RENEGOTIATION IN DEBT CONTRACTS

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> *"Hum! Hum!" said the king. "[...] You can judge this old rat. From time to time you will condemn him to death. Thus his life will depend on your justice. But you will pardon him on each occasion; for he must be treated thriftily. He is the only one we have."*
>
> In The Little Prince, by Antoine de Saint-Exupéry.

1 Introduction

This paper studies dynamic financial contracting when cash flows are not contractible. In such contracts, the prevention of default relies on the threat that the borrower be denied access to credit in the future. We identify the ability of parties to renegotiate the initial contract as an important constraint on the form and profitability of optimal contracts.

In credit transactions, the terms of the exchange are not simultaneous. For instance, the repayment of a loan falls due some time after the loan is granted. Because of their dynamic nature, credit arrangements have to take into account the possibility that one party, the borrower, does not deliver his term of the exchange in the future. At the time the borrower should repay the loan, he may be unable to do so, typically because the project financed by the loan failed, was delayed or did not generate enough returns. This is a case of *liquidity default.* However, the debtor may be able to repay the loan but choose not to do so. This is a case of *strategic default.*

Preventing strategic defaults is crucial to the profitability of credit transactions for lenders, and thus, in some cases, to the very existence of credit markets. The question is, how to make sure that the debtor repays whenever he can? One idea is to impose direct costs of default. A priori, criminal penalties could be used. Alternatively, the lender could be entitled to seize a collateral, for instance some physical assets financed through the loan. However, direct penalties may not be completely effective when it is costly to distinguish liquidity from strategic defaults. Severely punishing defaults irrespective of their being voluntary or not would lead to unjust punishments being enforced. This could in turn deter potential borrowers from taking loans, and so the credit market might collapse. Hence, the mere existence of a credit market is likely to necessitate moderate penalties and that the borrowers' liability be limited.

Several authors have emphasized that the repetition of lending can help discipline a borrower without collateral or when, as in the case of a sovereign country, courts have little power to enforce contracts. The general idea is that the borrower's current behavior can impact on the terms of his access to credit in the future. Hence, there may be indirect costs of default. For instance, a borrower with a bad track record is likely to face high interest rates in the future due to his bad reputation. Even absent reputation effects of default, the lenders can simply decide to deny further access to credit after too many defaults. Then, the current repayment is (less or) equal to the borrower's valuation of future access to credit. The more valuable future credit is, the higher the current repayment can be. In a way, the borrower is a prisoner of the profitability of his future projects.

This paper turns this idea on its head and shows how the lender can be the prisoner. We highlight the credibility problem that limits the effectiveness of such a threat. If the future relationship between the lender and the borrower is profitable, the lender has little incentive to put it to an end ex-post, i.e. after a default. Hence, ex-ante the threat of denying the borrower new capital is weakened.

We illustrate this idea and its implications by extending a simple model of lending without collateral developed by Bolton and Scharfstein (1990). Their model assumes that profits are not verifiable so that liquidity and strategic defaults are indistinguishable. Also, any reputation effect is assumed away. We first show that, in their framework, repeated lending can be interpreted as a sequence of gifts and sales from the lender to the borrower. This is easily illustrated in a two-period setting. The borrower has two consecutive projects. The optimal credit arrangement is as follows: lender invests in the first project and makes his funding the second one conditional on the borrower meeting the first loan's repayment. The repayment of the first loan is thus actually the price the borrower pays for the lender's second investment. The first loan can thus be interpreted as a gift. It is made so that the borrower invests in a profitable project, becomes rich and able to "buy" the second investment from the lender. The first stage can be seen as a *development phase* in which the lender invests in the borrower's project so that he becomes rich. Then he can be a customer of the lender in the second stage, which is thus a *surplus extraction phase* in that the borrower pays the lender with the profits he received in the development phase.

We show that, when contracts are binding, this two-phase feature extends to the case of more than two projects. A first sequence of projects are financed irrespective of whether the borrower repays or not. This corresponds to a *development phase*. Then, a second sequence is financed only conditional on repayments being made. This is a *surplus extraction phase* which relies on the termination threat, i.e. the threat that following one default too many, no new project will be financed.

However, the credibility of such a policy is an issue. If the termination threat was credible, it would prevent strategic defaults. Hence after a series of defaults, the lender would know that the borrower is simply unable to meet the repayments. However, bad fortune in the past does not mean that the future relationship is not profitable. The lender would then have an incentive to start a development phase again. The lender needs to fully commit to the termination threat via a contract. Suppose that, on the contrary, contracts can be renegotiated, that is they can be altered as soon as both parties agree on a new version. Then the termination threat loses force because both the lender and the borrower want to sign a new contract which makes the most of the remaining projects.

We try and capture this credibility issue in a crude version of the model. The incentive to renegotiate away a termination threat is increased by the assumption that profits disappear between two investment stages: at the beginning of each period the game is in the same state after a liquidity or a strategic default; hence, the incentive to start a development phase after a liquidity default extends completely to the case of a strategic default. (This assumption is not crucial for the effect to exist. It only makes it extreme and all the intuitions of the paper would remain valid without it). Indeed we show that, as soon as there are more than two projects, the optimal full commitment contract does not resist renegotiation. Hence, the ability of both parties to renegotiate contracts reduces the lender's surplus, sometimes to the point that lending collapses.

We then extend the analysis to the infinite horizon version of the model. In order to analyze the impact of renegotiation, we extend a concept of renegotiation-proofness developed by Farrell and Maskin (1989) in the context of infinitely repeated games. The main idea behind this approach is that an equilibrium outcome cannot rely on a threat that it Pareto dominates. We show that the investor makes zero profit along any renegotiation-proof contract. The result is very intuitive when the lender's only threat is the definitive termination of the relationship (as opposed to a temporary exclusion of the borrower from the credit market). The reasoning is by contradiction. Suppose that the lender was able to make a strictly positive expected profit. In the absence of a collateral, he has to use the termination threat. In the event of the threat becoming reality, both he and the borrower make zero profit. Hence, the threat is Pareto dominated by the outcome it is supposed to support, a contradiction with the Farrell and Maskin criterion. We show that this result extends to temporary and stochastic exclusions.

This paper belongs to a literature in which the costs of default determine a borrower's incentive to repay. Departing from the early works with exogenous costs, Allen (1981, 1983) studies the endogenous default cost from losing access to future credit. The value of access to credit itself decreases with the repayments on future loans. Hence he derives a closed-loop solution. Allen assumes that the termination threat is credible, suggesting conditions (such as reputation building by the lender) favoring its credibility.

The papers most related to ours are Farrell (1989), Hart and Moore (1991), and Thomas and Worral (1990). Hart and Moore present a dynamic theory of debt based on the ability of the borrower to withdraw from the relationship at any point. Since doing so would reduce the value of the project, his threat triggers the renegotiation of the original debt contract.

Farrell models repeated lending to a sovereign state as an infinitely repeated simultaneous move game to which he applies the concept of Renegotiation-Proof Subgame Perfect Equilibrium (see Farrell and Maskin, 1989). He proves that such an equilibrium exists, in which the efficient outcome is sustained, i.e., all projects are financed, even though the borrower defaults regularly. In contrast to our result, the lender makes a positive profit. However, as we show, his result relies on a simultaneous move model (see Section 5.4.2), an assumption that does not capture the dynamic nature of lending.

The rest of the paper proceeds as follows. Section 2.1 presents the model and reinterprets Bolton and Scharfstein's analysis. Sections 3 and 5 analyze the case of a finite and an infinite horizon respectively. Section 6 sketches some extensions. Section 7 concludes.

Some mathematical proofs are in the appendix.

2 A Framework to Analyze Repeated Lending

2.1 The Model

Our model of financial contracting based on the non-contractibility of cash flows extends Bolton and Scharfstein (1990)'s. A risk-neutral entrepreneur has one investment project in each of T periods, $t = 1, \ldots, T$. Within period t, the t^{th} project requires an investment outlay F and generates a cash flow $X_t \in \{X^L, X^H\}$ with $\Pr[X = X^L] \equiv \theta$ and $X^L < F < X^H$. Each project's expected cash flow $\hat{X} \equiv (1-\theta)X^H + \theta X^L$ exceeds the investment outlay, i.e., $\hat{X} > F$. Without the investment, the entrepreneur receives a cash flow $X^{\emptyset} = 0$. The T projects are thus identical and independent of each other. The entrepreneur is indispensable for each project, having no initial wealth, has to raise the necessary funds from a risk-neutral investor with a financial contract under two constraints.

Assumption 1 *The cash flows X_t are not contractible.*

The entrepreneur can divert corporate resources at the expense of the investor. This amounts to assuming prohibitively large monitoring costs.

Assumption 2 *The entrepreneur's liability is limited: he cannot be forced to make a repayment exceeding the cash flow he reports in the current period.*[1]

Additionally, we assume that collateral cannot be used either because there are no assets to be seized or because assets cannot be seized easily as in the case of a borrowing foreign country.

Let R_t denote the entrepreneur's payment to the investor in period t, let $\beta_t = 1$ if the t-th project is financed, and $\beta_t = 0$ otherwise, and let $\delta \in [0,1]$ be a constant discount rate. The entrepreneur and the investor's payoffs are

$$\sum_{t=1}^{T} \delta^{t-1}(X_t - R_t) \text{ and } \sum_{t=1}^{T} \delta^{t-1}(R_t - \beta_t F)$$

[1] A more general definition of limited liability is that the current repayment cannot exceed the current wealth implied by the entrepreneur's past and current reports net of his past repayments. Both definitions, though different in general, turn out to be equivalent in the contexts to which we will restrict our attention.

2.2 Lending = Giving + Selling

We first sketch a solution to Bolton and Scharfstein (1990)'s two-period model, developing the intuition that we will use repeatedly to analyze models with longer horizons. Consider first an isolated project (i.e., $T = 1$) and suppose that the investor has financed it. Once the project's cash flow X_1 is realized, the entrepreneur can always report X^L even if in reality $X_1 = X^H$. Under limited liability, the entrepreneur cannot be made to repay more than X^L. Since $X^L < F$, the investor makes a loss and is better off not financing the project.

Lemma 1 *For $T = 1$, the project is not financed.*

The no financing outcome stems from the entrepreneur's insufficient wealth. With wealth W, the entrepreneur would only need to raise $F - W$. For $F - W < X^L$, a contract involving a repayment of X^L would be enforceable and yield a positive net profit to the investor. This also implies that T projects cannot be financed independently of each other. The question arises whether repeated lending can provide incentives to repay.

Assumption 3 *The investor is or can be made indispensable for the realization of each project.*

This means that the investor has or can be given veto power over all projects. This may stem from a contract: when the fact that a project takes place is verifiable, the investor can be allocated veto power over the entrepreneur's future projects. For the sake of clarity, in the rest of the paper we will adopt this interpretation. Yet, such a contract can also be implemented if the investor owns (or is contractually given the ownership of) some input which is crucial to the project (this input can but need not be the investment F). For instance, the part of X_1 that is non-contractible does not cover the second investment cost, i.e., $X^H - X^L < F$ so that outside finance is needed for the second project. It must then also be ensured that the entrepreneur cannot raise funds from parties other than the initial investor. This may arise through a contract if transfers to third parties are contractible. This can also arise endogenously if the initial investor has an informational advantage over other potential investors. It must then be ensured that the entrepreneur cannot raise funds from parties other than the initial investor.

Bolton and Scharfstein derive the optimal contract in the case where the investor proposes the initial contract through a take-it-or-leave-it offer.

Assumption 4 *The investor has all the bargaining power in setting the initial contract.*

Consider now the case of two projects. Suppose that the investor has financed the first one. After the cash flow is realized, the entrepreneur cannot be forced to repay more than X^L. Since $X^H > F$, there is some positive probability that his wealth exceeds $F - X^L$. When this is the case, as explained above, it is possible to finance the second project as well.

The investor can sell to the entrepreneur his veto power over the second project and so extract part of X_1, which might allow him to break even over the two periods. What is the investor's payoff? The investor sinks F in the first project which yields him zero gross profit. Then, he sells veto power over the second project. Since he has full bargaining power, the price he sets equals the entrepreneur's valuation for that project, $\hat{X} - X^L$. When $X_1 = X^H$, he accepts and pays $\hat{X}$ (note that since $X^H > \hat{X}$, the entrepreneur's budget constraint is not binding). In that case, the investor extracts all the second period expected social surplus, $\hat{X} - F$. If $X_1 = X^L$, which happens with probability θ, the entrepreneur is unable to pay and the second project is not financed. The investor's surplus is thus

$$-F + \delta \left[(1-\theta)(\hat{X} - F) + \theta \cdot 0 \right] \tag{1}$$

For simplicity, we normalize the low cash flow to zero: $X^L = 0$.[2]

Proposition 1 *(Bolton and Scharfstein, 1990) The investor finances the first project if* $-F + \delta(1-\theta)(\hat{X} - F) > 0$. *At the end of the first period, the entrepreneur is given the choice between repaying* $R^L = 0$, *in which case the second project is not financed, and repaying* $R^H = \hat{X}$, *in which case the second project is financed.*

Lending à la Bolton and Scharfstein has two different phases. First comes a *development phase* in which, at a cost, the investor provides the entrepreneur with input (say capital). Then comes a *surplus extraction phase* in which he sells his participation to future projects. The point of the first phase is only to allow that with some sufficient probability the entrepreneur becomes sufficiently rich and so is able to pay the investor for future projects.[3]

[2] This assumption ensures that the optimal contract is unique. All our results extend the case $X^L > 0$. Indeed, for $X^L > 0$, one can renormalize the variables as follows. The low cash flow is $X^L = 0$, the high cash flow $X^H = X^H - X^L$, and the investment outlay $F - X^L$.

[3] In that respect, Bolton and Scharfstein's model tells a development aid story. A rich country subsidizes a poor one so that it develops into a trading partner.

In what follows we assume that, in the two-project case, the investor invests.

Assumption 5 $-F + (1-\theta)\delta(\hat{X} - F) > 0.$

In order to concentrate on the effect of the repetition of lending, we study a purely repeated situation. In particular, this requires that the cash flows retained by the entrepreneur do not transfer from one period to the next (see however Section 6).

Assumption 6 *Any cash flow retained by the entrepreneur in one period,* $X_t - R_t$, *does not transfer to the next.*

3 Finite Horizon: Full Commitment Contracts

We first use the intuition developed in the two-period case to characterize optimal contracts under full commitment, i.e., when contracts cannot be altered once signed.

3.1 "Simple Contracts"

We examine the example of $T = 3$ and $\delta = 1$ to derive some properties of optimal contracts.

Property 1 *The terms of a contract in period* t *depend only on history.*

A financial contract (is equivalent to a contract that) takes the following form. We represent histories up to and including period t as $h_t \in \{\emptyset, L, H\}^t$, where the τ-th term of the t-uple is $\emptyset$ if the τ-th project was not undertaken, and $\sigma \in \{L, H\}$, if it was financed and the entrepreneur reported X^σ. At the beginning of period $t+1$, the contract specifies the probability $\beta(h_t)$ with which the investor will finance the project, as well as the entrepreneur's repayment to the investor, $R(h_{t+1})$ which can take one of three values, $R^\emptyset(h_t), R^L(h_t), R^H(h_t)$, depending on whether the $(t+1)$-th project is not financed, and when it is financed, whether the entrepreneur reports $X_t = X^L$ or $X_t = X^H$.

Property 2 ***(Limited Liability)*** $R^L(h_t) = 0$ *and* $R^\emptyset(h_t) = 0.$

Suppose for now that the first project is financed. At the end of the first period, the entrepreneur can always report and repay $X^L = 0$. ΠE^L denote the entrepreneur's expected surplus after he reported X^L.[4] Instead, if the entrepreneur reports X^H at the end of the

[4] Note that we implicitly assume that the entrepreneur's surplus after he defaults is independent of whether the default was a liquidity default or a strategic one. In doing so, we use the assumption that profits are not transferable so that the past does not affect the outcome of the continuation contract.

first period, he is rich and known to be so. Hence, the investor can sell him his veto right over some of the remaining projects. Since the investor has all the bargaining power, he sells them for a price per project equal to the entrepreneur's valuation $\hat{X}$. More generally

Property 3 ***(Make them pay)***

$$R^H(h_t) = \sum_{s=1}^{T-t} \left[\beta_{t+s}(h_t, H) - \beta_{t+s}(h_t)\right] \cdot \delta^s \cdot \hat{X}$$

Since these projects are profitable, and the investor has all the bargaining power, he makes a profit on each veto right he sells. Hence, he will sell as many as the entrepreneur can buy. If the entrepreneur's budget constraint is not binding, the investor sells him the veto rights over the two remaining projects.

$$R_1^H = 2\hat{X} - \Pi E^L$$

This is true only as long as the entrepreneur's wealth X^H exceeds this price.

If instead

$$X^H < 2\hat{X} - \Pi E^L \tag{2}$$

then, the investor is better off not selling all remaining projects otherwise he would sell some of them at a loss. More generally, the investor should sell as many projects as the entrepreneur can buy.

Property 4 ***(Sell as much as possible)***

$$\textit{If } R^H(h_t) < X^H \textit{ then } \forall s \geq 0,\ \beta_{t+s}(h_t, H) = 1$$

However, which projects he sells is not indifferent. Suppose he had the choice between selling the second or the third project. By selling the third project, he only gets the price at which the entrepreneur values the project, $\hat{X}$. By selling the second project, not only does he get the same price but he also makes it possible that the entrepreneur will receive the high profit in this second project. If so, he is then able to buy the third project, which benefits the investor. In other words, when the budget constraint is binding, selling early projects preserves the chance of further trades, i.e., of selling the last projects.

Property 5 ***(Sell early projects)***

$$\textit{If } \beta_{t+s}(h_t, H) > \beta_{t+s}(h_t) \textit{ then } \forall s' < s,\ \beta_{t+s'}(h_t, H) = 1$$

Note however that since $X^H > \hat{X}$, the entrepreneur is able to buy the second project and a fraction of the third, Hence, the investor can sell to the entrepreneur the right to undertake the second project for sure, and the third one with probability $\beta \in (0,1)$ such that

$$X^H = (1+\beta)\hat{X} - \Pi E^L$$

These properties can be formalized and generalized so as to prove the following result.

Definition 1 *We call "simple" a contract satisfying Properties ??? to ???.*

[5]

Proposition 2 *The optimal full commitment contracts are simple contracts.*

Two results will be useful. First, property (iv) implies that when $X_t = X^H$, the entrepreneur's incentive compatibility constraint is binding: he is indifferent between defaulting strategically or not, i.e., reporting X^L or X^H.

Lemma 2 *In a simple contract, the entrepreneur's payoff is equal to that he would get by always reporting X^L (even when $X_t = X^H$).*

Second, the investor strictly prefers to avoid strategic defaults. Since he can commit to the continuation contract following a default, he is able to separate lucky from unlucky entrepreneurs by reducing all repayments R_t^H by an infinitesimal quantity so that the entrepreneur is induced to repay rather than default.

Lemma 3 *Simple contracts have the following properties:*

(i) are fully separating, i.e. there are no strategic defaults in equilibrium.

Lemma 4 *A "simple" contract is fully characterized by a sequence of T "zeros" and "ones", ending with a zero, in which the t-th term is the probability that the t-th project be financed given that the entrepreneur defaulted on all previous financed periods.*

Proof. Given the sequence, the contract can be constructed by backward induction: define $\Pi E(h_T, X^L) = 0$ for all histories h_T of the whole game; suppose that $\Pi E(h_t, X^L)$ is

[5]When $X^L > 0$, property (ii) can be rewritten as $R^L = X^L$ which is intuitive: to make strategic default as unattractive as possible and liquidity defaults the least costly possible, the investor should maximise the repayment in case of default. By limited liability, this is bounded from above by X^L.

defined; then, $R^H(h_t)$ and the continuation contract are defined by property (iii); hence, as a consequence, $\Pi E(h_{t-1}, X^L)$ is defined and so the induction can go one step backwards. ■

It is easy to check that $\langle 1, 0\rangle$ is the contract described in Proposition 1. Given Proposition 2, it is also easy to prove its optimality (and so Proposition 1 is a corollary of Proposition 2). The only other candidate is $\langle 0, 0\rangle$ in which no project is financed so that the investor's profit is zero. Hence, $\langle 1, 0\rangle$ being the only simple contract yielding a positive profit to the investor, it is optimal.

3.2 The Termination Threat

We can prove that the two-phase pattern of the optimal contract in the two-project case extends to more than two projects.

Proposition 3 *Optimal full commitment contracts have the following form*

$$\langle 1, ..., 1, 1, 0, 0, ...0\rangle$$

Under full commitment, optimal contracts have a simple form with two consecutive phases. In the first phase, the *development phase*, the renewal of investments by the investor is unconditional. More specifically, the investor finances the projects even if the entrepreneur has defaulted on past loans. Then, the investor's policy switches to a *surplus extraction phase* in which no new investment is made after a default. In other words, the investor disciplines the entrepreneur by using the threat to lose access to future credit, or *termination threat.*

We now characterize the optimal full commitment contract for $T = 3$. Lemma 2 provides an easy way to compute the entrepreneur's surplus; Lemma 3 does the same for the total surplus; the investor's surplus is the difference between the two.

Result 1 *For $T = 3$, the full commitment contract is $\langle 1, 0, 0\rangle$.*

We present here an intuitive proof in the case where the entrepreneur's budget constraint is never binding. [6] Also, for simplicity, we will focus on the case where there is no

[6] This amounts to assuming $X^H \geq (\delta + \delta^2)\hat{X} - 0$, i.e. $1 \geq \delta(1 + \delta)(1 - \theta)$. Remarkably, this is not a condition on X^H being large enough but rather on δ being small enough. This is a general feature of our model, that derives from it being a repetition of identical projects, and not from the normalization $X^L = 0$.

discounting. By Proposition 2, we only have to compare $\langle 1,0,0\rangle$ and $\langle 1,1,0\rangle$. (We have already shown that the first project is financed).

Suppose that the entrepreneur decides to always report X^L (even if $X_t = X^H$). He would get financed only once in $\langle 1,0,0\rangle$ and twice in $\langle 1,1,0\rangle$. Hence, by Lemma 2 the entrepreneur's surplus is respectively $\hat{X}$ and $2\hat{X}$. The total surplus from one project is $(\hat{X} - F)$. Hence, the social surplus in a contract is $(\hat{X} - F)$ times the expectation of the number of projects financed in equilibrium. In both contracts the first project is financed with certainty. In $\langle 1,0,0\rangle$ both the second and third projects are financed if and only if the entrepreneur does not default at the end of the first period. By Lemma 3, we know that in equilibrium there are only liquidity defaults. Hence, default occurs with probability θ. In $\langle 1,1,0\rangle$, the second project is financed with certainty, and the third is financed unless the entrepreneur defaulted twice which, by Lemma 3, occurs with probability θ^2. Hence the social surplus is respectively

$$[1 + 2(1-\theta)](\hat{X} - F) \qquad \text{and} \qquad [2 + (1-\theta^2)](\hat{X} - F)$$

The investor's surplus is respectively

$$-F + (1-\theta)2(\hat{X} - F) \text{ and } -2F + (1-\theta^2)(\hat{X} - F)$$

The trade-off between the two contracts is simple. $\langle 1,0,0\rangle$ emphasizes the extraction phase: the investor tries to sell two projects after one development stage, and the sale actually takes place with probability $(1-\theta)$. In contrast, $\langle 1,1,0\rangle$ emphasizes the development stage: the investor tries to sell only one project, but thanks to two (costly) development periods, the likelihood of the sale actually taking place is increased. However, the increase in probability does not even compensate for the fact that only one project is sold instead of two. Since, on top of this, the cost F of an additional development period is incurred, the investor is better off in $\langle 1,0,0\rangle$ than in $\langle 1,1,0\rangle$ by

$$F + [2(1-\theta) - (1-\theta^2)](\hat{X} - F) = F + (1-\theta)^2(\hat{X} - F) > 0$$

To give an intuition of the proof, we study the four-period model. Again, for simplicity, we focus on the case in which the budget constraint is never binding. [7] Let us show that $\langle 1,0,1,0\rangle$ is not optimal. By Lemma 2, and (only) when $\delta = 1$, the entrepreneur's surplus is only a function of the number of ones in the sequence representing the contract.

[7] This amounts to assuming $1 \geq \delta(1 + \delta + \delta^2)(1-\theta)$.

Hence, the entrepreneur gets the same surplus in $\langle 1,0,1,0\rangle$ and $\langle 1,1,0,0\rangle$ and we simply have to compare the social surplus. In equilibrium, the first and last projects are financed with the same probability in both contract (1 and $(1-\theta^2)$ respectively, by Lemma 2) and another project is financed with probability 1. Hence, as for payoffs, both contracts only differ in the probability with which the second and third projects respectively are financed in equilibrium. In $\langle 1,0,1,0\rangle$, the second project is financed with probability $(1-\theta)$. In $\langle 1,1,0,0\rangle$, the third one is financed with probability $(1-\theta^2)$. Hence, $\langle 1,1,0,0\rangle$ dominates $\langle 1,0,1,0\rangle$. [8]

The argument extends directly to other values of T. At the end of each financed period, the investor actually sells only (some of) those projects he would not if the entrepreneur defaulted. Hence, when there is no discounting, selling the last projects, maximizes the value of what is sold to the entrepreneur at the end of each financed period. Also, when $\theta > 0$, we have again the effect that financing early projects increases the probability of being able to sell the last ones.

One implication is that the optimal contract uses what we will henceforth call the *termination threat*: once one project has not been financed, no project will be financed in the future. [9]

Example: For $T = 4$, depending on the parameters, the optimal contract is either $\langle 1,0,0,0\rangle$ or $\langle 1,1,0,0\rangle$. In particular, when the entrepreneur's budget constraint is not binding, i.e. when $1 > \delta(1+\delta+\delta^2)$ then the optimal contract is

- $\langle 1,0,0,0\rangle$ if $\frac{F}{\hat{X}-F} > \delta(1-\theta)(\theta(1+\delta)-1)$;
- $\langle 1,1,0,0\rangle$ otherwise.

Lemma 5 *For all $T > 2$, the optimal full commitment contracts "end with (at least) two zeros".*

Proof. Compare $\langle 1,\ldots,1,0,0\rangle$ and $\langle 1,\ldots,1,0\rangle$. By Lemmas 2 and 3, the investor's surplus is respectively

$$-(1+\ldots+\delta^{T-3})F + (1-\theta^{T-2})(\delta^{T-2}+\delta^{T-1})(\hat{X}-F)$$

[8] As noted above, this reasoning holds when $\delta = 1$. It extends to values of δ close enough to one. For lower values of δ, $\langle 1,0,1,0\rangle$ may dominate $\langle 1,1,0,0\rangle$. But then it is dominated by $\langle 1,0,0,0\rangle$.

[9] In our framework, because cash flows disappear so that there is no point asking for a repayment after a non-financed project, this threat is equivalent to the *intransigence threat* stating that no new project will be financed before a high repayment is made. This would not be the case in a semi-separating contract, which can be optimal when profits transfer from one period to the next one(s).

and

$$-(1+\ldots+\delta^{T-2})F+(1-\theta^{T-1})\delta^{T-1}(\hat{X}-F)$$

Hence, the difference is

$$\delta^{T-2}F+\delta^{T-2}[(1-\theta^{T-2})(1+\delta)-\delta(1-\theta^{T-1})](\hat{X}-F)$$

The first term is positive and the second term, simplified by $\delta^{T-2}(\hat{X}-F)$ can be rewritten

$$1-(1+\delta)\theta^{T-2}(1-\theta)\geq 1-2\theta(1-\theta)>0$$

■

4 Finite Horizon: Renegotiation-Proof Contracts

Long-term agreements can be subject to an additional constraint that we have not so far taken into account: the initial contract is binding only as long as all the parties do not agree to alter it in the course of time. We model this renegotiation constraint by assuming that, at any stage, the investor can propose an alternative continuation contract through a take-it-or-leave-it offer to the entrepreneur. If the latter reflects the offer,the current contract is implemented.

Assumption 7 *The investor has full bargaining power in renegotiation.*

Note that for $T=2$, the optimal full commitment contract is renegotiation-proof. Indeed, the investor's threat not to finance the second project is credible since he would make a loss in the one-period subgame. Under Assumption 7, the entrepreneur cannot gain by renegotiating the repayment.[10]

First, as in the full commitment case, we can restrict our attention to the class of simple contracts. Then, going through the models with up to four periods, we study the implications of the renegotiation constraint and illustrate how the logics of the two phases conflict. The termination threat is not credible when the future relationship is profitable for the investor. As a direct consequence, optimal full commitment contracts are not renegotiation-proof. Instead, only temporary exclusions from access to credit are credible. With respect

[10] Actually, the argument we have developed implies that if renegotiation is costless, there is no need for an initial contract. The same outcome would be reached if the sale contract was signed at the end of the first period.

to the full commitment case, this lowers the profitability of the relationship for the investor and reduces the class of projects that are financed.

We are able to show that, as under full commitment, we can focus on simple contracts.

Proposition 4 *Optimal renegotiation-proof contracts are simple contracts.*

4.0.1 The Renegotiation Constraint is Binding

We show that, for $T > 2$, the optimal full commitment contract is not renegotiation-proof, and we characterize optimal renegotiation-proof contracts.

Consider the three-period model and suppose that initially the optimal full commitment contract $\langle 1, 0, 0 \rangle$ is signed. Suppose that the entrepreneur has defaulted on the first repayment. At the beginning of the second period, when renegotiation takes place, the cash flows generated by the first project have disappeared. Hence, the situation faced by both players is exactly the two-period problem. According to the initial contract, their relationship should end and so they would both make zero cash flow. However, there exist two-period contracts ($\langle 1, 0 \rangle$ for instance) that yield a strictly positive profit to both of them. Hence both the investor and the entrepreneur have an incentive to renegotiate the initial contract and agree on a profitable two-period contract. Since by assumption the investor has all the bargaining power in renegotiation, they will adopt his preferred renegotiation-proof two-period contract, $\langle 1, 0 \rangle$. Hence, contrary to what is specified in the initial contract, the second project is financed: the termination threat does not resist renegotiation, and so it is not credible ex-ante.

For $T = 3$, the optimal full commitment contract is not renegotiation-proof. As we have seen, the investor wants to be as tough as possible during the surplus extraction phase. His "toughest threat", i.e. the outcome that the entrepreneur dislikes most and will be ready to pay most so as to avoid it, is the termination threat, but it is not credible. The renegotiation-proof contract that yields the worst outcome to the entrepreneur is the optimal two-period renegotiation-proof contract, $\langle 1, 0 \rangle$. This actually constitutes a general result.

Proposition 5 *In the T-period optimal renegotiation-proof contract, if the entrepreneur defaulted at the end of all financed periods up to period t, the continuation contract is the (unique) optimal renegotiation-proof contract in the $(t-1)$-period model.*

Suppose that an optimal renegotiation-proof contract has been followed until at some point a default occurred. Now, since cash flows have disappeared, both players face a new game. Hence, the continuation contract suggested by the initial contract has to be a renegotiation-proof contract of the continuation game. The optimal renegotiation-proof contract of the continuation game is the best for two reasons. First, it maximizes the investor's continuation payoff which is good in itself. Second, since renegotiation-proof contracts are not Pareto comparable, it also minimizes the entrepreneur's continuation payoff, which in turn makes a strategic default less appealing.

In our framework, renegotiation makes the investor strictly worse off (i.e., $T = 2$ is a unique case). This illustrates our point that in a way the investor's position is weakened by the future profitability of his relationship with the entrepreneur. He cannot commit to extremely ex-post inefficient outcomes in order to discourage defaults ex-ante.

Proposition 6 *For all $T > 2$, the optimal full commitment contract is not renegotiation-proof.*

Proof. The optimal full commitment contract "ends with strictly more than one zero". However, the optimal renegotiation-proof contract in the two-period model is $\langle 1, 0\rangle$. Hence, by Proposition 5, the optimal full commitment contract is not renegotiation-proof. ■

Lemma 6 *Optimal renegotiation-proof contracts are fully separating, i.e., there are no strategic defaults in equilibrium.*

In our framework, in a renegotiation-proof contract, the investor is committed to all continuation contracts. This is because, by the stationarity of the model, all continuation contracts have to be renegotiation-proof contracts of the continuation game themselves, and so, by definition, resist renegotiation (in particular, this holds irrespective of the investor's belief about the nature of a default: liquidity or strategic). Again by slightly lowering high repayments he is able to break the entrepreneur's indifference in favor of not defaulting.

Example: We can derive the optimal renegotiation-proof contract in the three period model. By Proposition 5, it is either $\langle 0, 1, 0\rangle$ or $\langle 1, 1, 0\rangle$. In fact, it is $\langle 0, 1, 0\rangle$ if $\frac{F}{\hat{X}-F} > \theta(1-\theta)\delta^2$ and $\langle 1, 1, 0\rangle$ otherwise. The trade-off between the two contracts is the following. Consider the case where $\delta = 1$. $\langle 1, 1, 0\rangle$ maximizes the probability that the investor is able to sell his investment in the last period but this is done at a cost $2F$. When θ is close to zero or one, giving a "second chance" to the entrepreneur does not improve the probability by much. Only for intermediate values of θ might this option be valuable. A discount factor $\delta < 1$, by raising the cost of a development stage relative to the value of future projects makes $\langle 0, 1, 0 \rangle$ more attractive.

One implication is that, in our framework, the constraint of renegotiation can lead to more inefficiencies than under full commitment.

4.1 The Temporary Exclusion Threat

Consider the four-period model. First, consider the case in which $\langle 0, 1, 0 \rangle$ is the optimal renegotiation-proof contract in the three-period model. By Proposition 5 the optimal renegotiation-proof contract in the four-period model is either $\langle 0, 0, 1, 0 \rangle$ or $\langle 1, 0, 1, 0 \rangle$. However, it is easy to show that the first of the four projects is financed. The investor can always sign a sequence of two independent two-period contracts over the first two and the last two projects respectively. This is because $\langle 0, 1, 0 \rangle$ is renegotiation-proof. This contract obviously dominates $\langle 0, 0, 1, 0 \rangle$.

Second, consider the case in which $\langle 1, 1, 0 \rangle$ is optimal. Then we have to compare $\langle 1, 1, 1, 0 \rangle$ and $\langle 0, 1, 1, 0 \rangle$. The trade off is whether increasing by $(1-\theta)\theta^2$ the probability of getting $\delta^3(\hat{X} - F)$ is worth incurring a development cost F.

Result 2 *For $T = 4$, the optimal renegotiation-proof contract is:*

- $\langle 1, 0, 1, 0 \rangle$ *if* $\frac{F}{\hat{X}-F} > \theta(1-\theta)\delta^2$*;*
- $\langle 1, 1, 1, 0 \rangle$ *if* $\frac{F}{\hat{X}-F} < \theta^2(1-\theta)\delta^3$*;*
- $\langle 0, 1, 1, 0 \rangle$ *otherwise.*

Hence, the entrepreneur uses threats of temporary exclusion from access to capital of equal length (one period).

Since renegotiation takes place under symmetric information (because, even after a strategic default, the entrepreneur is poor and known to be so during the renegotiation phase), it cannot trigger ex-post inefficient outcomes, which could be used ex-ante as a threats. In our framework, the mere repetition of the game does not give more threat opportunities to the investor than in the two-project model. That is, only the finite horizon effect and the renegotiation-proof inefficient outcome when $T = 1$, which propagates backwards through the game, allows for a credible threats of inefficient outcomes.

5 Infinite Horizon

We now address the model with an infinite number of projects, i.e., $T = \infty$. More than a mere technical exercise, this is meant to capture best our main intuition that ex-post the logics of the development and the surplus extraction phases are conflicting, which reduces the effectiveness of threats ex-ante. This conflict is maximal in the infinitely repeated game: Because of the stationarity of the situation, any given period is not a priori one of investment or of extraction.

The optimal two-period contract $\langle 1, 0 \rangle$ suggests that the lender/borrower relationship continues "as a going concern" while the borrower makes sufficient repayments, but terminates in case of default. However, a finite version of the model cannot really capture the "going concern" aspect since the continuation game faced by the players is different in each period. This is transparent in Bolton and Scharfstein's result: the one-period game is not profitable for the investor whereas the two-period game is. This is actually what allows $\langle 1, 0 \rangle$ to be renegotiation-proof: if the one-stage game were profitable, the threat of termination would be ineffective. From a more technical viewpoint, the end-of-game effect is crucial. It ensures that an ex-post inefficient outcome can obtain, namely that the last stage is not financed, which can be used ex-ante as a threat.

5.1 The Renegotiation Concept

There is no well defined concept of renegotiation for contracts in an infinite horizon framework. For games, different concepts have been proposed.[11] We follow Farrell and Maskin (1989)'s approach to renegotiation because it captures best our intuition. Their concept of Weakly Renegotiation Proof Subgame Perfect Equilibrium (WRE) was developed for infinitely repeated simultaneous moves games. However, one crucial feature of credit captured in our model is its two-stage nature. Hence, in this section, we propose an extension of their definition to fit our framework.

Farrell and Maskin's main idea is that the set of outcomes that the players consider as sustainable should be internally consistent in the sense that no such outcome strictly Pareto dominates another. Otherwise, the players would renegotiate away from the Pareto dominated outcome, which contradicts its sustainability. Technically, to check whether a

[11] For a critical summary of the literature on renegotiation-proofness in games, see Fudenberg and Tirole (1991), p.174-182.

particular subgame-perfect equilibrium is weakly renegotiation-proof amounts to consider the set of payoff profiles in all its continuation equilibria, i.e., on and off the equilibrium path, and check whether none of these strictly Pareto dominates another. The concept of WRE constitutes a strict refinement of that of subgame-perfect equilibrium (see Section 5.4.2).

The fact that our framework is stationary makes it possible to use a similar approach. The stage game (described in Section 2.1) has potentially two stages. In stage A, the investor chooses whether to invest. In stage B, the cashflow is generated. When the project is not financed, only stage A is played. Hence, given that cash flows do not transfer between periods, the game is always in one of two states: the next stage to be played is either a stage A or a stage B.

Let h_t denote an observational history of the game such that the next stage to be played, be it A or B, is in period $t+1$. If the next stage to be played is a stage A then $h_t \in H_t^A = \{\emptyset, X^L, X^H\}^t$. Otherwise, $h_t \in H_t^B = H_t^A \times \{F\}$. By convention, $H_0^A = \{\emptyset\}$ and $H_0^B = \{(\emptyset, F)\}$.

Let C denote a generic contract. It yields payoffs I to the investor and E to the entrepreneur. $C(h)$ denotes the continuation contract after history h. It yields the continuation payoffs $I(h)$ and $E(h)$. Hence a contract C has two families of continuation payoff profiles depending on the stage to be played next. Let $K_A(C)$ (respectively $K_B(C)$) denote the set of all payoff profiles in continuation contracts after a history in $H_A = \bigcup_{t=0}^{\infty} H_A^t$ (respectively in $H_B = \bigcup_{t=0}^{\infty} H_B^t$). We adopt the notation $K_B(C) - (F, 0) = \{(I - F, E)$, with $(I, E) \in K_B(C)\}$. The following definition extends Farrell and Maskin's idea to make it applicable in our sequential moves framework.

Definition 2 *Let C be a full commitment contract. It is said to be weakly renegotiation-proof if and only if no element of $K_A(C) \bigcup [K_B(C) - (F, 0)]$ strictly Pareto dominates another, i.e.*

(i) $\forall (h, h') \in H_A \times H_A,\ [I(h) - I(h')]\,[E(h) - E(h')] \leq 0$

(ii) $\forall (h, h') \in H_B \times H_B,\ [I(h) - I(h')]\,[E(h) - E(h')] \leq 0$

(iii) $\forall (h, h') \in H_A \times H_B,\ [I(h) - I(h') + F]\,[E(h) - E(h')] \leq 0$

Point (i) is the application of Maskin and Farrell's criterion to contracts in $K_A(C)$. Again, the idea is that, due to the stationarity of the model, contracts in $K_A(C)$ can be signed and rule any subgame beginning with a stage A. Hence the set $K_A(C)$ should be self-consistent in the sense of Farrell and Maskin. Point (ii) applies the same idea to $K_B(C)$. Point (iii) introduces a consistency requirement between contracts in $K_A(C)$ and $K_B(C)$. It states that if a contract $C(h')$ in $K_B(C)$ is sustainable, so should the following contract starting in stage A: invest F with certainty and then follow $C(h')$.

5.2 The Investor Makes Zero Profit

We now characterize the weakly renegotiation-proof contracts of the infinitely repeated game and prove that in any of them the investor makes zero profit. In this section, we present a simple proof in the case where the contract cannot use randomization. In particular, contracts must specify whether, after a given history, a project is going to be financed with probability one or zero. Then, we extend the result when randomization is allowed.

Proposition 7 *In a weakly renegotiation-proof contract the investor's surplus is zero.*

Consider a contract C. Let $\mathbf{1}_t \in \{\emptyset, X^L\}^t \in H_t^A$ denote the history generated by contract C if the entrepreneur defaulted on all projects financed in periods 1 to t. Let $\beta(h)$ denote the probability that, after history $h_t \in H_t^A$, the project in period $(t+1)$ be financed. To establish the intuition of the proof, we first restrict to deterministic contracts, in which $\forall h,\ \beta(h) \in \{0,1\}$.

We proceed step by step to prove the proposition. To have the entrepreneur repay more than X^L, it is necessary for the investor to use the threat of an ex-post inefficient outcome. Any threat must incorporate non financing at some stage so that the entrepreneur does not keep on defaulting forever.

Lemma 7 *In an individually rational contract, there exists a finite rank t such that* $\beta(\mathbf{1}_t) \neq 1$.

Proof. By contradiction. Let C be an individually rational contract. Suppose that for all t, $\beta(\mathbf{1}_t) = 1$. Since the entrepreneur can always report the low cash flow, in which case he cannot be asked to pay more than X^L, we have:

$$\Pi E \geq \frac{\hat{X} - X^L}{1-\delta}$$

Since the first best social surplus is $\frac{\hat{X}-F}{1-\delta}$, we have

$$\Pi I \leq \frac{X^L - F}{1-\delta} < 0$$

So that the investor's individual rationality constraint is violated. ■

Suppose that there exists a weakly renegotiation-proof contract C which yields a strictly positive payoff $I > 0$ to the investor. At least one project is financed. Hence the entrepreneur's surplus E is strictly positive too.

Lemma 8 *The contract in which no project is financed is the only contract such that the entrepreneur's payoff is not strictly positive.*

Proof. The strategy to always default ensures a strictly positive payoff to the entrepreneur as soon as at least one project is financed. ■

Since both players make strictly positive profits in C, the contract along which no project is financed is strictly Pareto dominated by C and cannot be used to sustain C. In other words, a permanent exclusion from access to capital is not consistent with the existence of a profitable relationship. Hence, if there is any such exclusion, it must be temporary.

For a temporary exclusion, even of one period, credibility remains problematic. The exclusion phase must resist to renegotiation in at least two ways.

First, the players should not want to skip the exclusion phase and go straight to the continuation contract. In the relation after the exclusion, the entrepreneur makes strictly positive profits (Lemma 8). Hence, because of discounting, he is always ready to renegotiate and skip the exclusion phase so as to anticipate the future relationship. It must thus be the case that the investor does not want to do so. Hence he must commit to make a loss after the exclusion.

Lemma 9 *In a weakly renegotiation-proof contract, if* $\beta(h) = 0$ *then* $\Pi I(h) \leq 0$*.*

Proof. By contradiction. Let C be a weakly renegotiation-proof contract. Suppose $\beta(h) = 0$ and $\Pi I(h) > 0$. $\beta(h) = 0$ implies that

$$\Pi I(h) = \delta.\Pi I(h, \emptyset) \text{ and } \Pi E(h) = \delta.\Pi E(h, \emptyset)$$

$\Pi I(h) > 0$ implies that at least one period is financed. Hence, by Lemma 1, $\Pi E(h) > 0$. But then

$$\Pi I(h, \emptyset) > \Pi I(h) \text{ and } \Pi E(h, \emptyset) > \Pi E(h)$$

Hence, $C(h, \emptyset)$ Pareto dominates $C(h)$ which is inconsistent with C being weakly renegotiation-proof. ■

Second, the players should not want to renegotiate and reset and play the initial contract. Now, still under the hypothesis that $\Pi I > 0$, the investor makes a loss after the exclusion. But then, the investor would like to renegotiate to the initial contract, which yields positive profits instead. It has to be the case that the entrepreneur does not want to do so. Hence, the entrepreneur must make sufficiently large gains after the exclusion.

Lemma 10 *In a weakly renegotiation-proof contract such that* $\Pi I > 0$*, if* $\beta(h) = 0$ *then* $\Pi E(h) \geq \Pi E$*.*

Proof. By contradiction. Let C be a weakly renegotiation-proof contract. Suppose $\beta(h) = 0$ and $\Pi E(h) < \Pi E$. By Lemma 2, $\Pi I(h) \leq 0 < \Pi I$. Thus C strictly Pareto dominates $C(h)$ which is inconsistent with C being weakly renegotiation-proof. ■

We can now prove the proposition. To prevent renegotiation, the entrepreneur has to make such large gains that it is impossible for the investor to make a positive profit.

Proof. By contradiction. Let C be a weakly renegotiation-proof contract such that $\Pi I > 0$. By Lemma 7, there exists a t such that $\beta(\mathbf{1}_t) = 0$. Since the entrepreneur can always report X^L, in which case he cannot be asked to pay more than X^L, we have:

$$\Pi E \geq (1 + \delta + ... + \delta^n)(\hat{X} - X^L) + \delta^{n+1}\Pi E(\mathbf{1}_t)$$

By Lemma 10, $\Pi E(\mathbf{1}_t) \geq \Pi E$. Hence

$$\Pi E \geq (1 + \delta + ... + \delta^n)(\hat{X} - X^L) + \delta^{n+1}\Pi E$$

$$\Pi E \geq \frac{\hat{X} - X^L}{1 - \delta}$$

which as we have seen implies $\Pi I < 0$; a contradiction. Since the individual rationality constraint rules out the case $\Pi I < 0$, the result holds. ■

Note that so far the proof of Proposition 7 does not use requirement (iii). We have restricted our attention to the case with $\beta(h) \in \{0, 1\}$. In fact, the result extends to $\beta(h) \in [0, 1]$.

5.3 Optimal Renegotiation-Proof Contracts

Corollary 1 *All weakly renegotiation-proof contracts are strongly renegotiation-proof.*

Proof. Since they all yield a zero profit to the investor, none is Pareto dominated by another. ■

Corollary 2 *"Never invest" is a strongly renegotiation-proof contract.*

Proof. The outcome Don't invest/Default is a Nash Equilibrium of our stage game. Hence its infinite repetition constitutes a weakly renegotiation proof equilibrium of the infinitely repeated game. It is thus trivially sustained by a weakly renegotiation proof contract. ■

However, there are many other renegotiation-proof contracts. We only provide a simple example. Consider first the contract such that a default on the first loan triggers the termination of the relationship while a repayment ensures that all the remaining projects are financed. If the budget constraint is not binding the investor's profit is

$$-F + (1-\theta)(R^H - \frac{\delta}{1-\delta}F)$$

so that

$$R^H = (\frac{1}{1-\theta} + \frac{\delta}{1-\delta})F$$

The social surplus is then

$$\Pi S = [1 + (1-\theta)\frac{\delta}{1-\delta}](\hat{X} - F)$$

Since the investor's surplus is zero, the entrepreneur surplus equals the social surplus.

5.4 Discussion

5.4.1 Relation to Coasian Dynamics

In a debt contract, the repayment can be interpreted as the payment for some good or service delivered by the lender to the borrower *at the repayment date.* For instance, if the loan is fully collaterized, the borrower buys the lender's right to seize the collateral. When the repayment can be enforced by courts, the borrower buys the lender's right to sue him. As argued in Section 2.2, in Bolton and Scharfstein's model, the repayment (above the contractible part of the returns X^L), is the price paid by the lender in a transaction that can have "nothing to do with the first loan". In that respect, the second transaction is not a credit: the entrepreneur pays and the investor finances the second project *simultaneously.* (One easy way to see this is that it does not matter whether $X^L = 0$, i.e., it is not necessary that a repayment be made at the end of the second period.) [12] Actually what achieves the

[12] Only when the limited liability constraint in the first period can be binding is it necessary that any repayment be made in the second period in the optimal contract. Otherwise, even if $X^L > 0$, it is irrelevant whether the contractible part of the transfer is made at the end of the first or of the second period. In that

repetition of the relation is to reverse the order of the stages of investment and cash flows, i.e., the return stage of period 1 comes before the investment stage of period 2. In this model, the entrepreneur's willingness to pay is actually his ability to pay. This is the parameter over which there is asymmetric information between the two players. The investor-seller does not know the entrepreneur-buyer's willingness to pay. Hence, potentially, the seller could face a problem in setting the price, especially if he is going to try and sell the good several times. However, in the two-type version of the model we have used, these issues do not arise. The reason is that the price a low-ability-to-pay-type entrepreneur is able to pay, i.e. X^L, is lower than the production cost of the good, i.e. F. Hence, the investor would always propose a price equal to the high-ability-to-pay-type entrepreneur's willingness to pay, and this is what he does in the second period of the two-project game. Hence, under this version of the model, there is no scope for Coasian dynamics. Actually the originality of the model lies in the first period. The seller can, at the cost F of a transfer to the buyer, affect his ability to pay. Of course, it would not make sense to transfer money to the borrower so that he is able to transfer it back. What makes it a valuable option is that the borrower has exclusive access to a profitable project. Hence, after the project's returns are realized, his wealth can exceed the transfer, i.e., $X^H > \delta F$.

Let us start by assuming that the seller can produce at cost c one and only one unit of a good that the potential buyer values at v, with $0 < v < X^H - X^L$. Suppose first that he can observe the buyer's wealth. The seller will subsidize him, i.e. invest F "for free" (for X^L) in his projects until he has a lucky draw. Then, the *development phase* ends and the seller sets a price $p = v$ at which he sells the good. That is the *extraction phase*. In other words, the seller follows a simple rule: "Give to the poor, and sell to the rich". Suppose now that the seller does not observe the potential buyer's wealth. He cannot apply this rule as such. At the end of each development stage, he basically proposes a price without knowing the buyer's wealth. Suppose the buyer follows the myopic strategy of buying whenever the price is below his wealth (and his valuation). Then, the seller's strategy is simple: he just asks a high price $p = v$ at the end of each development stage and, in case the buyer does not buy, goes on with the development phase and invests again in the buyer's project. But this provides an incentive for the buyer not to follow this myopic strategy: if $p = v$, the buyer is

respect (only), the optimal contract is not unique. The contract derived in Bolton and Scharfstein is the one that makes the limited liability less constraining (i.e. when it is binding "enough", it is actually the unique optimal contract).

strictly better off not buying, even if he could, so that the development phase, in which his wealth increases (in expectations), goes on. Hence, the seller cannot always propose a price $p = v$. In other words, his monopoly power is weakened by his very ability to engage into a new development stage. Given that, as we argued, the description in terms of development phase and extraction phase suits most credit transactions, this phenomenon should prevail in most credit relationships. Our model in which the cash flows disappear from one stage to the other, is an extreme way to capture this idea. At the beginning of each period, if the entrepreneur has not bought the good yet, the investor knows he faces a poor entrepreneur. Hence, if he is to make a positive profit, he should start a development stage. He is always better off starting the development phase at the current period, delaying it would only lead to a discounting.

5.4.2 Relation to the Amnesty Dilemma

Farrell (1989) applies the concept of WRE to study the infinite repetition of Amnesty Dilemma games. It is meant to capture the fact that the only retaliation available to the investor is not to lend which, whatever the borrower's intentions, yields a zero payoff to both players.

$1, 1$	$-1, 3$
$3, -1$	$0, 0$

Prisoner's Dilemma

$1, 1$	$-1, 3$
$0, 0$	$0, 0$

Amnesty Dilemma

In the infinite repetition of the Prisoner's Dilemma, the usual subgame-perfect equilibria sustaining cooperation, i.e. each period's outcome is (1,1), are not weakly renegotiation-proof à la Farrell and Maskin: Both trigger strategies and tit-for-tat involve the outcome (0,0) as a threat which is strictly Pareto dominated by the outcome (1,1) it sustains. Nevertheless, cooperation is sustainable in a WRE. The punishment in case of a deviation from (1,1) to (-1,3) is to play (3,-1) for a while before reverting to cooperation. The threat is no longer Pareto dominated by (1,1).[13] Analyzing the infinitely repeated Amnesty Dilemma, Farrell shows that full cooperation is not sustainable. Obviously, the threat used in the Prisoner's Dilemma is not available. However, as long as there is not too much discounting, it is possible to sustain some of the Pareto efficient outcomes, i.e., in which all projects are financed. Also, it is possible for the investor to make some strictly positive gains. The idea is as follows. Along the equilibrium path, the investor always invests while the entrepreneur

[13] See Farrell and Maskin (1989) and van Damme (1989).

payments and defaults are in fixed proportions. Hence the investor gets an average stage payoff smaller than 1. In case the entrepreneur were to default more often than expected, both players switch to an inefficient subgame perfect equilibrium, combining the outcomes (0,0) and (1,1) in fixed proportions. The proportions can be chosen so that along the inefficient path, the investor gets at least as much as along the efficient one, so that he does not want to renegotiate and revert to the original path. However, the borrower is worse off because of the inefficiency so that the threat is effective.

Our model differs in that the players move in sequence. In that case, when he decides whether to default or not, the entrepreneur knows that the investment has been made. Hence, if he was able to make a positive profit by mixing between "Finance" and "Do not finance", this would imply that there is an equilibrium in which by following the "Finance" move the investor makes a profit. Hence, he would have an incentive to switch to the pure strategy "Finance".[14]

6 Extensions

6.1 Continuum of Cash Flows

6.2 Transferable Profits

The assumption that profits are not transferable across time is quite extreme, even though it might be relevant in some situations. It seems reasonable to assume that cash flows transfer from one period to the next. Then the asymmetric information can play a role, in favor of the lender. We present here the intuitions underlying some directions in which one could extend the study.

Suppose that the cash flows transfer from one period to the next. When deciding to make the current period a development stage or an extraction stage, the investor does not know the entrepreneur's wealth. Hence, he can decide to ask for a price $p > 0$ even though, in reality, the entrepreneur's wealth is zero. It is no longer so attractive for a rich entrepreneur to completely mimic a poor one: The renegotiation taking place under asymmetric information can yield an ex-post inefficient outcome, which in turn can be used ex-ante as a credible threat by the investor. Nevertheless, renegotiation-proofness remains

[14] Incidentally, since we allow for explicit contracts, for instance the investor is not necessarily a sovereign power, we enlarge the set of possible sustainable outcomes than in the analysis in terms of game theory. In particular, our result extends to the infinitely repeated game, i.e., without contracts.

a constraint. Typically, an equilibrium will be semi-separating, i.e., will incorporate some strategic defaults.

Consider for instance the contract $\langle 1,0,0,0\rangle$. If there was no strategic default it would not be renegotiation proof, even under asymmetric information about the entrepreneur's wealth. Having observed a default in the first period, and as a consequence not having financed the second project, the investor would infer that the entrepreneur has no wealth and propose the contract $\langle 1,0\rangle$ in the last two periods. It is possible to show that for $\langle 1,0,0,0\rangle$ to be renegotiation-proof, it is necessary that the entrepreneur defaults strategically with a probability $d \in [d_0, 1]$, with $d_0 > 0$.

6.3 Self-Financing

Suppose now that the cash flows produced by the investor's project accumulate and are a substitute for the investor's investment. The entrepreneur is able to sell his product and buy some inputs. There is no clear ranking for the investor between this situation and the previous one. Intuitively, there is a cost to the investor: he now operates under the threat that the entrepreneur becomes independent. This can lead to inefficiencies: if the high cash flow is enough to make the investor redundant in the second project, he will never start a development phase and the whole credit relationship will collapse. When development is important, the potential ability for the entrepreneur to gain independence can be socially detrimental.

However, the fact that the investor always fears to be subsidizing an already rich entrepreneur makes the threat of terminating the development phase "more credible". Again, an equilibrium will typically involve a semi-pooling strategy on the part of a rich entrepreneur. But in some circumstances, the same contract will involve less strategic default under the conditions of this section than under those of the previous one. For instance, it can be shown that the minimum probability of strategic default for $\langle 1,0,0,0\rangle$ to be renegotiation-proof is $d_1 < d_0$.

7 Conclusion

In a simple model of repeated lending, we have discussed the credibility of the threat of denying a lender access to future credit as a disciplinary device. Reversing the perspective often presented in the literature, we have shown how the lender's position, and not only the borrower's, can be weakened by the profitability of future projects. In an infinite horizon model, we obtained the extreme result that credibility considerations eroded the investor's profits down to zero. Of course, we have left aside many important theoretical and practical issues. For instance, we have not considered the effect of reputation building, both on the part of borrowers and lenders. Another important issue when there is no or not enough collateral is that of monitoring, either directly by the lender (as in Gale and Hellwig, 1985) or by other borrowers (see Armendariz de Aghion 1993). Nevertheless, we believe that the point is of some importance, in particular in the case of sovereign debt and that the approach to contract renegotiation in an infinite horizon framework has other potential applications.

In future research, one could try and endogenize the projects' characteristics such as their number, scale and order.

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APPENDIX

A Proof of Proposition 2 and Lemma 3

Denote ΠI_t and ΠE_t the investor's and entrepreneur's generic expected surplus in a t-period model. We are going to derive characteristics of the contract maximizing $\Pi I_t - \lambda_t \Pi E_t$, with $\lambda_t \geq 0$, under the model's constraints and given that it is known that the entrepreneur's wealth is zero at the beginning of the each period.

$$\begin{cases} \max \Pi I_t - \lambda_t \Pi E_t \\ \text{The outcome } (\Pi I_t, \Pi E_t) \text{ is feasible} \end{cases}$$

Case 1: The first project is financed. The investor's and the entrepreneur's expected surplus in the contingent continuation $(t-1)$-period contract are denoted by $\Pi I^{\sigma}_{t-1}$and ΠE^{σ}_{t-1} respectively. The total surplus is $\Pi S^{\sigma}_{t-1} = \Pi I^{\sigma}_{t-1} + \Pi E^{\sigma}_{t-1}$. The maximization is under the following constraints.

First, the entrepreneur's limited liability imposes $R^{\sigma} \leq X^{\sigma}$, $\sigma \in \{L, H\}$. Hence, there are two limited liability constraints, $(\mathrm{LL_L})$ and $(\mathrm{LL_H})$. Second, the we can restrict attention to revealing direct mechanisms. Hence two incentive compatibility constraints, $(\mathrm{IC_L})$ and $(\mathrm{IC_H})$. Third, the continuation outcomes $(\Pi I^{\sigma}_{t-1}, \Pi E^{\sigma}_{t-1})$ with $\sigma \in \{L, H\}$ should be feasible under the constraints of the $(t-1)$-period model given the information structure at the end of the first period. Hence two feasibility constraints $(\mathrm{F_L})$ and $(\mathrm{F_H})$.

$$\begin{cases} \max \Pi I_t - \lambda_t \Pi E_t \\ (\mathrm{LL_L}) \quad R^L \leq X^L \\ (\mathrm{LL_H}) \quad R^H \leq X^H \\ (\mathrm{IC_L}) \quad \delta \Pi E^L_{t-1} - R^L \geq \delta \Pi E^H_{t-1} - R^H \quad \text{or} \quad R^H \geq X^L \\ (\mathrm{IC_H}) \quad \delta \Pi E^H_{t-1} - R^H \geq \delta \Pi E^L_{t-1} - R^L \\ (\mathrm{F_L}) \quad (\Pi I^L_{t-1}, \Pi E^L_{t-1}) \text{ is feasible} \\ (\mathrm{F_H}) \quad (\Pi I^H_{t-1}, \Pi E^H_{t-1}) \text{ is feasible} \end{cases}$$

In what follows, we present the proof when the constraint ($\mathrm{LL_H}$) is not binding. The proof for the case in which ($\mathrm{LL_H}$) can be binding follows the same lines.

Let d denote the probability that the entrepreneur defaults strategically. Then we have

$$\begin{aligned} \Pi I_t - \lambda_t \Pi E_t \quad &= \quad -F + [\theta + (1-\theta)d][\delta \Pi I^L_{t-1} + R^L - \lambda_t(\delta \Pi E^L_{t-1} - R^L)] \\ &\quad + (1-\theta)(1-d)[\delta \Pi I^H_{t-1} + R^H - \lambda_t(\delta \Pi E^H_{t-1} - R^H)] \end{aligned}$$

Let us first assume that there are no strategic defaults in equilibrium. *Given that the investor can commit on the continuation contract following a default*, he can always make sure that this is the case by leaving ($\mathrm{IC_H}$) slack. We will check that he is indeed better off avoiding strategic defaults. Hence the modified program amounts to maximizing

$$\theta[\delta \Pi I^L_{t-1} + R^L - \lambda_t(\delta \Pi E^L_{t-1} - R^L)] + (1-\theta)[\delta \Pi I^H_{t-1} + R^H - \lambda_t(\delta \Pi E^H_{t-1} - R^H)]$$

under ($\mathrm{LL_L}$), ($\mathrm{IC_H}$), ($\mathrm{F_L}$) and ($\mathrm{F_H}$).

Because the coefficient of R^H is positive, ($\mathrm{IC_H}$) is binding. Hence, replacing ΠI^H_{t-1} by $\Pi S^H_{t-1} - \Pi E^H_{t-1}$ the modified program amounts to maximizing

$$\theta \delta \Pi I^L_{t-1} + [\lambda_t + (1-\theta)](R^L - \delta \Pi E^L_{t-1}) + (1-\theta)\delta \Pi S^H_{t-1}$$

under ($\mathrm{LL_L}$), ($\mathrm{F_L}$) and ($\mathrm{F_H}$). The coefficient of R^L being positive, we have

$$R^L = X^L = 0$$

Its coefficient being positive, it is optimal to maximize ΠS^H_{t-1} under ($\mathrm{F_H}$). It is clear that the efficient outcome,

$$\Pi S^H_{t-1} = (1 + ... + \delta^{t-2})(\hat{X} - F)$$

satisfies ($\mathrm{F_H}$). That is there is nothing preventing the investor from committing to finance all $(t-1)$ remaining projects. (Note that this is independent of the full commitment assumption). The only restriction imposed by ($\mathrm{F_H}$) is how this surplus is to be shared between the two players. Since the entrepreneur can (and will) always default, we have

$$\Pi I^H_{t-1} \leq -(1 + ... + \delta^{t-2})F$$

Since ($\mathrm{IC_H}$) is binding, we have

$$R^H + \delta \Pi I^H_{t-1} = \delta \Pi S^H_{t-1} - \delta \Pi E^L_{t-1}$$

Whether profits accrue to the investor though R^H or ΠI^H_{t-1} is indifferent. Hence we can always maximize ΠI^H_{t-1}. This, in particular, relieves maximally the pressure of ($\mathrm{LL_H}$). Also, since ΠS^H_{t-1} is efficient, we have

$$R^H + \delta \Pi I^H_{t-1} > R^L + \delta \Pi I^L_{t-1}$$

so that the investor is better off avoiding completely strategic defaults. Hence, the modified program amounts to maximizing

$$\theta \Pi I^L_{t-1} - (\lambda_t + 1 - \theta) \Pi E^L_{t-1}$$

under ($\mathrm{F_L}$). This is a program of the same form as the original one. Also, given that the entrepreneur's wealth disappears between two periods, the new program is under the condition that the entrepreneur's wealth is zero.

Case 2: The first project is not financed. If the first project is not financed then the amounts to:

$$\begin{cases} \max \Pi I_{t-1} - \lambda_t \Pi E_{t-1} \\ \text{(F) The outcome } (\Pi I_{t-1}, \Pi E_{t-1}) \text{ is feasible} \end{cases}$$

a program of the same form as the initial one.

B Proof of Proposition 3

We prove by contradiction that the optimal full commitment contract has the form $\langle 1, ..., 1, 0, ..., 0 \rangle$, i.e. the ones come first. Suppose that we have a sequence 0,1 corresponding to stages $T - t$ and $T - (t - 1)$. It means that in order to maximize $\Pi I_t - \lambda_t \Pi E_t$, one should not finance the first project but finance the second one. Hence $\max\{\Pi I_t - \lambda_t \Pi E_t\} = \delta \max\{\Pi I_{t-1} - \lambda_t \Pi E_{t-1}\} > 0$. But under full commitment, this is not possible. Under full commitment the investor can commit not to finance the last project. Thus, the model is truncated into a $(t - 1)$-period model. Hence, at least, $\max\{\Pi I_t - \lambda_t \Pi E_t\} = \max\{\Pi I_{t-1} - \lambda_t \Pi E_{t-1}\}$.

C Proof of Result

The proof in the case in which the budget constraint is not binding is provided in the text. Suppose now it is binding. Note first that it cannot be binding in $\langle 1,1,0\rangle$. Hence, the investor's surplus in $\langle 1,1,0\rangle$ is $-(1+\delta)F+(1-\theta^2)\delta^2(\hat{X}-F)$. In $\langle 1,0,0\rangle$, the entrepreneur's surplus is $\hat{X}$. If he does not default at the end of the first period, he repays X^H so that the second project is financed with certainty, and the third one with probability β such that $X^H=\delta(1+\beta\delta)\hat{X}=\delta(1+\beta\delta)(1-\theta)X^H$. Then, if the entrepreneur does not default in the second period, the third project is financed with certainty. Hence, the social surplus is

$$[1+(1-\theta)(\delta+\beta\delta^2+(1-\theta)(1-\beta)\delta^2](\hat{X}-F)$$

Given that $\delta(1+\beta\delta)(1-\theta)=1$, the entrepreneur's surplus is

$$-F+[1+(1-\theta)^2(1-\beta)\delta^2](\hat{X}-F)$$

which is greater than that under $\langle 1,1,0\rangle$.

D Proof of Result

We know that the optimal full commitment contract is $\langle 1,1,1,0\rangle$, $\langle 1,1,0,0\rangle$ or $\langle 1,0,0,0\rangle$. In $\langle 1,1,1,0\rangle$ the budget constraint is not binding so that the investor's surplus is

$$-(1+\delta+\delta^2)F+(1-\theta^3)\delta^3(\hat{X}-F)$$

In $\langle 1,1,0,0\rangle$ the entrepreneur's surplus is $(1+\delta)\hat{X}$. The social surplus is

$$(1+\delta)(\hat{X}-F)+(1-\theta^2)[\delta^2+\beta\delta^3+(1-\theta)(1-\beta)\delta^3](\hat{X}-F)$$

with $(\delta+\beta\delta^2)(1-\theta)=1$ so that the investor's surplus is

$$-(1+\delta)F+(1+\theta)[\delta+(1-\theta^2)(1-\beta)\delta^3](\hat{X}-F)$$

which is greater than that under $\langle 1,1,1,0\rangle$. Hence, the optimal contract is either $\langle 1,1,0,0\rangle$ or $\langle 1,0,0,0\rangle$.

Suppose that the budget constraint is not binding in $\langle 1,0,0,0\rangle$, that is

$$X^H\geq(\delta+\delta^2+\delta^3)\hat{X}$$

or simplifying by X^H

$$1\geq\delta(1+\delta+\delta^2)(1-\theta)$$

Then the investor's surplus in $\langle 1,1,0,0\rangle$ is

$$-(1+\delta)F + (1-\theta^2)(\delta^2+\delta^3)(\hat{X}-F)$$

whereas in $\langle 1,0,0,0\rangle$ it is

$$-F + (1-\theta)(\delta+\delta^2+\delta^3)(\hat{X}-F)$$

The difference is

$$-\delta F + (1-\theta)[(1+\theta)(\delta^2+\delta^3) - (\delta+\delta^2+\delta^3)](\hat{X}-F)$$

or

$$-\delta F + (1-\theta)\delta[\theta(1+\delta)-1](\hat{X}-F)$$

E Proof of Proposition 4 and Lemma 6

The proof of Proposition 3 holds. The only argument that uses specifically the full commitment assumption is that, after a default, the investor is committed to follow a continuation contract yielding a payoff profile $(\Pi I^L_{t-1}, \Pi E^L_{t-1})$. With renegotiation, the feasibility condition $(\mathrm{F_L})$ requires that $(\Pi I^L_{t-1}, \Pi E^L_{t-1})$ be obtained in a renegotiation-proof contract of the $(t-1)$-period model, given that the entrepreneur's wealth is known to be zero. By definition, this contract will not be renegotiated. Hence, the investor is committed to it.

F Proof of Proposition 5

We show by induction on T that:

1) there is a unique renegotiation-proof payoff profile

2) in an optimal renegotiation-proof contract, the continuation contract after a default in period t, is an optimal renegotiation-proof contract of the $(t-1)$-period model.

By Propositions 1 and **??**, the result is true for $T=2$. