, we trace this to two mechanisms generated by a capacity increase: a bonus effect and a demand differentiation effect.)

Defining,

$$S(\Delta \kappa) \equiv \{ q \in [q_{1}^{FB}, q_{1}^{\text{max}}) \text{ s.t. } L(q, \Delta \kappa) \geq 0 \}, \quad (26)$$

the firm’s problem boils down to the following. If $S(\Delta \kappa) = \emptyset$, the optimal capacity choice is $q^* = \inf S(\Delta \kappa)$. Indeed, objective (15) is strictly decreasing for $q \geq q_{1}^{FB}$. If instead $S(\Delta \kappa) = \emptyset$, the firm cannot fund a capacity $q \in [q_{1}^{FB}, q_{1}^{\text{max}}]$ and retain incentives to exert $e = 1$. In that case, condition (4) implies it optimally abandons the project ($q^* = 0$). This leads to our main result.

**Theorem 1.** The firm’s optimal choice of capacity $q^*$ and effort level $e^*$ solutions to problem (15) are as follows. If the firm undertakes the project ($q^* > 0$), then it exerts effort $e^* = 1$.

- If $W \geq cq_{1}^{FB}$, the first-best capacity $q^* = q_{1}^{FB}$ is funded with cash.
- If $cq_{1}^{FB} > W \geq (c - s)q_{1}^{FB}$, the first-best capacity $q^* = q_{1}^{FB}$ is funded with cash and riskless debt.
- If $(c - s)q_{1}^{FB} > W$, two thresholds, $\Delta \kappa$ and $\Delta \kappa$, exist such that we have the following:
Figure 1. (Color online) Optimal Capacity $q^*$ as a Function of the Moral Hazard Problem’s Severity $\Delta\kappa$

Note. $W = 0$, $r = 11$, $c = 10$, $s = 0$, $D_1 \sim \text{Gamma}(10, 10)$, and $D_0 \sim \text{Gamma}(8, 10)$.

(i) If $0 \leq \Delta\kappa \leq \Delta\kappa_1$, the first-best capacity $q^* = q_{1FB}^*$ is funded with cash and risky debt.

(ii) If $\Delta\kappa < \Delta\kappa \leq \Delta\kappa_1$, a capacity strictly above the first-best level ($q^* > q_{1FB}^*$) is funded with cash and risky debt. Furthermore, $q^*$ increases monotonically with $\Delta\kappa$.

(iii) If $\Delta\kappa > \Delta\kappa_1$, the firm abandons the project ($q^* = 0$).

Figure 1 displays differential effort cost $\Delta\kappa$’s impact on optimal capacity $q^*$ for $W = 0$, $r = 11$, $c = 10$, $s = 0$, and demands $D_1$ and $D_0$ following gamma distributions with shape and scale parameters $(10, 10)$ and $(8, 10)$. Theorem 1 applies, as gamma distributions satisfy the MLRP with respect to the shape parameter, holding the scale parameter constant. As the figure shows, the optimal capacity equals the first-best capacity $q_{1FB}^*$ for $\Delta\kappa \leq \Delta\kappa_1 = 8.98$. It is then strictly increasing above $q_{1FB}^*$, until $\Delta\kappa = \Delta\kappa_1 = 12.75$, beyond which the project is abandoned.

The intuition is as follows. If possible, the firm sets up first-best capacity $q_{1FB}^*$ and works ($e = 1$) to maximize objective (15). With enough cash ($W \geq (c - s)q_{1FB}^*$), it can fund $q_{1FB}^*$ with cash and risk-free debt, thus maintaining its incentives to work. Say cash now is lower. For $\Delta\kappa = 0$, incentive compatibility constraint (25) holds for any $q$ and so the firm sets up $q_{1FB}^*$. As $\Delta\kappa$ increases, the constraint tightens until it binds, which occurs for threshold $\Delta\kappa_1$, but the firm sticks to $q_{1FB}^*$. Beyond that point, the constraint is violated for $q_{1FB}^*$ but holds for some capacities in $(q_{1FB}^*, q_{1max}^*)$. Objective (15) being strictly decreasing in $q$ over that interval, the firm will set up the smallest such capacity; that is, $q^* = \inf S(\Delta\kappa)$. As $\Delta\kappa$ increases, the firm must increase capacity $q$ further beyond $q_{1FB}^*$ to retain incentives; that is, $q^*$ increases with $\Delta\kappa$. Finally, for $\Delta\kappa$, constraint (25) is binding for $q = q_{1max}^*$. Beyond that threshold, it cannot be satisfied, and the firm must abandon the project.

