

The Dynamics of Financially Constrained Arbitrage

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ABSTRACT

We develop a model in which financially constrained arbitrageurs exploit price discrepancies across segmented markets. We show that the dynamics of arbitrage capital are self-correcting: following a shock that depletes capital, returns increase, which allows capital to be gradually replenished. Spreads increase more for trades with volatile fundamentals or more time to convergence. Arbitrageurs cut their positions more in those trades, except when volatility concerns the hedgeable component. Financial constraints yield a positive cross-sectional relationship between spreads/returns and betas with respect to arbitrage capital. Diversification of arbitrageurs across markets induces contagion, but generally lowers arbitrageurs' risk and price volatility.

THE ASSUMPTION OF FRICTIONLESS ARBITRAGE is central to finance theory and all of its practical applications. It is hard to reconcile, however, with the large body of evidence on so-called market anomalies, in particular those concerning price discrepancies between assets with almost identical payoffs. Such discrepancies arise during both crises and more tranquil times. For example, large and persistent violations of covered interest parity (CIP) have been documented for all major currency pairs, during and after the global financial crisis. Price discrepancies that are hard to reconcile with frictionless arbitrage have also been documented for stocks, government bonds, corporate bonds, and credit default swaps (CDS).¹

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¹References to the empirical literature are provided in Sections I.B.3 and III.C.2. In these sections, we also explain how to map our model and results to the empirical settings.

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To address these anomalies, one approach is to abandon the assumption of frictionless arbitrage and study the constraints faced by real-world arbitrageurs, for example, hedge funds or trading desks in investment banks. Arbitrageurs have limited capital, which can constrain their activity and ultimately affect market liquidity and asset prices. Empirical studies show that various measures of arbitrage capital are related to the magnitude of the anomalies.

Since arbitrage capital can be targeted at multiple anomalies, returns to investing in the anomalies are interdependent and hence so are arbitrageurs' positions. This interdependence raises a number of questions. How should arbitrageurs allocate their limited capital across anomalies, and how should this allocation respond to shocks to capital? Which anomalies' returns are more sensitive to changes in arbitrage capital? How do the expected returns offered by the different anomalies relate to sensitivity to arbitrage capital and other characteristics? How do the expected returns offered by anomalies evolve over time, and how do these dynamics relate to those of arbitrage capital? In this paper, we develop a model to address these questions.

We consider a discrete-time, infinite-horizon economy with a riskless asset and a number of "arbitrage opportunities" (the anomalies within our model) that consist of a pair of risky assets with correlated payoffs. Each risky asset is traded in a different segmented market by risk-averse investors who can trade only that asset and the riskless asset. Investors experience endowment shocks that generate a hedging demand for the risky asset in their market. Shocks are opposites within each pair, so a positive hedging demand for one asset in the pair is associated with a negative hedging demand of equal magnitude for the other. This simplifying assumption ensures that arbitrageurs trade only on the price discrepancy between the two assets. Market segmentation is exogenous in our model, but could arise because of regulation, agency problems, or a lack of specialized knowledge.

We make two key assumptions. First, we assume that, unlike other investors, arbitrageurs can trade all assets. They therefore have better opportunities than other investors. By exploiting price discrepancies between paired assets, arbitrageurs intermediate trade between otherwise segmented investors, providing them liquidity: they buy cheap assets from investors with negative hedging demand, and sell expensive assets to investors with positive hedging demand. We refer to the price discrepancies that arbitrageurs seek to exploit as "arbitrage spreads" and use them as an inverse measure of liquidity.

Second, we assume that arbitrageurs are constrained in their access to external capital. We derive their financial constraint following the logic of market segmentation and assuming that they can walk away from their liabilities unless they are backed by collateral. Consider an arbitrageur who wishes to buy an asset and short the other asset in its pair. The arbitrageur could borrow the cash required to buy the former asset, but the loan must be backed by collateral. Posting the asset as collateral would leave the lender exposed to a decline in its value. The arbitrageur could post as additional collateral the short position in the other asset, which can offset declines in the value of the long position. Market segmentation, however, prevents investors other than arbitrageurs from

dealing in multiple risky assets, which implies that the additional collateral must be a riskless asset position. We assume that collateral must be sufficient to protect the lender fully against default. This implies that positions in assets with more volatile payoffs require endogenously more collateral so that lenders are protected against larger losses. The need for collateral limits the positions that an arbitrageur can establish, and that constraint is a function of his wealth. The positions that arbitrageurs can establish as a group are constrained by their aggregate wealth, which we refer to as arbitrage capital.²

When assets in each pair have identical payoffs, arbitrage is riskless. We analyze this case, which is a natural benchmark, first. If spreads are positive, then the riskless return offered by arbitrage opportunities exceeds the riskless rate. Arbitrageurs, however, may not be able to scale up their positions to exploit that return because of their financial constraint. Their optimal policy is to invest in the opportunities that offer maximum return per unit of collateral. Equilibrium is characterized by a cutoff return per unit of collateral: arbitrageurs invest in opportunities above the cutoff, which drives their return down to the cutoff, and do not invest in opportunities below the cutoff. The cutoff is inversely related to arbitrage capital. For example, when capital increases, arbitrageurs become less constrained and can hold larger positions. This drives down the returns of the opportunities they invest in.

The inverse relationship between returns and capital implies self-correcting dynamics and a deterministic steady state. If arbitrage capital is low, then arbitrageurs hold small positions, returns are high, and capital gradually increases. Conversely, if capital is high, then returns are low and capital decreases because of arbitrageurs' consumption. In steady state, arbitrage remains profitable enough to offset the natural depletion of capital due to consumption.

We next analyze the case in which payoffs within each asset pair consist of a component that is identical across the two assets and hedgeable by arbitrageurs and a component that differs across the two assets. Because asset payoffs are not identical, arbitrage is risky. As in the case of riskless arbitrage, arbitrageurs invest in the opportunities that offer maximum return per unit of collateral. Unlike in that case, however, the relevant return is the expected return net of a risk adjustment that depends on arbitrageur risk aversion and position size. The financial constraint binds when the risk-adjusted return exceeds the riskless rate.

To compute the equilibrium under risky arbitrage in closed form, we specialize our analysis to the case in which asset payoffs are near-identical and hence arbitrage risk is small. In the stochastic steady state, the financial constraint always binds and arbitrage capital follows an approximate AR(1) process. Moreover, the first-order effect of arbitrage risk on equilibrium variables operates

² In Internet Appendix Section II, we formulate equilibrium in our model under general collateralized contracts. The Internet Appendix may be found in the online version of this article. We allow, for example, loan repayments to be state-contingent and to extend over several periods. We show that, under some conditions, contracts can be restricted to the ones that we consider in our main model. See Section I.C.2.

through the financial constraint rather than through risk aversion. Indeed, price movements caused by shocks to arbitrage capital represent an additional source of risk for a collateralized position. The required collateral must then increase by an amount proportional to the standard deviation of these movements. On the other hand, the risk adjustment induced by risk aversion is proportional to the variance because it is an expectation of gains and losses weighted by marginal utility.

Using our closed-form solutions, we can determine the cross-section of expected returns and arbitrageur positions. We show that expected returns are high for arbitrage opportunities involving assets with volatile payoffs because these opportunities require more collateral. They are also high for “long-horizon” opportunities, that is, opportunities for which price discrepancies take longer to disappear because endowment shocks have longer duration. Indeed, because spreads for these opportunities are more sensitive to shocks to arbitrageur wealth, the losses that arbitrageurs can incur are larger, implying higher collateral requirements.

The characteristics associated with high expected returns are also associated with high sensitivity of spreads to arbitrage capital, that is, high “arbitrage-capital betas.” Since opportunities with volatile payoffs require more collateral, they must offer higher expected returns. Since, in addition, changes in capital impact the return per unit of collateral, arbitrage-capital betas for the same opportunities are high. In the case of long-horizon opportunities, the causal channel is different: high arbitrage-capital betas result in high collateral requirements, which in turn result in high expected returns. Our results are consistent with the relationship between expected returns or spreads on the one hand and arbitrage-capital betas on the other being increasing in the cross-section, as documented in Avdjiev et al. (2016) in the context of covered interest arbitrage and Cho (2016) in the context of stock market anomalies.

The cross-section of arbitrageur positions differs from that of expected returns. Arbitrageurs hold larger positions in opportunities where the hedgeable component of payoff volatility is larger, but smaller positions in opportunities where the unhedgeable component is larger or where horizon is longer. Intuitively, volatility has two countervailing effects on arbitrageur positions: it lowers them because it raises collateral requirements, but it raises them because it raises investors’ hedging demand and need for intermediation. The effect of each component of volatility on collateral requirements is proportional to its standard deviation, while that on hedging demand is proportional to its variance. The former is larger in the case of small unhedgeable volatility, that is, small arbitrage risk. The effect of volatility on the dynamics of positions parallels that on average positions. Following drops in arbitrage capital, positions in opportunities with higher unhedgeable volatility are cut by more, while positions in opportunities with higher hedgeable volatility are cut by less.

We finally use our model to study how the degree of mobility of arbitrage capital affects market stability: does capital mobility stabilize markets, or does it propagate shocks causing contagion and instability? To do so, we consider the possibility that any given arbitrageur can allocate his wealth to exploit only

one opportunity. That is, arbitrage markets themselves are segmented so that arbitrage capital cannot be reallocated from one opportunity to another. For simplicity, we take opportunities to be symmetric with independent payoffs. If an arbitrageur can diversify across opportunities but others remain undiversified, then the variance of his wealth decreases because spreads are independent. If instead all arbitrageurs can diversify, then spreads become perfectly correlated, as arbitrageurs act as conduits transmitting shocks in one market to all markets—a contagion effect. We show, however, that because collective diversification causes the variance of each spread to decrease, the variance of each arbitrageur's wealth decreases. In fact, collective diversification lowers wealth variance by as much as individual diversification. In that sense, capital mobility stabilizes markets.

Our paper belongs to a growing theoretical literature on the limits of arbitrage, and more precisely to the strand emphasizing arbitrageurs' financial constraints.³ We contribute to this literature by deriving the cross-section and dynamics of arbitrageur returns and positions in a setting in which arbitrageurs exploit price discrepancies between assets with similar payoffs.

Shleifer and Vishny (1997) are the first to derive a two-way relationship between arbitrage capital and asset prices. Gromb and Vayanos (2002) introduce some of our model's building blocks: arbitrageurs intermediate trade across segmented markets, and a collateral-based financial constraint limits the liquidity they can provide. That model assumes a single arbitrage opportunity and finite horizon. These assumptions rule out, respectively, cross-sectional effects and self-correcting dynamics.

Our result that arbitrage opportunities with higher collateral requirements offer higher returns is related to a number of papers. In Geanakoplos (2003), Garleanu and Pedersen (2011), and Brumm et al. (2015), multiple risky assets differ in their collateral value, that is, the amount that agents can borrow using the asset as collateral. Assets with low collateral value must offer higher expected returns, and violations of the law of one price can arise.⁴ These violations, however, are different in nature from those in our model: we assume that both assets in a pair have the same collateral value but differ in investors' hedging demand. Empirical studies confirm that hedging demand (or more generally, demand unrelated to collateral value) is a key driver of arbitrage spreads.⁵

In Brunnermeier and Pedersen (2009), collateral-constrained arbitrageurs invest in assets with maximum return per unit of collateral. Since volatile assets require more collateral, their returns are higher and more sensitive to changes in arbitrage capital. However, in their paper there is no segmentation and the law of one price holds. Moreover, the analysis does not address dynamic issues such as the effect of horizon or the recovery from shocks.

³ For a survey of this literature, see Gromb and Vayanos (2010).

⁴ Detemple and Murthy (1997), Basak and Croitoru (2000, 2006), and Chabakauri (2013) derive related results for more general portfolio constraints.

⁵ See, for example, the literature on covered interest arbitrage, summarized in Section I.B.3.

Our results on self-correcting dynamics are related to several papers. In Duffie and Strulovici (2012), capital recovers following adverse shocks because new capital enters the market. In Xiong (2001), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014), recovery occurs because existing capital grows faster—the same channel as in our model. However, in these papers arbitrageurs invest in a single risky asset. This rules out cross-sectional effects and violations of the law of one price.⁶

Finally, our analysis of integration versus segmentation relates to Wagner (2011), who shows that investors choose not to hold the same diversified portfolio because this exposes them to the risk that they all liquidate at the same time, and to Guembel and Sussman (2015) and Caballero and Simsek (2017), who show that segmentation generally raises volatility and reduces investor welfare. Contagion effects resulting from changes in arbitrageur capital or portfolio constraints are also derived in, for example, Kyle and Xiong (2001) and Pavlova and Rigobon (2008).

The rest of the paper is organized as follows. Section I presents the model. Section II derives the equilibrium when arbitrage is riskless because assets in each pair have identical payoffs. Section III analyzes risky arbitrage and derives the cross-sectional properties of prices and positions, as well as the effects of capital mobility. Section IV concludes. Proofs are in the Internet Appendix.