6.2. Managerial Implications

As per Theorem 1, the firm sets up the first-best capacity if financial frictions are mild ($\Delta\kappa$ small) or absent ($\Delta\kappa = 0$).\footnote{de Véricourt and Gronb: Financing Capacity Investment} Thus, the first implication of our analysis is that financing needs per se (i.e., $W < cq_{1FB}^*$) do not require deviating from efficient capacity investment. Hence, before distorting operational decisions for financial reasons, managers may want to challenge the financial policy and, more generally, check whether adjustments to financial contracts can avoid distortions.

When financial frictions are stronger, however, distortions from the efficient capacity are needed. The second implication is that, when this happens, overinvestment is optimal.

Corollary 1. If the firm runs the project ($q^* > 0$), the installed capacity level $q^*$ is at least as high as first best capacity ($q^* \geq q_{1FB}^*$) with $q^* > q_{1FB}^*$ if and only if $W < (c - s)q_{1FB}^*$ and $\Delta\kappa > \Delta\kappa_1$.

Furthermore, the level of overinvestment becomes larger as the financial frictions worsen.

Corollary 2. If the firm runs the project ($q^* > 0$), the installed capacity level $q^*$ is nondecreasing in $\Delta\kappa$.

This suggests that overinvestment should arise, and be more important, where incentive provision is more of a concern, for example, when human capital is a relatively more important input, when auditing and benchmarking are more difficult, when the legal protection of investors is weaker, etc.

Third, our analysis has implications for the role of cash in capacity choices. Indeed, provided the project is undertaken, capacity is nonincreasing in cash. This contrasts with standard finance intuition and stems from frictions being milder at larger capacity levels.

Proposition 3. Two nonnegative thresholds, $W$ and $\bar{W}$, exist such that

(i) if $W < W$, the project is abandoned ($q^* = 0$);

(ii) if $W \in [W, \bar{W})$, capacity exceeds the first best ($q^* > q_{1FB}^*$) and decreases with cash $W$;

(iii) if $W \geq \bar{W}$, the first-best capacity is set up ($q^* = q_{1FB}^*$).

For low cash levels, the firm abandons the project, as it cannot both raise enough funds and maintain incentives. For high cash levels, the firm can do both but at the cost of setting capacity above the efficient level. As cash increases within that range, the need for external funds drops, as does the debt’s face value, which also relaxes the incentive compatibility constraint, and capacity can be lowered. The firm sets capacity at the first-best level only when cash is large enough.

Figure 2 depicts the impact of cash $W$ on optimal capacity $q^*$ for the settings of Figure 1 with $\Delta\kappa = 13.9$.\footnote{de Véricourt and Gronb: Financing Capacity Investment}
The first one, which we call the bonus effect, stems from the monotonicity condition for financial claims. To see this, note that when capacity increases from \( q_1^{FB} \) to \( q > q_1^{FB} \), more demands can be met, which increases the firm’s payoff. This amounts to the firm receiving a bonus (equal to \( E[P_{1,q} - P_{1,q_1^{FB}} | D_1 \geq q_1^{FB}] \)) if demand exceeds first-best capacity. Demand being more likely to exceed first-best capacity when the firm exerts effort, this bonus boosts the firm’s incentive do so.

Note that in typical (e.g., salesforce) contracting problems, the incentive contract would simply pay the agent a bonus if realized demand exceeded first-best capacity, with no need for overinvesting. Crucially, however, such a bonus scheme is impossible with financial claims. Indeed, \( R(\cdot) \) would clearly decrease by the bonus amount at \( p = r q_1^{FB} \), violating monotonicity, a key condition in finance. Instead, only investing can implement a bonus and generate the corresponding incentives.

Overinvesting has another incentive effect due to the noncontractibility of unmet demands. A contract motivates the firm by rewarding it differently for different demand realizations. This requires the ability to differentiate between these realizations, which is impossible when demands exceed capacity, as unmet demands are not contractible. However, a capacity increase from \( q_1^{FB} \) to \( q > q_1^{FB} \) makes demand realizations in \( (q_1^{FB}, q] \) contractible. This demand differentiation effect allows financial claims to specify different repayments across these realizations. The claim can then reward the firm for higher demand realizations, which boosts incentives because of the MLRP.

Note, finally, that at the first-best capacity level, constraint (25) is relaxed only by the bonus effect, as the demand differentiation effect is second order and hence negligible. (Online Appendix B.4 establishes this and demonstrates formally the existence of these effects.)

7. Strategic Investor
So far, we have considered a competitive investor, as is standard in finance. Yet, financiers sometimes enjoy substantial market power over fund-seeking firms. This may stem from privileged information vis-à-vis rival financiers, or other entry barriers. In this section, we extend the model to the case of a monopolist investor and show that the overinvestment result still holds.

This setup is also consistent with the operations-and-finance literature, which often considers a monopolist financier as a Stackelberg leader proposing financial terms to which the borrowing firm responds by choosing a loan size (and possibly other project characteristics).
7.1. The Model

In the game’s first stage, the investor, the Stackelberg leader, chooses size-contingent financing terms, that is, a repayment function $R(\cdot, \cdot, \cdot)$, such that if the firm chooses to raise an amount $I$ from the investor and installs capacity $q$, and if revenue $p$ realizes subsequently, the firm must repay $R(I, q, p)$, where $R(I, q, \cdot)$ is a feasible claim.

In the game’s second stage, the firm, the Stackelberg follower, reacts to the offered repayment terms $R(\cdot, \cdot, \cdot)$ by choosing whether to undertake or abandon the project. If it undertakes the project, it also chooses the funding amount $I$, the capacity $q$, and its effort level $e \in (0, 1)$. For brevity, we assume the firm has no cash ($W = 0$). (The case $W > 0$ is an easy extension.)

In the last stage, both players receive their payoffs: demand is drawn from distribution $f_\epsilon$, revenue is realized, and the firm makes its repayment $R(I, q, p)$ to the investor.

Thus, in the second stage, given the investor’s choice of $R(\cdot, \cdot, \cdot)$, the firm chooses funding amount $I$, capacity $q$, and effort $e$ to maximize its expected payoff:

$$
\max_{I, q, e} \left\{ E[P_{e, q} - R(I, q, P_{e, q})] + I - c q - \kappa_{\epsilon}(q > 0) \right\}
$$

(27)

(where $\mathbb{1}(\cdot)$ is the indicator function) subject to the funding constraint

$$
I \geq c q. \quad (28)
$$

In the first stage, the investor chooses a schedule of feasible claims $R(\cdot, \cdot, \cdot)$ to maximize his expected payoff:

$$
\max_{R(\cdot, \cdot, \cdot)} E[R(I, q, P_{e, q})] - I, \quad (29)
$$

where $q$, $I$, and $e$ are the solution to problem (27).\(^{15}\)

We show next that our main result continues to hold in this setup.

**Theorem 2.** In the Stackelberg game above, given $\kappa_0$, two thresholds, $\Delta \kappa$ and $\Delta \bar{\kappa}$, exist with $0 < \Delta \kappa \leq \Delta \bar{\kappa} \leq \kappa_0^{\text{max}} = \pi_1(q_{1h}^F) - \kappa_0$ such that in any Nash equilibrium outcome, the following hold:

- For $\Delta \kappa < \Delta \bar{\kappa}$, the first best obtains: $q^* = q_{1h}^F$ and $e^* = 1$.
- For $\Delta \kappa \in (\Delta \bar{\kappa}, \Delta \bar{\kappa}^*)$, the optimal capacity exceeds the first-best level: $q^* > q_{1h}^F$ and $e^* = 1$.
- For $\Delta \kappa > \Delta \bar{\kappa}^*$, the project is abandoned: $q^* = 0$.

Moreover, whenever the project is undertaken in equilibrium (i.e., $q^* > 0$), an equilibrium exists in which the financial claim $R(I^*, q^*, \cdot)$ is debt.

The intuition is as follows. For $\Delta \kappa$ low enough, the rent needed to induce the firm to exert $e = 1$ for $q = q_{1h}^F$ is below its reservation utility. Hence, there is no cost for the investor to leave the rent to the firm and “top it up” with a transfer $I > c q_{1h}^F$ to ensure the firm’s participation.

For larger values of $\Delta \kappa$, the rent exceeds the firm’s reservation utility and is thus costly for the investor. The investor can cut it by increasing capacity above the first-best level. The cost of doing so is to lower the project’s total surplus. However, at $q = q_{1h}^F$, that cost is second order and it is therefore optimal for the investor to increase capacity.

For $\Delta \kappa$ large enough, for any capacity level, the rent exceeds the project’s value. Hence, inducing effort is incompatible with the investor’s participation constraint, and the project is abandoned.

In short, Theorem 2 shows that our overinvestment result still holds for a monopolist investor. It also establishes the optimality of debt in a Stackelberg game where the investor is the leader, thus extending Innes (1990) to a setup more typical of the operations-and-finance literature.

7.2. Implications

Theorem 2 sheds light onto the role of monopoly power in capacity financing problems.

**Corollary 3.** Absent financial frictions (i.e., $\Delta \kappa = 0$), the first best obtains: $q^* = q_{1h}^F$.

Thus, the result implies that, considering optimal contracting, monopoly power per se is not a financial friction and is thus irrelevant for financing and investment decisions. This result generalizes Dada and Hu’s (2008) Proposition 4, which states that in the Stackelberg game where the investor can set loan-size-contingent interest rates, the first best obtains. Indeed, in their model, there is no effort problem, which amounts to $\Delta \kappa = 0$ in ours. In fact, this result derives more fundamentally from the Coase theorem (Coase 1960, Posner 1993) stating that frictionless negotiation by strategic agents leads to the efficient outcome via compensating transfers among agents.

A second implication of our analysis is that, combined with frictions ($\Delta \kappa > 0$), monopoly power leads the investor to be more conservative than in the competitive case when deciding whether to undertake the project. If it funds the project, however, the installed capacity is larger than in the competitive case. Specifically, denote by $q_{1c}^*$ and $q_{1c}$ the optimal installed capacity levels corresponding to the competitive and strategic cases, given by Theorems 1 and 2, respectively.

**Proposition 5.** If $q_{1c}^* > 0$, then $q_{1c}^* > 0$ and $q_{1c}^* \geq q_{1c}^F$.

The reason is that incentive compatibility may require that the firm derive a rent, that is, a payoff above its reservation utility. A monopolist investor does not internalize that rent in its decisions. Hence, it may be willing to increase $q$ not only to ensure incentive compatibility, but also to reduce the firm’s rent (hence $q_{1c}^* \geq q_{1c}^F$). By the same token, its decision to abandon the project does not account for the loss of rent this implies for the firm (hence $q_{1c}^* > 0 \Rightarrow q_{1c}^F > 0$).
8. Conclusion
This paper follows an optimal contracting approach to study how a firm's capacity choice under demand uncertainty interacts with its decision of how to finance that capacity. In that approach, the firm optimizes over a set of feasible sources of funds derived endogenously from assumptions about fundamentals. This contrasts with the existing literature on financing capacity, which assumes exogenously restricted sets of feasible claims. Such restrictions can lead one to conclude capacity should be distorted away from the first best, when in fact need would disappear at no cost with a simple contract change. Ours is the first paper to propose results for the problem of financing capacity under demand uncertainty that are robust in this sense.

Overall, our work suggests that before deviating from efficient capacity investment, operations managers should challenge the financial policy and, more generally, check whether adjustments to financial contracts can avoid these distortions. When no contract adjustment can eliminate the problem, overinvestment, rather than underinvestment, may be optimal.

Our approach yields results that run counter to general finance intuition. Besides our overinvestment result, we find that the firm increases its investment when it is low in cash, and can decrease its investment if the project’s unit revenue is high. This further demonstrates that operations do matter in finance problems.

Our model can be extended in many directions. For instance, other types of moral hazard are possible: effort may affect the quality or yield of installed capacity. In this case, we conjecture that the optimal capacity is typically below the efficient level. Indeed, this form of financial friction worsens with the capacity level and thus implies higher costs of external capital at higher capacity levels. (Instead, our bonus and demand differentiation effects cause frictions to milder for higher capacity.)

We have focused on the natural assumption that higher demand realizations are more indicative of high effort. This need not be the case. For instance, effort could reduce the variance of demand. If so, the MLRP would not hold, and equity could be a better funding source than debt.

Finally, we focus on incentive-based frictions, but others exist. In our model, the firm uses its capacity (and financing) choice to commit to exerting effort once the project is underway, and this commitment is costly when it entails overinvestment. This logic points to an alternative model of capacity financing under asymmetric information about the distribution of demand. We conjecture that, as per a similar logic, a firm with a superior demand distribution might overinvest as a costly signal to separate itself from firms with inferior demand distributions.

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Endnotes
1 Dada and Hu (2008), who study a capital-constrained newsvendor problem similar to ours, offer an illustration of these issues. In Section 2 of that paper, a monopolist bank sets a fixed interest rate and the newsvendor chooses how much to borrow. The result (Proposition 2) is underinvestment. Yet, as the model features no financial friction, we know that the result is not robust to a simple contract change and that in fact optimal contracting would restore efficiency. (Recall that under optimal financial contracting, the investor’s monopoly power is not a friction; see Section 7.) And indeed, Dada and Hu’s (2008) Proposition 4 shows that loan-size-contingent interest rates achieve first best.
2 Risk neutrality is standard and important for the optimality of debt financing. No discounting is for simplicity only.
3 The assumption that no cost is incurred if \( q = 0 \) is for simplicity. It amounts to assuming that \( \kappa_0 \) is a fixed cost, which will allow us to focus on cases where the firm exerts \( e = 1 \) or sets \( q = 0 \).
4 Following a demand realization \( d \), an agent would revise his prior \( \nu = \Pr[e = 1] \) to posterior \( \Pr[e = 1 | d]/\Pr[d] = \nu f_f(d)(\nu f_f(d) + (1 - \nu) f_f(d))^{-1} \) which, by the MLRP, is strictly increasing with \( d \).
5 Note that the MLRP is stronger than first-order stochastic dominance and increasing hazard ratios.
6 Inflating revenues amounts to using the funds raised secretly for buying units for \( r \) per unit, thus inflating revenues artificially, before reselling them at their salvage value. An alternative foundation for monotonicity is that the investor could engage in sabotage and reduce the payoff. (Innes 1990 discusses both foundations.) The first foundation does not directly rule out nonmonotonic claims, but the revelation principle implies that nonmonotonic claims are equivalent to monotonic claims; that is, one can restrict to monotonic claims without loss of generaliity. The second foundation implies that nonmonotonic claims are strictly suboptimal, as they imply inefficient sabotage, which monotonic claims avoid.
7 We ignore the distinction between interest and principal payments, which is irrelevant in our model.
8 Xu and Birge’s (2004) Proposition 2.1 also applies the Modigliani–Miller theorem to the newsvendor model.
9 Note that although the firm retains no economic interest in the project, it is compensated by the investor’s investment \( I = \mathbb{E}P_{1-s} \), which exceeds the funding needs \( (cq - W)^+ \).
10 In that case, \( K < sq \) so that \( \tilde{d} = 0 \). This means that (25) is implied by (4).
11 This is enough to maintain the incentive to work as long as (25) holds for \( q = q_1^\alpha \). When \( \Delta \kappa \) exceeds threshold \( \Delta \kappa_0 \), that is, when \( L(q_1^\alpha, \Delta \kappa) < 0 \), the firm must raise \( q \) above \( q_1^\alpha \) to relax constraint (25), that is, increase \( L(q, \Delta \kappa) \).
12 This standard effect also implies that in incentive-based frictions where the newsvendor chooses how much to borrow, the result (Proposition 2) is underinvestment. Yet, as the model features no financial friction, we know that the result is not robust to a simple contract change and that in fact optimal contracting would restore efficiency. (Recall that under optimal financial contracting, the investor’s monopoly power is not a friction; see Section 7.) And indeed, Dada and Hu’s (2008) Proposition 4 shows that loan-size-contingent interest rates achieve first best.
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21 In that case, \( K < sq \) so that \( \tilde{d} = 0 \). This means that (25) is implied by (4).
22 This is enough to maintain the incentive to work as long as (25) holds for \( q = q_1^\alpha \). When \( \Delta \kappa \) exceeds threshold \( \Delta \kappa_0 \), that is, when \( L(q_1^\alpha, \Delta \kappa) < 0 \), the firm must raise \( q \) above \( q_1^\alpha \) to relax constraint (25), that is, increase \( L(q, \Delta \kappa) \).
23 This standard effect also implies that in incentive-based frictions where the newsvendor chooses how much to borrow, the result (Proposition 2) is underinvestment. Yet, as the model features no financial friction, we know that the result is not robust to a simple contract change and that in fact optimal contracting would restore efficiency. (Recall that under optimal financial contracting, the investor’s monopoly power is not a friction; see Section 7.) And indeed, Dada and Hu’s (2008) Proposition 4 shows that loan-size-contingent interest rates achieve first best.
monotonicity condition. This key difference implies that instances exist in which overinvestment arises in our model but not in a salesforce model. For illustration, say \( f_1/f_0 \) is strictly increasing over \([0, q_{FB}^1]\) but constant for \( q \geq q_{FB}^1 \). In that case, overinvestment arises in our model but there is no (reason for) overinvesting in a salesforce model (because demand realizations beyond first best are not more indicative of effort).

\[14\] See also some operations-and-finance papers, for example, Xu and Birge (2004).

\[15\] Thus, our model is a Stackelberg game. In the first stage, the investor strategically chooses its action, taking into account that the firm will choose its best response to it. To see this, note that in its problem (29), the investor takes into account decisions \((q, I, e)\) determined by the firm’s problem (27). This game structure is identical to that of Dada and Hu (2008), only the action sets are different. (The firm’s action set is different because we introduce effort for the firm, and the investor’s action set is different because we do not a priori restrict the investor to debt contracts with fixed interest rates.)

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