I. The Model

A. Assets

There is an infinite number of discrete periods indexed by $t \in \mathbb{N}$. There is one riskless asset with exogenous return $r > 0$, and a continuum \mathcal{I} of infinitely lived risky assets, all in zero supply. Risky assets come in pairs. Asset i 's payoff per share in period t is

$$d_{i,t} \equiv \bar{d}_i + \epsilon_{i,t} + \eta_{i,t}, \quad (1)$$

where \bar{d}_i is a positive constant, and $\epsilon_{i,t}$ and $\eta_{i,t}$ are random variables distributed symmetrically around zero in the intervals $[-\bar{\epsilon}_i, \bar{\epsilon}_i]$ and $[-\bar{\eta}_i, \bar{\eta}_i]$, respectively. The other asset in i 's pair is denoted by $-i$ and its payoff per share in period t is

$$d_{-i,t} \equiv \bar{d}_i + \epsilon_{i,t} - \eta_{i,t}. \quad (2)$$

If $\bar{\eta}_i = 0$, then assets i and $-i$ have identical payoffs and a trade consisting of a one-share long position in one asset and a one-share short position in the other

⁶ Kondor and Vayanos (2017) derive self-correcting dynamics in a setting in which arbitrageurs can invest in multiple risky assets. However, arbitrageurs in their setting do not intermediate trades because there is no segmentation, and the law of one price holds. Greenwood, Hanson, and Liao (2015) assume gradual rebalancing of arbitrageur portfolios across markets, in the spirit of Duffie (2010) and Duffie and Strulovici (2012), and allow for multiple risky assets within each market. However, arbitrageurs in their setting face no financial constraints.

involves no risk. If instead $\bar{\eta}_i > 0$, then payoffs are not identical and the long-short trade is risky. In both cases, we refer to asset pair $(i, -i)$ as an arbitrage opportunity. This corresponds to textbook arbitrage when $\bar{\eta}_i = 0$ and the two assets trade at different prices.

We assume that the variables $\frac{\epsilon_{i,t}}{\bar{\epsilon}_i}$ are i.i.d. over time and identically distributed across asset pairs. (Correlation across pairs is possible.) We make the same assumption for the variables $\frac{\eta_{i,t}}{\bar{\eta}_i}$, which we also assume are independent of $\frac{\epsilon_{i,t}}{\bar{\epsilon}_i}$. Because distributions are identical across asset pairs, $\bar{\epsilon}_i$ and $\bar{\eta}_i$ are proportional to the standard deviations of $\epsilon_{i,t}$ and $\eta_{i,t}$, respectively, and we refer to them as volatilities. We restrict \bar{d}_i to be larger than $\bar{\epsilon}_i + \bar{\eta}_i$ so that asset payoffs are nonnegative. We denote by $p_{i,t}$ the ex-dividend price of asset i in period t , and we define the asset's price discount as

$$\phi_{i,t} \equiv \frac{\bar{d}_i}{r} - p_{i,t}, \tag{3}$$

that is, the present value of expected future payoffs discounted at the riskless rate r , minus the price.

B. Outside Investors

B.1. Market Segmentation

For some agents, who we refer to as outside investors, the markets for the risky assets are segmented. Each outside investor can invest in only two assets: the riskless asset and one specific risky asset. We refer to the outside investors who can invest in risky asset i as i -investors. We assume that i -investors are competitive and infinitely lived, form a continuum with measure μ_i , consume in each period, and have negative exponential utility

$$- \mathbf{E}_t \left[\sum_{s=t+1}^{\infty} \gamma^{s-t} \exp(-\alpha c_{i,s}) \right], \tag{4}$$

where α is the coefficient of absolute risk aversion and γ is the subjective discount factor. In period t , an i -investor chooses positions $\{y_{i,s}\}_{s \geq t}$ in asset i and consumption $\{c_{i,s}\}_{s \geq t+1}$ to maximize (4) subject to a budget constraint. We denote the investor's wealth in period t by $w_{i,t}$. We study optimization in period t after consumption $c_{i,t}$ has been chosen, which is why we optimize over $c_{i,s}$ for $s \geq t + 1$. Accordingly, we define $w_{i,t}$ as the wealth net of $c_{i,t}$. We assume that i - and $-i$ -investors are identical in terms of their measure, that is, $\mu_i = \mu_{-i}$. Negative exponential utility of outside investors simplifies our analysis because it ensures that their demand for risky assets is independent of their wealth. The only wealth effects in our model concern the arbitrageurs.

B.2. Endowment Shocks

Outside investors receive random endowments, which affect their appetite for risky assets. In period t , each i -investor receives an endowment equal to

$$u_{i,t-1}(\epsilon_{i,t} + \eta_{i,t}), \quad (5)$$

where $u_{i,t-1}$ is known in period $t - 1$. We assume that $u_{i,t}$ is equal to zero, except over a sequence of M_i periods $t \in \{h_i - M_i, \dots, h_i - 1\}$ during which it can become equal to a constant u_i . When this occurs, we say that i -investors experience an endowment shock of intensity u_i and duration M_i .

An endowment shock in market i is accompanied by an endowment shock in market $-i$. If the shocks were identical, then assets i and $-i$ would be trading at the same price in the absence of arbitrageurs because of symmetry. To ensure a difference in prices and hence a role for arbitrageurs, we assume that endowment shocks differ. We further restrict the shocks to be opposites, that is, $u_i = -u_{-i}$. This assumption, together with that of zero supply, ensures that the price discounts of assets i and $-i$ are opposites in equilibrium. With opposite price discounts, arbitrageurs (described in Section I.C) find it optimal to hold opposite positions in the two assets, and hence trade only on the price discrepancy between them. This simplifies the equilibrium because arbitrageurs are not exposed to the shock $\epsilon_{i,t}$, and thus they earn a riskless return when assets i and $-i$ have identical payoffs.

In exploiting price discrepancies, arbitrageurs intermediate trade between, and provide liquidity to, investors. Suppose, for example, that i -investors experience a shock $u_i > 0$. Their endowment then becomes positively correlated with $\epsilon_{i,t} + \eta_{i,t}$ and hence with asset i 's payoff. As a consequence, asset i becomes less attractive to them. Conversely, asset $-i$ becomes more attractive to $-i$ -investors, who experience a shock $u_{-i} < 0$. In the absence of arbitrageurs, the equilibrium price of asset i would decrease and that of asset $-i$ would increase. Arbitrageurs can exploit this price discrepancy by buying asset i from i -investors and selling asset $-i$ to $-i$ -investors. In doing so, they intermediate trade between the two sets of investors, which market segmentation otherwise prevents. Because of arbitrageurs, prices are less sensitive to endowment shocks and price discrepancies are smaller.⁷

⁷ If endowment shocks for i - and $-i$ -investors were not opposites, then arbitrageurs would not hold opposite positions in assets i and $-i$. They would still intermediate trade between investors, however, if they are sufficiently risk averse. Suppose, for example, that i -investors experience a shock $u_i > 0$ but $-i$ -investors do not, that is, $u_{-i} = 0$. Arbitrageurs would buy asset i from i -investors to benefit from its positive price discount. If they are sufficiently risk-averse, they would hedge that position by selling asset $-i$ to $-i$ -investors, thereby intermediating trade between i - and $-i$ -investors. Because buying asset i yields a higher expected excess return than selling asset $-i$, arbitrageurs would choose not to be fully hedged, and hence would be exposed to the risk that $\epsilon_{i,t}$ is low. If assets i and $-i$ were in positive rather than in zero supply, then arbitrageurs would hold a larger long position in asset i and would be more exposed to the risk that $\epsilon_{i,t}$ is low. If assets without endowment shocks were also in positive supply, then they would trade at a positive price discount but one that would be smaller than asset i 's. Because positions in the no-endowment-shock assets require a comparable level of collateral as in assets i and $-i$ but earn

When investors i and $-i$ experience endowment shocks, we say that arbitrage opportunity $(i, -i)$ is active. We identify active opportunities with the assets with positive endowment shocks: we set

$$\mathcal{A}_t \equiv \{i \in \mathcal{I} : u_{i,t} > 0\}$$

and refer to active opportunity $(i, -i)$ for $i \in \mathcal{A}_t$ as opportunity i . We assume that the set \mathcal{A}_t of active opportunities is finite. We also assume that the probability of an opportunity becoming active (an event that may occur in period $h_i - M_i$ for opportunity i) is arbitrarily small. This is consistent with opportunities forming a continuum and a finite number being active in each period. This also ensures that endowment shocks do not affect prices until they actually hit investors.

B.3. Interpretation

Our assumptions fit settings in which assets with similar payoffs trade in partially segmented markets. These include Siamese twin stocks, covered interest arbitrage across currencies, government bonds, corporate bonds, and CDS.

Siamese twin stocks have identical dividend streams but differ in the country where most of their trading occurs. Rosenthal and Young (1990) find that price differences between Siamese twins can be significant. Dabora and Froot (1999) show that a stock tends to appreciate relative to its Siamese twin when the aggregate stock market in the country where the stock is mostly traded goes up. They argue that one reason Siamese twin stocks differ in their main trading venue is that each stock belongs to a different country's main stock index. Thus, index funds in each country can invest only in one of the stocks. Index funds in that setting correspond to our model's outside investors, flows in or out of these funds correspond to our model's endowment shocks, and market segmentation arises from restricted fund mandates (which are possibly a response to agency problems).

Covered interest arbitrage exploits violations of CIP, the relationship implied by the absence of arbitrage between the spot and forward exchange rates for a currency pair and the interest rates on the two currencies. Violations of CIP can be measured by the cross-currency basis (CCB). Taking one of the currencies to be the dollar, the CCB is the difference between the dollar interest rate minus its CIP-implied value. A negative CCB indicates that the dollar is cheaper in the forward market than its CIP-implied value.

Violations of CIP were small between 2000 and the beginning of the global financial crisis, but were large during the crisis and have remained large since. Explanations of CIP violations during the crisis have focused on increased

a lower expected return, arbitrageurs would not trade those assets if their wealth were small enough.

counterparty risk and difficulty borrowing in dollars.⁸ These factors subsided after the crisis, however. Explanations of CIP violations after the crisis have focused instead on hedging pressure in the forward market combined with financially constrained arbitrage. Borio et al. (2016) construct measures of the hedging demand of banks, institutional investors (such as pension funds and insurance companies), and nonfinancial firms. Consistent with the hedging pressure explanation, they find that a negative CCB is more likely when these institutions seek to hedge against a drop in the dollar. Du, Tepper, and Verdellhan (2016) argue that the demand for hedging against a drop in the dollar should be high for currencies with low interest rates relative to the dollar, and find that a negative CCB is indeed more likely for those currencies. They also relate the CCB to measures of arbitrageurs' financial constraints.⁹

Our model can be applied to covered interest arbitrage by interpreting the two assets in a pair as a currency forward and its synthetic counterpart. Outside investors in the forward market are the hedgers that Borio et al. (2016) consider: these agents may lack the specialized knowledge or trading infrastructure to access synthetic forwards. Likewise, outside investors in the synthetic forward market may be prevented from trading forwards because of restricted mandates.¹⁰

Bonds with similar coupon rates and times to maturity can trade at significantly different yields. Fontaine and Garcia (2012) and Hu, Pan, and Wang (2013) aggregate such deviations across the nominal term structure by fitting it to a smooth curve. They find that the fit worsens when arbitrageurs' financial constraints tighten, for example, during financial crises or when the leverage of shadow banks decreases. In that context, outside investors can represent investors who must hold bonds with specific coupon rates and times to maturity. Such investors might be insurance companies or pension funds, and their preferences could be driven by asset-liability management or tax considerations.

Fleckenstein, Longstaff, and Lustig (2014) find that nominal government bonds tend to be significantly more expensive than their synthetic counterparts formed by inflation-indexed bonds, inflation swaps, and zero-coupon bonds. Moreover, price discrepancies become larger when arbitrage capital, measured by hedge-fund assets, is depleted. The additional finding that nominal and inflation-indexed bonds are owned by different types of institutions suggests a degree of market segmentation.

⁸ See, for example, Baba and Packer (2009), Coffey, Hrung, and Sarkar (2009), and Mancini Grifoli and Rinaldo (2012).

⁹ Other related work on CIP violations after the crisis includes Avdjiev et al. (2016), Iida, Kimura, and Sudo (2016), Liao (2016), and Sushko et al. (2016).

¹⁰ Consider, for example, U.S. nonfinancial firms that issue debt in euros to benefit from lower credit spreads in the euro area relative to the United States (Borio et al. (2016)). Those firms seek to hedge against a drop in the dollar as they earn most of their profits in dollars but must pay euro-denominated debt. They can hedge in the forward market, but trading synthetic forwards may be too complicated or impossible for them—in particular, they would have to borrow dollars without paying a credit spread. Conversely, bond mutual funds can invest in euro- or dollar-denominated bonds but may be prevented by their mandates from trading currency forwards. Liao (2016) links CIP violations to the hedging demand of nonfinancial firms using a segmented-markets model.

Duffie (2010) documents price discrepancies between corporate bonds and matched CDS. These discrepancies became particularly large during the global financial crisis, but remained significant afterward. One driver of market segmentation in that setting is that individual investors can trade corporate bonds but not CDS.

C. Arbitrageurs

C.1. Better Investment Opportunities

Arbitrageurs can invest in all risky assets and in the riskless asset. Hence, they are the only agents who can overcome market segmentation. We assume that they are competitive and infinitely lived, form a continuum with measure one, consume in each period, and have logarithmic utility

$$\mathbf{E}_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \log(c_s) \right], \quad (6)$$

where β is the subjective discount factor.¹¹ In period t , an arbitrageur chooses positions $\{x_{i,s}\}_{i \in \mathcal{I}, s \geq t}$ in all risky assets and consumption $\{c_s\}_{s \geq t+1}$ to maximize (6). The arbitrageur is subject to a financial constraint (see Section I.C.2) and a budget constraint. We denote the arbitrageur's wealth in period t by W_t and assume that $W_0 > 0$. Since arbitrageurs have measure one, W_t is also their aggregate wealth, which we refer to as arbitrage capital. Logarithmic utility of arbitrageurs simplifies our analysis because it ensures that their consumption is a constant fraction of their wealth.

C.2. Financial Constraint

We assume that agents must collateralize their asset positions. Consider an agent who wants to establish a long position in a risky asset. If the agent needs to borrow cash to buy the asset, then he must post collateral as a commitment to repay the cash loan. Consider next an agent who wants to establish a short position in a risky asset. The agent must borrow the asset so that he can sell it, and must post collateral as a commitment to return the asset. We assume that i -investors have enough wealth to collateralize any position they may want to establish, that is, up to $\mu_i u_i$. However, arbitrageurs may be constrained by their wealth.¹²

¹¹ By fixing the measure of arbitrageurs, we are ruling out entry and focusing on the evolution of existing arbitrageurs' wealth as the driver of price dynamics. Duffie and Strulovici (2012) study how entry impeded by search frictions affects price dynamics. Their analysis provides a complementary perspective to ours. Note that the duration M_i of endowment shocks can be interpreted as the time it takes for enough new arbitrageurs to enter the market for arbitrage opportunity ($i, -i$) and eliminate that opportunity.

¹² Our assumption that outside investors are unconstrained does not necessarily imply that they are wealthier than arbitrageurs because their positions could be smaller. This could be the case

Standard asset pricing models assume that agents can establish any combination of asset positions provided they have sufficient wealth to cover any liabilities that their positions generate. One interpretation of this constraint is that a central clearinghouse registers all positions and prevents agents from undertaking liabilities that they cannot cover. The constraint is formally equivalent to requiring wealth to be nonnegative, and thus is redundant for agents with logarithmic utility.

We assume that arbitrageurs are subject to a stronger constraint. We require that they have sufficient wealth *in each market* to cover any liabilities that their positions generate in that market. The positions of arbitrageurs in market i consist of a position in asset i and a position in the riskless asset. We require that this combined position always have nonnegative value. Thus, liability is calculated market-by-market rather than by aggregating across all markets. This is in the spirit of market segmentation: the same informational or regulatory frictions that prevent i -investors from investing in risky assets other than asset i could also be preventing arbitrageurs' lenders in market i from accepting such assets as collateral.¹³

To derive the financial constraint of an arbitrageur, we denote by $x_{i,t}$ his position in asset i , by $z_{i,t}^0$ his investment in the riskless asset held in market i , and by $z_{i,t} = x_{i,t}p_{i,t} + z_{i,t}^0$ the value of his combined position in market i , all in period t . The value of the arbitrageur's combined position in market i in period $t + 1$ is

$$\begin{aligned} z_{i,t+1} &= z_{i,t}^0(1+r) + x_{i,t}[d_{i,t+1} + p_{i,t+1}] \\ &= z_{i,t}(1+r) + x_{i,t}[d_{i,t+1} + p_{i,t+1} - (1+r)p_{i,t}] \end{aligned} \quad (7)$$

and must be nonnegative. Requiring (7) to be nonnegative for all possible realizations of uncertainty in period $t + 1$ yields

$$z_{i,t} \geq \max_{\{\epsilon_{j,t+1}, \eta_{j,t+1}\}_{j \in \mathcal{I}}} \left\{ x_{i,t} \left(p_{i,t} - \frac{d_{i,t+1} + p_{i,t+1}}{1+r} \right) \right\}. \quad (8)$$

The right-hand side of (8) represents the maximum loss, in present value terms, that the arbitrageur can realize in market i between periods t and $t + 1$. This loss must be smaller than the value of the arbitrageur's combined position in market i in period t . Thus, the arbitrageur can finance a long position in

for two reasons. First, the position that arbitrageurs as a group establish in asset i is the opposite of that of i -investors. Therefore, if arbitrageurs are in smaller measure than i -investors, they hold a larger position per capita in asset i . Second, each arbitrageur can trade more risky assets than each outside investor, leading to a larger aggregate position.

¹³ Using one asset as collateral for a position in the other is known as cross-netting. One situation in which cross-netting is generally not possible is when one asset is traded over-the-counter and the other in an exchange, for example, U.S. bonds are traded over the counter and U.S. bond futures are traded in the Chicago Mercantile Exchange. For a more detailed description of the frictions that hamper cross-netting, see, for example, Gromb and Vayanos (2002) and Shen, Yan, and Zhang (2014). While our model rules out cross-netting, it can be generalized to allow for partial cross-netting.

asset i by borrowing cash with the asset as collateral, but must contribute enough cash of his own to cover against the most extreme price decline. Conversely, the arbitrageur can borrow and short-sell asset i using the cash proceeds as collateral for the loan, but must contribute enough cash of his own to cover against the most extreme price increase.

Aggregating (8) across markets yields the financial constraint

$$W_t = \sum_{i \in \mathcal{I}} z_{i,t} \geq \sum_{i \in \mathcal{I}} \max_{\{\epsilon_{j,t+1}, \eta_{j,t+1}\}_{j \in \mathcal{I}}} \left\{ x_{i,t} \left(p_{i,t} - \frac{d_{i,t+1} + p_{i,t+1}}{1+r} \right) \right\} \quad (9)$$

since the value of the arbitrageur's positions summed across markets is his wealth W_t . The constraint (9) requires that the arbitrageur have enough wealth to cover his maximum loss in each market.¹⁴

Our formulation of the financial constraint assumes that the only assets that arbitrageurs can trade with i -investors, or use as collateral in market i , are asset i and the riskless asset. Under a more general formulation, arbitrageurs could trade with i -investors any contracts that are contingent on future uncertainty. These contracts could be collateralized by the riskless asset, by asset i , or by any other contracts traded in market i . Moreover, contracts could extend over any number of periods. In Internet Appendix Section II, we formulate equilibrium in our model under such general contracts. The only restrictions that we maintain are that i -investors cannot contract directly with j -investors for $j \neq i$ (they can contract only indirectly through arbitrageurs) and arbitrageurs cannot use contracts traded with j -investors as collateral for contracts traded with i -investors. These restrictions are consistent with the logic of market segmentation.

We show in Internet Appendix Section II that, without loss of generality, contracts can be assumed to be fully collateralized and hence default-free. Moreover, when assets in each pair have identical payoffs ($\bar{\eta}_i = 0$) and distributions are binomial ($\epsilon_{i,t}$ binomial), contracts can be restricted to those studied in this section—only asset i and the riskless asset are traded and used as collateral in market i . This generalizes, within our setting, the no-default result of Fostel and Geanakoplos (2015) shown under the assumption that contracts extend over one period.¹⁵ Thus, the financial constraint (9) can be derived under general contracts that are consistent with market segmentation.

¹⁴ The constraint (9) can extend to a continuous-time limit of our model if that limit involves jumps. With jumps, the support of $\{\epsilon_{j,t+1}, \eta_{j,t+1}\}_{j \in \mathcal{I}}$ does not converge to zero and neither does the maximum loss in period $t + 1$. If instead the support of $\{\epsilon_{j,t+1}, \eta_{j,t+1}\}_{j \in \mathcal{I}}$ converges to zero, as it would in a Brownian limit, then the maximum loss converges to zero and (9) is always met. For a derivation of a financial constraint in a continuous-time limit with jumps, see Chabakauri and Han (2017).

¹⁵ Besides allowing for dynamic contracts, we allow a contract to serve as collateral for other contracts, in a recursive manner. A similar recursive construction can be found in Gottardi and Kubler (2015). Simsek (2013) characterizes default rates in collateral equilibrium for general distributions in a static setting. For more references on leverage and collateral equilibrium, see the survey by Fostel and Geanakoplos (2015).

D. Symmetric Equilibrium

We look for a competitive equilibrium that is symmetric in the sense that price discounts and agents' positions are opposites for the assets in each pair.

DEFINITION 1: A competitive equilibrium consists of prices $p_{i,t}$ and positions in the risky assets $y_{i,t}$ for the i -investors and $x_{i,t}$ for the arbitrageurs, such that positions are optimal given prices and the markets for all risky assets clear:

$$\mu_i y_{i,t} + x_{i,t} = 0. \quad (10)$$

DEFINITION 2: A competitive equilibrium is symmetric if for the assets $(i, -i)$ in each pair the price discounts are opposites ($\phi_{i,t} = -\phi_{-i,t}$), the positions of outside investors are opposites ($y_{i,t} = -y_{-i,t}$), and the positions of arbitrageurs are opposites ($x_{i,t} = -x_{-i,t}$).

Symmetry implies that the price discount of each asset is one-half of the difference between the price of the other asset and its own price:

$$\phi_{i,t} = \frac{p_{-i,t} - p_{i,t}}{2}.$$

Since the price discount measures the price difference between paired assets, we also refer to it as the spread. The spread is an inverse measure of the liquidity that arbitrageurs provide to outside investors.

E. Optimization Problems

E.1. Outside Investors

The budget constraint of an i -investor is

$$\begin{aligned} w_{i,t+1} = & y_{i,t}(d_{i,t+1} + p_{i,t+1}) + (1+r)(w_{i,t} - y_{i,t}p_{i,t}) + u_{i,t}(\epsilon_{i,t+1} + \eta_{i,t+1}) \\ & - c_{i,t+1}. \end{aligned} \quad (11)$$

The investor holds $y_{i,t}$ shares of asset i in period t , and these shares are worth $y_{i,t}(d_{i,t+1} + p_{i,t+1})$ in period $t + 1$. The investor also holds $w_{i,t} - y_{i,t}p_{i,t}$ units of the riskless asset in period t , that is, wealth $w_{i,t}$ minus the investment $y_{i,t}p_{i,t}$ in asset i . This investment is worth $(1+r)(w_{i,t} - y_{i,t}p_{i,t})$ in period $t + 1$. Finally, the random endowment $u_{i,t}(\epsilon_{i,t+1} + \eta_{i,t+1})$ is added to the investor's wealth in period $t + 1$, while consumption $c_{i,t+1}$ lowers wealth.

We can simplify (11) by introducing the return per share of asset i in excess of the riskless asset. This excess return is

$$\begin{aligned} R_{i,t+1} & \equiv d_{i,t+1} + p_{i,t+1} - (1+r)p_{i,t} \\ & = (1+r)\phi_{i,t} - \phi_{i,t+1} + \epsilon_{i,t+1} + \eta_{i,t+1}, \end{aligned} \quad (12)$$

where the second step follows from (1) and (3). The expected excess return of asset i is

$$\Phi_{i,t} \equiv \mathbf{E}_t(\mathbf{R}_{i,t+1}) = (1+r)\phi_{i,t} - \mathbf{E}_t(\phi_{i,t+1}). \quad (13)$$

Using (12) and (13), we can write (11) as

$$w_{i,t+1} = (1+r)w_{i,t} + y_{i,t}\Phi_{i,t} + (y_{i,t} + u_{i,t})(\epsilon_{i,t+1} + \eta_{i,t+1}) + y_{i,t}[\mathbf{E}_t(\phi_{i,t+1}) - \phi_{i,t+1}] - c_{i,t+1}. \quad (14)$$

The investor's wealth in period $t+1$ is uncertain as of period t because of the payoff shock $\epsilon_{i,t+1} + \eta_{i,t+1}$ and the price discount $\phi_{i,t+1}$. The investor's exposure to the payoff shock is the sum of her asset position $y_{i,t}$ and endowment shock $u_{i,t}$, while her exposure to the price discount is $y_{i,t}$.

We conjecture that the investor's value function in period t is

$$V_{i,t}(w_{i,t}) = -\exp(-Aw_{i,t} - F_{i,t}), \quad (15)$$

where $F_{i,t}$ is a possibly stochastic function and A is a constant. The value function is negative exponential in wealth because the utility function depends on consumption in the same manner.

E.2. Arbitrageurs

The budget constraint of an arbitrageur is

$$W_{t+1} = \sum_{i \in \mathcal{I}} x_{i,t}(d_{i,t+1} + p_{i,t+1}) + (1+r) \left(W_t - \sum_{i \in \mathcal{I}} x_{i,t}p_{i,t} \right) - c_{t+1}. \quad (16)$$

The differences with the budget constraint (11) of an i -investor are that the arbitrageur can invest in all assets and receives no endowment. We next simplify the budget constraint (16) and the financial constraint (9) by using properties of a symmetric equilibrium.

A first simplifying property is that $\phi_{i,t} = 0$ for assets that are not part of active opportunities. This property holds in equilibrium, as we explain here and show formally in Sections II and III. An implication of this property is that $\mathbf{E}_t(\phi_{i,t+1}) = \phi_{i,t} = 0$ since the probability of an opportunity becoming active is arbitrarily small. Since $\Phi_{i,t} = 0$, investing in assets that are not part of active opportunities exposes arbitrageurs to risk that is not compensated in terms of expected excess return. Investing in those assets also tightens the financial constraint (9). Hence, the optimal position is zero. Outside investors' optimal position is also zero because they have zero endowment and hence would hold a nonzero position only if the expected excess return were nonzero. Therefore, the markets for assets that are not part of active opportunities clear with zero price discount, confirming our conjecture that $\phi_{i,t} = 0$. Using that property as well as (12), (13), $\epsilon_{i,t+1} = \epsilon_{-i,t+1}$, $\eta_{i,t+1} = -\eta_{i,t+1}$, and $\phi_{i,s} = -\phi_{-i,s}$ for $s = t, t+1$,

we can write the budget constraint (16) as

$$W_{t+1} = (1+r)W_t + \sum_{i \in \mathcal{A}_t} (x_{i,t} - x_{-i,t}) [\Phi_{i,t} + \eta_{i,t+1} + \mathbf{E}_t(\phi_{i,t+1}) - \phi_{i,t+1}] + \sum_{i \in \mathcal{A}_t} (x_{i,t} + x_{-i,t}) \epsilon_{i,t+1} - c_{t+1}, \quad (17)$$

and the financial constraint (9) as

$$W_t \geq \sum_{i \in \mathcal{A}_t} \left[\max_{\{\epsilon_{j,t+1}, \eta_{j,t+1}\}_{j \in \mathcal{I}}} \frac{x_{i,t} [-\Phi_{i,t} - \epsilon_{i,t+1} - \eta_{i,t+1} - \mathbf{E}_t(\phi_{i,t+1}) + \phi_{i,t+1}]}{1+r} + \max_{\{\epsilon_{j,t+1}, \eta_{j,t+1}\}_{j \in \mathcal{I}}} \frac{x_{-i,t} [\Phi_{i,t} - \epsilon_{i,t+1} + \eta_{i,t+1} + \mathbf{E}_t(\phi_{i,t+1}) - \phi_{i,t+1}]}{1+r} \right]. \quad (18)$$

Two further simplifying properties are that $\phi_{i,t+1}$ is independent of $\epsilon_{i,t+1}$ and that $x_{i,t}$ and $x_{-i,t}$ must have opposite signs. The first property holds in equilibrium, as we show in Sections II and III. Intuitively, when arbitrageurs hold opposite positions in assets i and $-i$, their wealth W_{t+1} is independent of $\epsilon_{i,t+1}$ and the same is true of spreads, which depend on wealth. The second property follows from arbitrageurs' optimization. Indeed, if $x_{i,t}$ and $x_{-i,t}$ had the same sign, then an arbitrageur would be able to reduce both in absolute value while holding $x_{i,t} - x_{-i,t}$ constant. That would reduce his risk without affecting his expected excess return, as can be seen from the budget constraint (17), and would relax the financial constraint (18). Using the two simplifying properties, we can write (18) as

$$W_t \geq \sum_{i \in \mathcal{A}} \frac{(|x_{i,t}| + |x_{-i,t}|) \bar{\epsilon}_i + 2 \max_{\{\eta'_{j,t+1}\}_{j \in \mathcal{I}}} \{(x_{i,t} - x_{-i,t}) [-\Phi_{i,t} - \eta_{i,t+1} - \mathbf{E}_t(\phi_{i,t+1}) + \phi_{i,t+1}]\}}{1+r}. \quad (19)$$

A final simplifying property is that $x_{i,t}$ and $x_{-i,t}$ must be (exact) opposites. Indeed, if $x_{i,t} + x_{-i,t} \neq 0$, then an arbitrageur could set $x_{i,t} + x_{-i,t} = 0$ while holding $x_{i,t} - x_{-i,t}$ constant. That would eliminate his exposure to $\epsilon_{i,t+1}$ without affecting his expected excess return, his exposure to $\eta_{i,t+1}$ and $\phi_{i,t+1}$, and the financial constraint (19). Using this property, we can simplify the budget constraint (17) to

$$W_{t+1} = (1+r)W_t + 2 \sum_{i \in \mathcal{A}_t} x_{i,t} [\Phi_{i,t} + \eta_{i,t+1} + \mathbf{E}_t(\phi_{i,t+1}) - \phi_{i,t+1}] - c_{t+1}, \quad (20)$$

and the financial constraint (19) to

$$W_t \geq 2 \sum_{i \in \mathcal{A}} \frac{|x_{i,t}| \bar{\epsilon}_i + \max_{\{\eta_{j,t+1}\}_{j \in \mathcal{I}}} \{x_{i,t} [-\Phi_{i,t} - \eta_{i,t+1} - \mathbf{E}_t(\phi_{i,t+1}) + \phi_{i,t+1}]\}}{1+r}. \quad (21)$$

The arbitrageur's optimization problem reduces to choosing positions in assets $i \in \mathcal{A}_t$, that is, those with positive endowment shocks. Positions in the

corresponding assets $-i$ are opposites, and positions in assets that are not part of active opportunities are zero. We conjecture that the value function of an arbitrageur in period t is

$$V_t(W_t) = B \log(W_t) + G_t, \quad (22)$$

where G_t is a possibly stochastic function and B is a constant.

II. Riskless Arbitrage

In this section, we solve for equilibrium when assets in each pair have identical payoffs ($\bar{\eta}_i = 0$). With identical payoffs, arbitrageur wealth W_t does not depend on the payoff realizations because arbitrageurs hold opposite positions in the assets in each pair. Hence, the return that arbitrageurs earn from one period to the next is riskless. That riskless return could change stochastically, however, over time. We rule out stochastic variation by assuming that the set

$$\mathcal{C}_t \equiv \{(\bar{\epsilon}_i, \bar{\eta}_i, u_i, \mu_i, h_i - t) : i \in \mathcal{A}_t\},$$

describing the characteristics of active opportunities, is deterministic. Thus, while arbitrageurs are uncertain as to which opportunities will become active, they know what their overall return will be. With a deterministic \mathcal{C}_t , the dynamics of arbitrageur wealth, arbitrageur positions, and spreads are deterministic. With deterministic spreads, the expected excess return of asset i simplifies to

$$\Phi_{i,t} = (1 + r)\phi_{i,t} - \phi_{i,t+1}. \quad (23)$$

One setting that yields a deterministic \mathcal{C}_t and that we emphasize later is as follows. The universe \mathcal{I} of risky assets is divided into $2N$ disjoint families \mathcal{I}_n for $n = 1, \dots, N$, with the assets in each family forming a continuum and having the same characteristics $(\bar{\epsilon}_i, \bar{\eta}_i, u_i, \mu_i, M_i)$. Moreover, one asset from each family is randomly drawn in each period to form an active opportunity (together with the other asset in its pair). Under these assumptions, \mathcal{C}_t is not only deterministic, but constant.

The case of riskless arbitrage is a natural benchmark. It is highly tractable and yields useful results and intuitions, which facilitate the analysis of risky arbitrage in Section III. We start by deriving the first-order conditions of outside investors and arbitrageurs in an equilibrium of the conjectured form, that is, symmetric with deterministic price discounts. We then impose market clearing and show that such an equilibrium exists.

A. First-Order Conditions

A.1. Outside Investors

Since $\eta_{i,t+1} = 0$ and $\phi_{i,t+1}$ is deterministic, the budget constraint (14) of an i -investor simplifies to

$$w_{i,t+1} = (1 + r)w_{i,t} + y_{i,t}\Phi_{i,t} + (y_{i,t} + u_{i,t})\epsilon_{i,t+1} - c_{i,t+1}. \quad (24)$$

The only risk borne by the investor between periods t and $t + 1$ is the payoff shock $\epsilon_{i,t+1}$, and the investor's exposure to that risk is $y_{i,t} + u_{i,t}$.

PROPOSITION 1: *The value function of an i -investor in period t is given by (15), where $A = r\alpha$ and $F_{i,t}$ is deterministic. The investor's optimal position in asset i is given by the first-order condition*

$$\Phi_{i,t} - \bar{\epsilon}_i f'[(y_{i,t} + u_{i,t})\bar{\epsilon}_i] = 0, \quad (25)$$

where the function $f(y)$ is defined by

$$\exp\left[\frac{\alpha A f(y)}{\alpha + A}\right] \equiv E\left[\exp\left(-\frac{\alpha A y \epsilon_{i,t}}{(\alpha + A)\bar{\epsilon}_i}\right)\right]. \quad (26)$$

The first-order condition (25) takes an intuitive form. The first term in the left-hand side, $\Phi_{i,t}$, is the expected excess return of asset i . The second term, $\bar{\epsilon}_i f'[(y_{i,t} + u_{i,t})\bar{\epsilon}_i]$, is a risk adjustment, reflecting the investor's risk from holding the position. Since the function $f(y)$ is convex, as shown in Lemma 1, the risk adjustment is increasing in the investor's exposure $y_{i,t} + u_{i,t}$. The investor's first-order condition amounts to setting the risk-adjusted expected excess return that she derives from asset i to zero. This yields a standard downward-sloping demand: the investor's position y_i in asset i is increasing in the asset's expected excess return $\Phi_{i,t}$ and hence is decreasing in the asset's price $p_{i,t}$.¹⁶

LEMMA 1: *The function $f(y)$ is nonnegative, symmetric around the vertical axis ($f(-y) = f(y)$), and strictly convex. It also satisfies $f'(-y) = -f'(y)$, $f'(0) = 0$, and $\lim_{y \rightarrow \infty} f'(y) = 1$.*

The function $\frac{\alpha A f(y)}{\alpha + A}$ is the cumulant-generating function of $-\frac{\alpha A y \epsilon_{i,t}}{(\alpha + A)\bar{\epsilon}_i}$. Cumulant-generating functions are convex. Symmetry around the vertical axis follows because $\epsilon_{i,t}$ is distributed symmetrically around zero. Symmetry implies $f'(-y) = -f'(y)$ and $f'(0) = 0$.

The first-order condition of $-i$ -investors yields an optimal position that is the opposite of that of i -investors. This follows from (25), the observations that price discounts, expected excess returns, and endowment shocks are opposites for assets i and $-i$, and the fact that $f'(y) = -f'(-y)$.

A.2. Arbitrageurs

Since $\eta_{i,t+1} = 0$ and $\phi_{i,t+1}$ is deterministic, the budget constraint (20) of an arbitrageur simplifies to

$$W_{t+1} = (1 + r)W_t + 2 \sum_{i \in A_t} x_{i,t} \Phi_{i,t} - c_{t+1}, \quad (27)$$

¹⁶ The function $\bar{\epsilon}_i f'[(y_{i,t} + u_{i,t})\bar{\epsilon}_i]$ can be interpreted as a "pricing function," yielding the expected excess return $\Phi_{i,t}$ as a function of the position $y_{i,t}$ of the "marginal investor."

and the financial constraint (21) simplifies to

$$W_t \geq 2 \sum_{i \in \mathcal{A}_t} \frac{|x_{i,t}| \bar{\epsilon}_i - x_{i,t} \Phi_{i,t}}{1+r}. \quad (28)$$

Equation (27) confirms that the dynamics of arbitrageur wealth are deterministic. The per-share return of an active opportunity i is $2\Phi_{i,t}$, that is, twice the expected excess return $\Phi_{i,t}$ of asset i . This return is nonnegative in equilibrium, as we show in Section II.B. While i -investors earn $\Phi_{i,t}$ as compensation for risk, arbitrageurs earn it risk-free because they can combine a position in asset i with one in asset $-i$. Thus, when $\Phi_{i,t} > 0$, arbitrageurs can earn a riskless return above the riskless rate r .

Equation (28) shows that the collateral required to hold a position in an active opportunity i is larger when asset payoffs are more volatile, that is, $\bar{\epsilon}_i$ is larger, and when the opportunity offers a lower return, that is, $\Phi_{i,t}$ is smaller. In both cases, this is because the maximum loss of the position is larger.

PROPOSITION 2: *The value function of an arbitrageur in period t is given by (22), where $B = \frac{\beta}{1-\beta}$ and $G_{i,t}$ is deterministic. The arbitrageur's optimal consumption is*

$$c_t = \frac{1-\beta}{\beta} W_t. \quad (29)$$

- *If all active opportunities offer zero return, that is, $\Phi_{i,t} = 0$ for all $i \in \mathcal{A}_t$, then the arbitrageur is indifferent between any combination of positions in these opportunities.*
- *If instead some active opportunities offer a positive return and the remainder zero return, then the arbitrageur holds nonzero positions only in opportunities with maximum return per unit of collateral:*

$$i \in \operatorname{argmax}_{j \in \mathcal{A}_t} \frac{\Phi_{j,t}}{\bar{\epsilon}_j - \Phi_{j,t}}. \quad (30)$$

For these opportunities, positions are long in assets $i \in \mathcal{A}_t$, that is, those with positive endowment shocks. Moreover, the financial constraint (28) binds.

Arbitrageurs' optimal investment policy can be derived intuitively as follows. Substituting the optimal consumption (29) into the budget constraint (27), we can write the latter as

$$W_{t+1} = \beta \left[(1+r)W_t + 2 \sum_{i \in \mathcal{A}_t} x_{i,t} \Phi_{i,t} \right]. \quad (31)$$

Since assets $i \in \mathcal{A}_t$ offer nonnegative expected excess returns, arbitrageurs do not benefit from shorting them. Therefore, we can write the financial constraint (28) as

$$W_t \geq 2 \sum_{i \in \mathcal{A}_t} \frac{x_{i,t}(\bar{\epsilon}_i - \Phi_{i,t})}{1+r}. \quad (32)$$

Maximizing W_{t+1} in (31) subject to (32) and $x_{i,t} \geq 0$ is a simple linear programming problem. Arbitrageurs invest only in those opportunities that offer the maximum return $\Phi_{i,t}$ per unit of collateral $\bar{\epsilon}_i - \Phi_{i,t}$. Moreover, when some opportunities offer a nonzero return, arbitrageurs “max out” their financial constraint (32) because they can earn a riskless return above the riskless rate r . Maximizing return per unit of collateral, $\frac{\Phi_{i,t}}{\bar{\epsilon}_i - \Phi_{i,t}}$, is equivalent to maximizing return per unit of volatility $\frac{\Phi_{i,t}}{\bar{\epsilon}_i}$. We focus on the latter from now on.

B. Equilibrium

B.1. Arbitraging Arbitrage

Combining arbitrageurs’ optimal investment policy with that of outside investors and imposing market clearing, we can derive a sharp characterization of equilibrium returns and positions. We denote by

$$\mathcal{T}_t \equiv \{i \in \mathcal{A}_t : x_{i,t} > 0\}$$

the set of active opportunities that arbitrageurs actually trade in period t , that is, those in which they hold nonzero positions.

PROPOSITION 3: *In equilibrium, there exists $\Pi_t \in [0, 1)$ such that in period t :*

- *Arbitrageurs trade only active opportunities i such that $f'(u_i \bar{\epsilon}_i) > \Pi_t$, that is,*

$$\mathcal{T}_t = \{i \in \mathcal{A}_t : f'(u_i \bar{\epsilon}_i) > \Pi_t\}.$$

- *All active opportunities that arbitrageurs trade offer the same return Π_t per unit of volatility, while those that they do not trade offer return $f'(u_i \bar{\epsilon}_i) \in (0, \Pi_t]$ per unit of volatility. That is,*

$$i \in \mathcal{T}_t \Rightarrow \frac{\Phi_{i,t}}{\bar{\epsilon}_i} = \Pi_t,$$

$$i \in \mathcal{A}_t / \mathcal{T}_t \Rightarrow \frac{\Phi_{i,t}}{\bar{\epsilon}_i} = f'(u_i \bar{\epsilon}_i) \in (0, \Pi_t].$$

Proposition 3 implies that active opportunities can be ranked according to $f'(u_i \bar{\epsilon}_i)$. As can be seen by setting $y_{i,t} = 0$ in outside investors’ first-order condition (25), $f'(u_i \bar{\epsilon}_i)$ is the return per unit of volatility that opportunity i would offer in the absence of arbitrageurs. Arbitrageurs trade only those opportunities for which $f'(u_i \bar{\epsilon}_i)$ is above a cutoff $\Pi_t \in (0, 1)$. Their activity causes the

return per unit of volatility offered by these opportunities to decrease to the common cutoff. Opportunities for which $f'(u_i \bar{\epsilon}_i)$ is below that cutoff are not traded, and their return per unit of volatility remains equal to $f'(u_i \bar{\epsilon}_i)$.

Since the function $f(y)$ is convex, it follows that $f'(u_i \bar{\epsilon}_i)$ is increasing in the endowment shock u_i and in volatility $\bar{\epsilon}_i$. Thus, arbitrageurs are more likely to trade opportunities with higher volatility and higher endowment shocks: these are the opportunities offering higher return per unit of volatility in the absence of arbitrageurs.

The equalization of returns across traded opportunities can be interpreted as “arbitraging arbitrage.” If a traded opportunity offered lower return per unit of volatility than another opportunity, arbitrageurs could raise their profit by redeploying their scarce capital to the latter. The arbitraging-arbitrage result is at the basis of the contagion effects derived in Section III. Suppose, for example, that arbitrageurs experience losses in opportunity i . This would force them to scale back their position in that opportunity, causing its return to increase. In turn, arbitraging arbitrage would induce them to redeploy capital to that opportunity and away from others, causing the return of others to increase as well.

B.2. Dynamics of Arbitrage Capital

Using Proposition 3, we can determine the dynamics of arbitrageur wealth and the relationship between wealth and Π_t .

PROPOSITION 4: *In equilibrium, arbitrageur wealth evolves according to*

$$W_{t+1} = \beta \frac{1+r}{1-\Pi_t} W_t. \quad (33)$$

- If $W_t > W_{c,t} \equiv \frac{2}{1+r} \sum_{i \in \mathcal{A}_t} \mu_i u_i \bar{\epsilon}_i$, then the financial constraint is slack, arbitrageurs earn the riskless rate r , all active opportunities are traded, and their return Π_t per unit of volatility is zero.
- If $W_t < W_{c,t}$, then the financial constraint binds and arbitrageurs earn a riskless return above the riskless rate r . The return Π_t per unit of volatility offered by all traded opportunities is the unique positive solution of

$$2 \frac{1-\Pi_t}{1+r} \sum_{i \in \mathcal{I}_t} \mu_i [u_i \bar{\epsilon}_i - (f')^{-1}(\Pi_t)] = W_t, \quad (34)$$

and decreases in W_t .

When the variables $\epsilon_{i,t}$ have a binomial distribution, Π_t is a convex function of W_t .

The financial constraint is slack when all active opportunities offer zero return, that is, $\Phi_{i,t} = 0$ for all $i \in \mathcal{A}_t$. This happens when arbitrageurs fully absorb the endowment shocks of outside investors, that is, $x_{i,t} = \mu_i u_i$ for all

$i \in \mathcal{A}_t$. Setting $\Phi_{i,t} = 0$ and $x_{i,t} = \mu_i u_i$ in (32), we find that W_t must exceed the threshold $W_{c,t}$ defined in Proposition 4. Since all active opportunities offer zero return, $\Pi_t = 0$ and arbitrageurs earn the riskless rate r .

When instead $W_t < W_{c,t}$, arbitrageurs cannot fully absorb the endowment shocks of outside investors. Therefore, all active opportunities offer a positive return, the return Π_t per unit of volatility offered by all traded opportunities is also positive, and arbitrageurs earn a riskless return above r . Moreover, when W_t decreases, Π_t increases because arbitrageurs are less able to absorb the endowment shocks.

Convexity of Π_t means that a given drop in W_t causes a larger increase in Π_t when it occurs in a region in which W_t is smaller. Clearly, this comparison holds between the constrained and the unconstrained regions: a drop in W_t raises Π_t when $W_t < W_{c,t}$, but has no effect on Π_t when $W_t > W_{c,t}$. The intuition for why the comparison can also hold within the constrained region is as follows. When W_t is less than but close to $W_{c,t}$, all active opportunities are traded and hence a drop in W_t causes arbitrageurs to reduce their positions in all opportunities. Since the effect is spread out across many opportunities, the reduction in each position is small and thus the increase in Π_t is small. When instead W_t is close to zero, arbitrageurs concentrate their investment in a small number of opportunities, in which case a drop in W_t triggers a large reduction in each position. Proposition 4 confirms the convexity of Π_t under the sufficient condition that the variables $\epsilon_{i,t}$ that describe asset payoffs have a binomial distribution. Proposition 5 shows that an equilibrium with all the properties conjectured or shown in this section exists.

PROPOSITION 5: *A symmetric equilibrium exists in which price discounts $\phi_{i,t}$, outside investors' positions $y_{i,t}$, and arbitrageurs' positions $x_{i,t}$ and wealth W_t are deterministic. In this equilibrium, price discounts are zero for assets that are not part of active opportunities, and expected excess returns $\Phi_{i,t}$ are nonnegative for assets with positive endowment shocks.*

C. Steady State and Convergence Dynamics

We next derive a steady state by specializing our model to the stationary “asset family” setting described at the beginning of Section II. This steady state is deterministic and used as a basis for constructing a stochastic steady state in Section III. We index the $2N$ families by $n \in \{-N, \dots, -1, 1, \dots, N\}$, where for an asset in family n , the other asset in its pair belongs to family $-n$, and where families $n = 1, \dots, N$ comprise the assets with the positive endowment shocks. We denote by $(\bar{\epsilon}_n, \bar{\eta}_n, u_n, \mu_n, M_n)$ the characteristics $(\bar{\epsilon}_i, \bar{\eta}_i, u_i, \mu_i, M_i)$ for all assets i in family n . (As in the rest of Section II, we assume $\bar{\eta}_i = 0$.) The set of active opportunities in period t is

$$\mathcal{A} = \{(n, m) : n \in \{1, \dots, N\}, m \in \{1, \dots, M_n\}\}.$$

Opportunity (n, m) consists of one asset in family $n \in \{1, \dots, N\}$ and one asset in family $-n$, and it remains active for $m - 1$ more periods. We denote the former

asset by (n, m) and the latter by $(-n, m)$, and refer to m as the horizon of opportunity (n, m) . The expected excess returns of assets (n, m) and $(-n, m)$ do not depend on m (Proposition 3), nor do arbitrageurs' and outside investors' positions (equations (10) and (25)). Accordingly, we index these quantities by the family subscript, n or $-n$, and the time subscript, t . The price discounts of the two assets depend on m , and thus we index them by the additional subscript m . Since arbitrageurs' positions do not depend on m , we can write the set of active opportunities traded in period t as

$$\mathcal{I}_t = \{(n, m) : n \in \mathcal{N}_t, m \in \{1, \dots, M_n\}\},$$

where we denote by \mathcal{N}_t the subset of families in $\{1, \dots, N\}$ whose assets are traded. We drop the time subscript for steady-state values.

PROPOSITION 6: *In equilibrium, the wealth W_t of arbitrageurs and the return Π_t per unit of volatility offered by all traded opportunities converge over time monotonically to steady-state values W and Π .*

- If $\beta(1+r) > 1$, then W_t increases toward $W = \infty$ and Π_t decreases toward $\Pi = 0$.
- If $\beta(1+r) < 1 - \bar{\Pi}$, where $\bar{\Pi} \equiv \max_{n=1, \dots, N} f'(u_n \bar{\epsilon}_n) < 1$, then W_t decreases toward $W = 0$ and Π_t increases toward $\Pi = \bar{\Pi}$.
- Otherwise, the steady-state values are given by

$$W = 2 \frac{1 - \Pi}{1 + r} \sum_{n \in \mathcal{N}} \mu_n M_n [u_n \bar{\epsilon}_n - (f')^{-1}(\Pi)] \in (0, W_c), \quad (35)$$

$$\Pi = 1 - \beta(1+r) \in (0, \bar{\Pi}), \quad (36)$$

where $W_c \equiv \frac{2}{1+r} \sum_{n=1}^N \mu_n M_n u_n \bar{\epsilon}_n$. If $W_t < W$, then W_t increases toward W and Π_t decreases toward Π . If $W_t > W$, then W_t decreases toward W and Π_t increases toward Π .

The dynamics in Proposition 6 can be derived by specializing Proposition 4 to the stationary case. According to Proposition 4, the wealth of arbitrageurs increases between periods t and $t + 1$ if $\beta \frac{1+r}{1-\Pi_t} > 1$. Intuitively, wealth increases if the return earned by arbitrageurs exceeds the rate at which they consume. Arbitrageurs earn the riskless (net) return $\frac{1+r}{1-\Pi_t} - 1$ and consume at the rate $\frac{1-\beta}{\beta}$ as shown in (29).

Using Proposition 4, we can characterize how the return $\frac{1+r}{1-\Pi_t} - 1$ earned by arbitrageurs depends on their wealth W_t . When $W_t > W_c$, all active opportunities offer zero return, $\Pi_t = 0$, and arbitrageurs earn the riskless rate r . When instead $W_t < W_c$, Π_t is positive and arbitrageurs earn a riskless return above r . Decreases in W_t within that region raise Π_t and in turn arbitrageurs' return, which reaches its maximum value (corresponding to the maximum value of Π_t) when W_t goes to zero. Setting $y_{i,t} = 0$ in outside investors' first-order condition (25), we find that the return per unit of volatility from an active opportunity

$(i, -i)$ in the absence of arbitrageurs is equal to $f'(u_i \bar{\epsilon}_i)$. Therefore, the maximum value of Π_t is $\bar{\Pi} \equiv \max_{n=1, \dots, N} f'(u_n \bar{\epsilon}_n)$. Specializing Proposition 4 to the stationary case ensures that the function linking Π_t to W_t , and in particular the parameters W_c and $\bar{\Pi}$, are constant over time.

The dynamics of wealth in the stationary case follow from the above observations. When $\beta(1+r) > 1$, arbitrageurs consume at a rate smaller than the riskless rate. Hence, their wealth increases over time even if $\Pi_t = 0$, that is, all active opportunities offer zero return, and becomes arbitrarily large. When instead $\beta(1+r) < 1 - \bar{\Pi}$, arbitrageurs consume at a rate larger than the maximum return that their trades can offer. Hence, their wealth decreases over time and converges to zero.

In the intermediate case $1 - \bar{\Pi} < \beta(1+r) < 1$, arbitrageurs' wealth converges to an interior steady-state value. Indeed, when wealth is large, all active opportunities offer zero return, and wealth decreases because $\beta(1+r) < 1$. When instead wealth is close to zero, active opportunities offer close to their maximum return, and wealth increases because $1 - \bar{\Pi} < \beta(1+r)$. Dynamics are self-correcting: wealth decreases when it is large because arbitrageurs earn a low return, and wealth increases when it is small because arbitrageurs earn a high return. The steady-state value W implied by these dynamics is smaller than W_c because the steady-state return earned by arbitrageurs must exceed r to offset consumption. An increase in the subjective discount factor raises consumption, and thus raises the steady-state return and lowers the steady-state wealth.

III. Risky Arbitrage

Most real-life arbitrage involves some risk. To introduce arbitrage risk in our model, we allow assets in each pair to have nonidentical payoffs ($\bar{\eta}_i > 0$). The return that arbitrageurs earn from one period to the next is then risky and arbitrageurs' wealth W_t is stochastic. We look for a symmetric equilibrium in which W_t is the only stochastic state variable. We maintain the assumption that the set \mathcal{C}_t describing the characteristics of active opportunities is deterministic. This prevents characteristics from becoming additional state variables. Because arbitrageurs hold opposite positions in the two assets in each pair, W_t does not depend on the realizations of $\epsilon_{i,t}$; rather, it depends only on the realizations of $\eta_{i,t}$. The same is true for positions and spreads.

We start by deriving the first-order conditions of outside investors and arbitrageurs in an equilibrium of the conjectured form. We then specialize our analysis to the case in which arbitrage risk is small ($\bar{\eta}_i$ small) and we compute the equilibrium in closed form. Using our analytical solution, we study how spreads, expected excess returns, and arbitrageur positions depend on cross-sectional characteristics, how they respond to shocks, and whether the mobility of arbitrage capital across markets makes them more stable.

A. First-Order Conditions

A.1. Outside Investors

An i -investor bears more risk than in the riskless-arbitrage case ($\bar{\eta}_i = 0$). This is because asset i 's payoff includes the additional component $\eta_{i,t+1}$, and because the asset's price discount $\phi_{i,t+1}$ varies stochastically following changes in arbitrageur wealth. As in the riskless-arbitrage case, the investor sets the risk-adjusted expected excess return that she derives from asset i to zero. The risk adjustment, however, includes a term reflecting the additional risk. This is the third term in the left-hand side of (37).

PROPOSITION 7: *The value function of an i -investor in period t is given by (15), where $A = r\alpha$ and $F_{i,t}$ is a function of W_t . The investor's optimal position in asset i is given by the first-order condition*

$$\Phi_{i,t} - \bar{\epsilon}_i f'[(y_{i,t} + u_{i,t})\bar{\epsilon}_i] + E_t \left[\frac{\mathcal{M}_{i,t+1}}{E_t[\mathcal{M}_{i,t+1}]} (\eta_{i,t+1} + E_t(\phi_{i,t+1}) - \phi_{i,t+1}) \right] = 0, \quad (37)$$

where

$$\mathcal{M}_{i,t+1} \equiv \exp \left(- \frac{\alpha \{ A[(y_{i,t} + u_{i,t})\eta_{i,t+1} - y_{i,t}\phi_{i,t+1}] + F_{i,t+1} \}}{\alpha + A} \right).$$

A.2. Arbitrageurs

An arbitrageur bears the risk represented by $\eta_{i,t+1}$ and $\phi_{i,t+1}$. Because of that risk, he may benefit from shorting assets $i \in \mathcal{A}_t$, and thus may amplify the price discrepancies induced by outside investors' endowment shocks. Short positions in assets $i \in \mathcal{A}_t$ may be beneficial even though they earn negative expected excess returns because they hedge long positions in other such assets. In the rest of this section, we assume that short positions in assets $i \in \mathcal{A}_t$ do not arise in equilibrium. This assumption is satisfied, for example, when arbitrage risk is small and the expected excess returns $\Phi_{i,t}$ of assets $i \in \mathcal{A}_t$ exceed a (strictly) positive bound. This result is shown in Proposition 8 and extends a result shown in Proposition 2 under riskless arbitrage.

When arbitrageurs hold long positions in asset $i \in \mathcal{A}_t$, an increase in $\eta_{i,t+1}$ raises their wealth and hence lowers the spread $\phi_{i,t+1}$ (which is positive). When instead arbitrageurs hold no position in the asset, $\phi_{i,t+1}$ is independent of $\eta_{i,t+1}$. In either case, the minimum value $-\bar{\eta}_i$ of $\eta_{i,t+1}$ can be realized jointly with the maximum value of $\phi_{i,t+1}$, which we denote by $\bar{\phi}_{i,t+1}$. Conversely, the maximum value $\bar{\eta}_i$ of $\eta_{i,t+1}$ can be realized jointly with the minimum value of $\phi_{i,t+1}$, which we denote by $\underline{\phi}_{i,t+1}$. Using these observations, we can simplify arbitrageurs' financial constraint (21) because we can determine the maximum loss scenario. A long position in asset $i \in \mathcal{A}_t$ suffers its maximum loss when $\eta_{i,t+1} = -\bar{\eta}_i$ and $\phi_{i,t+1} = \bar{\phi}_{i,t+1}$ (maximum spread and hence maximum price discount for asset

i). Conversely, a short position in the asset suffers its maximum loss when $\eta_{i,t+1} = \bar{\eta}_i$ and $\phi_{i,t+1} = \underline{\phi}_{i,t+1}$. Equation (21) thus becomes

$$W_t \geq 2 \sum_{i \in \mathcal{A}_t} \frac{|x_{i,t}|(\bar{\epsilon}_i + \bar{\eta}_i) + x_{i,t}^+[\bar{\phi}_{i,t+1} - \mathbf{E}_t(\phi_{i,t+1})] - x_{i,t}^-[\mathbf{E}_t(\phi_{i,t+1}) - \underline{\phi}_{i,t+1}] - x_{i,t}\Phi_{i,t}}{1+r}, \quad (38)$$

where $x_{i,t}^+ = \max\{x_{i,t}, 0\}$ and $x_{i,t}^- = \min\{x_{i,t}, 0\}$. Moreover, when $x_{i,t} \geq 0$ for all $i \in \mathcal{A}_t$, (38) simplifies to

$$W_t \geq 2 \sum_{i \in \mathcal{A}_t} \frac{x_{i,t}[\bar{\epsilon}_i + \bar{\eta}_i + \bar{\phi}_{i,t+1} - \mathbf{E}_t(\phi_{i,t+1}) - \Phi_{i,t}]}{1+r}. \quad (39)$$

The constraint is more stringent than in the case of riskless arbitrage because of the additional risks that the arbitrageur is subject to.

PROPOSITION 8: *The value function of an arbitrageur in period t is given by (22), where $B = \frac{\beta}{1-\beta}$ and G_t is a function of aggregate arbitrageur wealth. The arbitrageur's optimal consumption is given by (29). When short positions in assets $i \in \mathcal{A}_t$ are not optimal, as is the case when $\min_{i \in \mathcal{A}_t} \Phi_{i,t}$ exceeds a positive bound and $\max_{i \in \mathcal{A}_t} \{\bar{\eta}_i, \bar{\phi}_{i,t+1} - \mathbf{E}_t(\phi_{i,t+1})\}$ is small:*

- *The arbitrageur's optimal positions are nonzero only in opportunities with maximum risk-adjusted return per unit of collateral*

$$i \in \operatorname{argmax}_{j \in \mathcal{A}_t} \frac{\Phi_{j,t} + \mathbf{E}_t \left[\frac{\mathcal{M}_{t+1}}{\mathbf{E}_t[\mathcal{M}_{t+1}]} (\eta_{j,t+1} + \mathbf{E}_t(\phi_{j,t+1}) - \phi_{j,t+1}) \right]}{\bar{\epsilon}_j + \bar{\eta}_j + \bar{\phi}_{j,t+1} - \mathbf{E}_t(\phi_{j,t+1}) - \Phi_{j,t}}, \quad (40)$$

where

$$\mathcal{M}_{t+1} \equiv \frac{1}{(1+r)W_t + 2 \sum_{j \in \mathcal{A}_t} x_{j,t} [\Phi_{j,t} + \eta_{j,t+1} + \mathbf{E}_t(\phi_{j,t+1}) - \phi_{j,t+1}]}.$$

For these opportunities, positions are long in assets $i \in \mathcal{A}_t$.

- *The financial constraint (39) is slack if the maximum risk-adjusted return per unit of collateral is zero, and binds if it is positive.*

As in the case of riskless arbitrage, the arbitrageur invests only in those opportunities that offer the maximum return per unit of collateral. When arbitrage is risky, however, that return includes a risk adjustment that depends on arbitrageur risk aversion and position size. The risk adjustment for arbitrageurs is different from that for outside investors because market segmentation implies that arbitrageurs bear different risks.

The arbitrageur's portfolio problem combines two objectives: allocate scarce capital to the most profitable opportunities, and trade off risk and return. When the financial constraint is slack, only the second objective is present. The arbitrageur invests in any given arbitrage opportunity i to take advantage of its positive expected excess return $2\Phi_{i,t}$. As he increases his position in that

opportunity, he bears more risk and hence his risk-adjusted return decreases. The optimal position renders the risk-adjusted return equal to zero. This is the standard first-order condition of a risk-averse investor.¹⁷

If the optimal positions under a slack financial constraint violate that constraint, then the first objective of the portfolio problem kicks in. To meet the financial constraint, the arbitrageur scales down his positions. As a consequence, he bears less risk and risk-adjusted returns become positive. Positions are scaled down until the constraint is met, and in such a way as to maintain risk-adjusted returns per unit of collateral equal across all opportunities for which positions remain positive.

The two objectives of the arbitrageur's portfolio problem can be completely separated in two special cases. When asset payoffs are identical, only the first objective is present because the arbitrageur faces no risk. When instead the common payoff shock $\epsilon_{i,t+1}$ to both assets in a pair $(i, -i)$ is zero for all pairs ($\bar{\epsilon}_i = 0$), only the second objective is present: the arbitrageur determines his positions by trading off risk and return, and the financial constraint is always slack.

COROLLARY 1: *When $\bar{\epsilon}_i = 0$ for all $i \in \mathcal{A}_t$ and short positions in assets $i \in \mathcal{A}_t$ are not optimal, arbitrageurs' financial constraint is slack.*

The intuition for Corollary 1 is that when the common shock is not present, a long position in asset i and a short position in asset $-i$ can achieve their maximum loss at the same time. (That is not possible when the common shock is present because the long position achieves its maximum loss when $\epsilon_{i,t+1} = -\bar{\epsilon}_i$ while the short position achieves its maximum loss when $\epsilon_{i,t+1} = \bar{\epsilon}_i$.) Hence, requiring arbitrageurs to have sufficient wealth in each market to cover any liabilities in that market is equivalent to requiring that they be able to cover any overall liability. Moreover, arbitrageurs choose their positions so as to keep positive wealth and hence cover any overall liability because logarithmic utility makes zero consumption prohibitively costly.

B. Equilibrium for Small Arbitrage Risk

To study the case in which arbitrage risk is small, we set $\bar{\eta}_i = \lambda_i \bar{\eta}$ and take $\bar{\eta}$ to be small holding λ_i constant. We also specialize our model to the stationary "asset family" setting described at the beginning of Section II, and focus on parameters for which the steady state derived in Section II.C is interior, that

¹⁷ The standard first-order condition in a one-period setting is

$$E(\mathcal{M}R) = 0 \Leftrightarrow E(R) + E\left(\frac{\mathcal{M}}{E[\mathcal{M}]}\right)[R - E(R)] = 0,$$

where R is an asset's excess return over the riskless asset and \mathcal{M} is the pricing kernel. The risk adjustment is $E\left(\frac{\mathcal{M}}{E[\mathcal{M}]}\right)[R - E(R)]$. It depends on the size of the position, which is why the investor diversifies across assets. (That is, if the investor invested all his wealth in only one asset, the risk-adjusted return of that asset would decline below that of other assets, inducing the investor to also invest in the other assets.)

is, arbitrageur wealth does not converge to zero or infinity. Since arbitrageur wealth W_t converges to the steady-state value W when $\bar{\eta} = 0$, it converges to a stationary probability distribution with support close to W when $\bar{\eta}$ is small. When W_t moves within the support of the stationary distribution, as we assume from now on, the subset \mathcal{N}_t of families whose assets are traded in equilibrium could change over time. However, if we rule out the nongeneric case in which $f'(u_n \bar{\epsilon}_n) = \Pi$ for some $n \in \{1, \dots, N\}$, then for $\bar{\eta}$ small enough, \mathcal{N}_t remains constant over time and so does the set \mathcal{T}_t of traded active opportunities. We therefore drop the time subscript from both sets. Since the support of the stationary distribution is close to W , expected excess returns $\Phi_{i,t}$ of assets $i \in \mathcal{A}_t$ exceed a positive bound. Hence, short positions in assets $i \in \mathcal{A}_t$ are not optimal, arbitrageurs' first-order condition is (40), and the financial constraint (39) binds.

We look for spreads, expected excess returns, and positions of arbitrageurs and outside investors that take the form

$$\phi_{n,m,t} = \phi_{n,m,t}^0 + \phi_{n,m,t}^1 \bar{\eta} + o(\bar{\eta}), \quad (41)$$

$$\Phi_{n,m,t} = \Phi_{n,t}^0 + \Phi_{n,m,t}^1 \bar{\eta} + o(\bar{\eta}), \quad (42)$$

$$x_{n,m,t} = x_{n,t}^0 + x_{n,m,t}^1 \bar{\eta} + o(\bar{\eta}), \quad (43)$$

$$y_{n,m,t} = y_{n,t}^0 + y_{n,m,t}^1 \bar{\eta} + o(\bar{\eta}) \quad (44)$$

for asset (n, m) . The superscript 0 denotes the value of the corresponding variable, as a function of W_t , in the case of riskless arbitrage (Section II). The superscript 1 denotes a first-order term in $\bar{\eta}$ introduced by risk. As in Section II.C, we drop the time subscript from the zeroth- and first-order terms in (41) to (44) when these are evaluated at the steady-state value W .

We look for dynamics of wealth, within the support of the stationary distribution, that take the form

$$W_{t+1} = W + \rho(W_t - W) + v\bar{\eta} + \sum_{(n,m) \in \mathcal{T}} \sigma_n \eta_{n,m-1,t+1} + o(\bar{\eta}), \quad (45)$$

where $(\rho, v, \{\sigma_n\}_{n \in \mathcal{N}})$ are constants. We compute these constants, as well as the first-order terms in (42) to (44), in the Internet Appendix (Proposition IA1). We confirm in particular, that $\rho \in (0, 1)$. Thus, to a first order in $\bar{\eta}$, W_t follows an AR(1) process. The mean to which this process reverts is equal to W plus a first-order term in $\bar{\eta}$. Variation around the mean is caused by the shocks

$\eta_{n,m-1,t+1}$ for $(n, m) \in \mathcal{T}$. The coefficient σ_n describing how $\eta_{n,m-1,t+1}$ affects W_{t+1} is given by

$$\sigma_n = \frac{2\beta x_n^0}{1 + 2\beta \sum_{(n',m) \in \mathcal{T}} x_{n'}^0 \frac{d\phi_{n',m-1,t+1}^0}{dW_{t+1}}}, \quad (46)$$

where the derivatives in the denominator are evaluated at $W_{t+1} = W$. The intuition for (46) can be seen from the budget constraint (20). Substituting the optimal consumption (29) and changing asset subscripts from i to (n, m) , we can write (20) as

$$W_{t+1} = \beta \left[(1+r)W_t + 2 \sum_{(n,m) \in \mathcal{T}} x_{n,m,t} [\Phi_{n,m,t} + \eta_{n,m-1,t+1} + \mathbf{E}_t(\phi_{n,m-1,t+1}) - \phi_{n,m-1,t+1}] \right]. \quad (47)$$

A negative shock $\eta_{n,m-1,t+1}$ reduces the wealth of arbitrageurs through the term $2\beta x_{n,m,t} \eta_{n,m-1,t+1}$. This is the direct effect of the shock, holding spreads $\phi_{n',m-1,t+1}$ constant, and corresponds to the numerator in (46). There is also an indirect *amplification* effect that operates through a change in spreads: because W_t decreases due to the direct effect, spreads increase and this amplifies the reduction in W_t . The indirect effect corresponds to the second term in the denominator in (46), which is negative and thus lowers the denominator and raises σ_n .

C. Economic Implications

C.1. Spreads, Returns, and Positions in the Cross-Section

An arbitrage opportunity (n, m) is described by five characteristics: the volatility $\bar{\epsilon}_n$ of the payoff shock that is common to the two assets and that arbitrageurs can hedge via a long-short trade; the volatility $\bar{\eta}_n$ of the payoff shock that arbitrageurs cannot hedge away; the endowment shock u_n of outside investors, which determines their relative demand for the two assets; the measure μ_n of outside investors; and the horizon m of the opportunity. We examine how these characteristics affect spreads $\phi_{n,m,t}$, expected excess returns $\Phi_{n,m,t}$, and arbitrageur positions $x_{n,m,t}$.

PROPOSITION 9: *Suppose that arbitrage risk $\bar{\eta}$ is small. Holding constant all other characteristics as well as arbitrageur wealth:*

- *An opportunity with higher hedgeable volatility $\bar{\epsilon}_n$ has a higher spread and expected excess return, and attracts more investment by arbitrageurs.*
- *An opportunity with higher nonhedgeable volatility $\bar{\eta}_n$ has a higher spread and expected excess return, and attracts less investment by arbitrageurs. The same holds for an opportunity with longer horizon m .*
- *An opportunity with larger endowment shock u_n attracts more investment by arbitrageurs. It has a higher spread and expected excess return, except*

when the comparison is between traded opportunities, in which case spreads and expected excess returns are the same to the first order in $\bar{\eta}$.

- An opportunity with larger measure of outside investors μ_n attracts more investment by arbitrageurs.

Outside investors in more volatile opportunities, that is, those with higher $\bar{\epsilon}_n$ or $\bar{\eta}_n$, are more eager to share risk. Therefore, those opportunities offer higher expected excess returns in arbitrageurs' absence. Their expected excess returns remain higher even if arbitrageurs invest in them. Indeed, arbitrageurs invest in opportunities with the highest return per unit of collateral, and more volatile opportunities require more collateral. In turn, higher returns imply higher spreads. Indeed, spreads are a present value of future expected excess returns discounted at the riskless rate,

$$\phi_{n,m,t} = \mathbb{E}_t \left(\sum_{s=0}^{m-1} \frac{\Phi_{n,m-s,t+s}}{(1+r)^{s+1}} \right), \quad (48)$$

as can be seen by solving (13) backward with the terminal condition $\phi_{n,0,t+m} = 0$.¹⁸

The source of volatility, $\bar{\epsilon}_n$ or $\bar{\eta}_n$, matters for arbitrageurs' positions. When hedgeable volatility is higher (higher $\bar{\epsilon}_n$), positions are larger. When instead nonhedgeable volatility is higher (higher $\bar{\eta}_n$), positions are smaller. The intuition comes from higher volatility having two countervailing effects on arbitrageur positions: it raises positions because expected excess returns increase, but it lowers positions because collateral requirements increase. Expected excess returns in arbitrageurs' absence are proportional to the return variance, which for small $\bar{\eta}$ is proportional to $\bar{\epsilon}_n^2$ plus second-order terms in $\bar{\eta}$.¹⁹ Collateral requirements are equal to a position's maximum possible loss, which is $\bar{\epsilon}_n + \bar{\eta}_n + \bar{\phi}_{n,m-1,t+1} - \mathbb{E}_t(\phi_{n,m-1,t+1}) - \Phi_{n,m,t}$ from (39). The ratio of these quantities (return per unit of collateral) is increasing in $\bar{\epsilon}_n$ but decreasing in $\bar{\eta}_n$. In particular, an increase in $\bar{\eta}_n$ has a second-order effect on the variance but a first-order effect on the collateral requirement.

More generally, when arbitrage risk is small, its dominant effect on equilibrium variables is through the financial constraint rather than through risk aversion. Arbitrage risk raises the maximum possible loss of a position by a first-order term in $\bar{\eta}$. The risk adjustments that it induces in (37) and (40), however, are second order in $\bar{\eta}$, as they involve an expectation of gains and losses weighted by marginal utility. It is because of the effect through the financial

¹⁸ Because (13) is linear in spreads and expected excess returns, (48) is exact and not a small-risk approximation.

¹⁹ The return variance can be computed using (12), the independence between $\epsilon_{n,t}$ and $\{\eta_{n',t}\}_{n' \in \mathcal{N}}$, the fact that W_t is the only stochastic state variable affecting spreads, and the dynamics of W_t in (45). Note that since the effect of arbitrage risk on the return variance is second order in $\bar{\eta}$, our analysis of the equilibrium for small arbitrage risk would not change if instead of holding $\bar{\epsilon}_n$ constant when $\bar{\eta}_n$ varies, we also vary $\bar{\epsilon}_n$ so that the return variance in the absence of arbitrageurs remains constant. Indeed, the required variation in $\bar{\epsilon}_n$ would be second order in $\bar{\eta}$.

constraint that the equilibrium variables in (41) to (44) include a first-order term in $\bar{\eta}$.

The effect of horizon m on spreads follows from (48). Spreads of opportunities with a longer horizon are the present value of a longer sequence of future returns and hence are higher. The effect of horizon on expected excess returns and arbitrageur positions is more subtle. Spreads of opportunities with a longer horizon are more sensitive to shocks to arbitrageur wealth, as we show in Section III.C.2. As a consequence, these opportunities require more collateral (term $\bar{\phi}_{n,m-1,t+1} - \mathbb{E}_t(\phi_{n,m-1,t+1})$ in (39)), and higher collateral requirements push expected excess returns up and arbitrageur positions down. Note that horizon has no effect on expected excess returns or positions under riskless arbitrage. The effect instead arises because arbitrage risk impacts collateral requirements and is included in the first-order terms in (42) to (44).

The effect of the endowment shock u_n on spreads, expected excess returns, and arbitrageur positions is the same as under riskless arbitrage. In particular, spreads and expected excess returns increase with u_n for nontraded opportunities and become flat when u_n is large enough so that opportunities are traded. The flat part arises because traded opportunities offer the same return per unit of collateral and because u_n does not affect the collateral requirement. The effect of the measure μ_n of outside investors on arbitrageur positions is also the same as under riskless arbitrage: arbitrageurs hold larger positions in an opportunity where there are more outside investors.

C.2. Response to Shocks

We next examine how spreads, expected excess returns, and arbitrageur positions respond to shocks to arbitrageur wealth, and how these dynamics depend on the characteristics of arbitrage opportunities. We focus on traded opportunities; nontraded opportunities are not affected by shocks.

For any given traded opportunity, spreads, expected excess returns, and arbitrageur positions change over time both because arbitrageur wealth changes and because the opportunity's horizon shortens. We focus on the wealth-induced variation, keeping horizon constant. Hence, we compare spreads, expected excess returns, and arbitrageur positions across opportunities within the same family and with the same horizon (same (n, m)) but at different points in time.

PROPOSITION 10: *Suppose that arbitrage risk $\bar{\eta}$ is small and that a shock in period t lowers arbitrageur wealth below its mean.*

- *The immediate effect is that spreads and expected excess returns increase, and arbitrageurs scale down their positions.*
- *Following this immediate reaction, there is a recovery phase, during which spreads, expected excess returns, and positions are expected to revert gradually toward their original values.*

The dynamics in Proposition 10 follow from the self-correcting dynamics (45) of arbitrageur wealth. Following a shock that lowers wealth in period t ,

wealth is expected to increase gradually back to its mean. Spreads and expected excess returns are decreasing functions of wealth, and hence increase instantly and then decrease gradually. By contrast, positions are increasing functions of wealth, and hence decrease instantly and then increase gradually.

PROPOSITION 11: *Suppose that arbitrage risk $\bar{\eta}$ is small and that a shock in period t lowers arbitrageur wealth below its mean. Holding constant all other characteristics:*

- *For an opportunity with higher hedgeable volatility $\bar{\epsilon}_n$, the immediate increase in spread and expected excess return is larger, and the immediate decrease in arbitrageur positions is smaller.*
- *For an opportunity with higher nonhedgeable volatility $\bar{\eta}_n$, the immediate increase in spread and expected excess return is larger, and so is the immediate decrease in arbitrageur positions.*
- *For an opportunity with longer horizon m , the immediate increase in spread is larger. The immediate increase in expected excess return and the immediate decrease in arbitrageur positions are also larger if the steady-state value W of arbitrageur wealth W_t is large enough.*

These comparisons remain the same during the recovery phase.

Spreads and expected excess returns of more volatile opportunities, that is, those with higher $\bar{\epsilon}_n$ or $\bar{\eta}_n$, are more sensitive to changes in arbitrageur wealth. This is because changes in wealth impact the return per unit of collateral, which arbitrageurs equalize across all opportunities in which they invest. The resulting impact on returns, and hence on spreads, is stronger for more volatile opportunities because they require more collateral.

Arbitrageur positions in more volatile opportunities can be more or less sensitive to changes in wealth, depending on the source of volatility. Positions are less sensitive when volatility is higher because of $\bar{\epsilon}_n$, but more sensitive when it is higher because of $\bar{\eta}_n$. The intuition parallels that in Proposition 9. Following a drop in arbitrageur wealth, positions in all opportunities must be scaled down, and the return per unit of collateral is equalized across opportunities at a higher level. The increase in expected excess returns resulting from a cut in arbitrageur positions (i.e., the inverse elasticity of the demand of outside investors) is proportional to the return variance, and hence to $\bar{\epsilon}_n^2$ plus second-order terms in $\bar{\eta}$. The ratio of this quantity to the collateral required $\bar{\epsilon}_n + \bar{\eta}_n + \bar{\phi}_{n,m-1,t+1} - \mathbf{E}_t(\phi_{n,m-1,t+1}) - \Phi_{n,m,t}$ is increasing in $\bar{\epsilon}_n$, and so positions in opportunities with higher $\bar{\epsilon}_n$ must be cut by less to achieve the new higher level of return per unit of collateral. The ratio is instead decreasing in $\bar{\eta}_n$, and so positions in opportunities with higher $\bar{\eta}_n$ must be cut by more. Put differently, by cutting positions in opportunities with higher $\bar{\eta}_n$, arbitrageurs save a larger amount of collateral, and the returns of these opportunities do not increase sufficiently following the cuts to compensate for that.

Spreads of opportunities with a longer horizon are more sensitive to changes in wealth because they depend on a longer sequence of future returns, all of

which are sensitive to wealth changes. Because their spreads are more sensitive to changes in wealth, collateral requirements are higher for long-horizon opportunities. In turn, this implies that expected excess returns and arbitrageur positions for long-horizon opportunities tend to be more sensitive to wealth changes. At the same time, wealth changes affect the required collateral as a function of horizon because they affect the volatility of equilibrium prices. If the incremental collateral required by longer-horizon opportunities decreases following a wealth drop, then expected excess returns and arbitrageur positions for these opportunities can be less sensitive to changes in wealth. This possibility is ruled out if W is large enough.

Taken together, the results of Sections III.C.1 and III.C.2 imply an increasing cross-sectional relationship between spreads or expected excess returns on the one hand and betas with respect to arbitrageur wealth on the other. Indeed, Proposition 9 shows that spreads and expected excess returns are higher for more volatile opportunities and for opportunities with a longer horizon. Moreover, Proposition 11 shows that spreads for the same opportunities increase more following a drop in wealth. Hence, their realized returns decrease more and their wealth betas are higher.

The cross-sectional relationship implied by our model is consistent with recent empirical evidence. Avdjiev et al. (2016) study the behavior of the CCB, which is defined in Section I.B.3 and is an arbitrage spread associated with violations of CIP. They find that currencies with a higher basis are also those for which the basis increases more when the dollar exchange rate increases. Moreover, increases in the exchange rate are associated with tighter funding conditions of currency market arbitrageurs. Cho (2016) studies the relationship between alphas of a wide cross-section of stock market anomalies and the sensitivities of these anomalies' returns to the funding conditions of broker-dealers. He finds no relationship before 1993. After 1993, however, anomalies with higher alphas are also those that yield lower returns when funding conditions tighten. These findings are consistent with arbitrageurs becoming more active in trading the anomalies in the more recent sample.²⁰

We show that an increasing cross-sectional relationship between spreads/returns and wealth betas arises because of two mutually reinforcing mechanisms. High collateral requirements cause spreads and expected excess returns to be high (through the equalization of return per unit of collateral) and wealth betas to be high (through the dynamics implied by same channel). Moreover, high wealth betas cause collateral requirements to be high (because the maximum possible loss of a position increases) and feed back into high spreads and expected excess returns. Exogenous characteristics can “activate” either of the two mechanisms, setting off the mutually reinforcing cycle. Payoff shock volatility ($\bar{\epsilon}_n$ and $\bar{\eta}_n$) activates the first mechanism, as higher volatility pushes

²⁰ Cho (2016) also finds that anomalies with higher profitability before 1993 tend to have higher wealth betas after 1993. This is consistent with arbitrageurs allocating more wealth to the more profitable anomalies: anomalies with small u_n do not attract any investment and are not sensitive to changes in wealth, while the opposite holds when u_n exceeds a threshold.

collateral requirements up. Horizon (m) activates the second mechanism, as a longer horizon renders spreads more sensitive to wealth. Versions of the first mechanism have been shown in a number of papers (e.g., Gromb and Vayanos (2002), Geanakoplos (2003), Brunnermeier and Pedersen (2009), Garleanu and Pedersen (2011), Brumm et al. (2015)). The second mechanism is new to our model.

D. Mobility of Arbitrage Capital

Finally, we use our model to study how the mobility of arbitrage capital affects market stability. Our maintained assumption so far is that all arbitrageurs can trade all assets and hence arbitrage capital is fully mobile across markets. We contrast full mobility to the case in which the assets in each family pair ($n, -n$) for $n = 1, \dots, N$ are traded by a separate set of arbitrageurs. We refer to the former case as integrated arbitrage markets and to the latter as segmented arbitrage markets. These notions of integration and segmentation are distinct from the asset-level segmentation concerning outside investors and collateral requirements, which we continue to assume. Integration of arbitrage markets could be triggered, for example, by deregulation of international capital flows.

When arbitrage is riskless, integration and segmentation of arbitrage markets yield the same steady state. Indeed, Proposition 6 applied to each segmented arbitrage market implies that arbitrageurs in market n have nonzero wealth in steady state if $f'(u_n \bar{\epsilon}_n) > 1 - \beta(1 + r)$. Moreover, the return per unit of volatility is $\Pi = 1 - \beta(1 + r)$ in the markets where arbitrageur wealth is nonzero, and $f'(u_n \bar{\epsilon}_n) \leq \Pi$ in the markets where it is zero. Since this return is the same across the nonzero-wealth markets and is lower in the zero-wealth markets, lifting the segmentation restriction has no effect: arbitrageurs are indifferent between staying in their market or diversifying into other nonzero-wealth markets, and the return per unit of volatility in all markets does not change.

COROLLARY 2: *Suppose that $\bar{\eta} = 0$ (riskless arbitrage). In steady state, integration of arbitrage markets has no effect on spreads and returns.*

When arbitrage is risky, arbitrageurs strictly prefer diversifying across arbitrage markets as long as risks are imperfectly correlated. We assume that risks are independent across markets, that is, the shocks $\eta_{n,m,t}$ are independent across n . But while diversification is beneficial for any given arbitrageur assuming that others do not diversify, it has a countervailing effect when they all diversify: it induces correlation between arbitrage markets, which makes diversification less effective. This is because arbitrageurs who hold positions in multiple markets react to negative shocks in one market by cutting positions in all markets—a contagion effect.

To examine whether diversification remains effective despite the correlation that it induces, we compute the variance of arbitrageur wealth. We also compute the variance of spreads, which can be interpreted as a measure of market stability. We first compare integration and segmentation when arbitrage

opportunities are symmetric, that is, $(\bar{\epsilon}_n, \bar{\eta}_n, \mu_n, u_n, M_n)$ is independent of n . We then consider the asymmetric case.

PROPOSITION 12: *Suppose that arbitrage risk $\bar{\eta}$ is small and that arbitrage opportunities are symmetric.*

- *The variance of each arbitrageur's wealth under integration is $\frac{1}{N}$ times that under segmentation. It is also equal to the wealth variance of an arbitrageur who diversifies across markets when other arbitrageurs do not.*
- *The variance of each spread under integration is $\frac{1}{N}$ times that under segmentation.*

An arbitrageur who diversifies across markets when other arbitrageurs do not enjoys a reduction in the variance of his wealth by a factor of N . This follows from a standard result in portfolio theory: optimal diversification across N assets with i.i.d. returns results in a variance that is N times lower than without diversification. Surprisingly, diversification lowers the variance by a factor of N even when all other arbitrageurs diversify and hence markets become correlated. This is because while spreads become perfectly correlated across markets, their variance is divided by N .

The intuition for why the variance of the spreads is divided by N under integration is as follows. Since the aggregate position of arbitrageurs in each market is the same under integration as under segmentation when arbitrage is riskless, the same holds for small arbitrage risk to the highest order in $\bar{\eta}$. Hence, a negative shock $\eta_{n,m,t}$ causes the same drop in aggregate arbitrageur wealth under integration as under segmentation, holding spreads constant. Under segmentation, only arbitrageurs in market n are affected, and they cut their positions in that market. Under integration, all arbitrageurs are affected, and they cut their positions in all markets. Because the drop in aggregate wealth is the same in both cases and because markets are symmetric, the cut in each market under integration is $\frac{1}{N}$ times the cut in market n under segmentation. Hence, spreads in all markets under integration increase by $\frac{1}{N}$ times the increase in the spreads in market n under segmentation. Moreover, the contribution of the shock $\eta_{n,m,t}$ to the variance of the spreads in market n under integration is $\frac{1}{N^2}$ of its contribution under segmentation. Because, however, N times as many shocks (i.e., all shocks) contribute to the variance under integration, the variance of the spreads is $N \times \frac{1}{N^2} = \frac{1}{N}$ times that under segmentation.

PROPOSITION 13: *Suppose that arbitrage risk $\bar{\eta}$ is small and that arbitrage opportunities are asymmetric. Spreads of traded opportunities $n \in \mathcal{N}$ for which $\frac{u_n \bar{\epsilon}_n - (f')^{-1}(\Pi)}{\max_{n' \in \mathcal{N}} [u_{n'} \bar{\epsilon}_{n'} - (f')^{-1}(\Pi)]}$ or $\frac{\bar{\eta}_n}{\max_{n' \in \mathcal{N}} \bar{\eta}_{n'}}$ are sufficiently small have higher variance under integration than under segmentation.*

Integration can raise the variance of spreads of opportunities with small endowment shocks (small u_n) or low volatility (small $\bar{\epsilon}_n$ or $\bar{\eta}_n$). For example, because arbitrageurs hold small positions in opportunities with small u_n , payoff

shocks $\eta_{n,m,t}$ have small effects on their wealth and on spreads, resulting in low variances under segmentation. Instead, under integration, these opportunities are exposed to shocks coming from other markets, so variances can increase.

IV. Conclusion

We develop a model in which arbitrageurs' limited access to capital affects the functioning of financial markets. Arbitrageurs in our model are uniquely able to exploit price discrepancies between assets traded in segmented markets, but face financial constraints limiting their ability to do so. We compute the equilibrium in closed form when arbitrage is riskless and when arbitrage risk is small. In the latter case, arbitrage capital follows AR(1) dynamics in a stochastic steady state. We determine how arbitrageurs allocate their limited capital across mispriced assets in steady state, and how this allocation changes following shocks to capital. We also determine which arbitrage trades offer the highest expected returns and how these returns relate to the trades' sensitivity to arbitrage capital and other characteristics. We finally examine how the diversification of arbitrageurs across markets affects the risk they bear and the volatility of spreads.

When arbitrage risk is small, its first-order effect on equilibrium variables is through the financial constraint rather than through risk aversion. Hence, our results on how characteristics such as volatility, horizon, and investor demand affect expected returns and arbitrageur positions are driven by the characteristics' effects on the financial constraint. For general arbitrage risk, risk aversion would come into play, and its effects might differ from those of the financial constraint. Determining the combined effects of risk aversion and the financial constraint on the cross-section of expected returns and arbitrageur positions is a natural extension of our research. This would require computing the equilibrium for general arbitrage risk. Analysis of this equilibrium would shed light on the role of arbitrage capital as a risk factor, and on how the significance of this factor is affected by the capitalization and diversification of arbitrageurs.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.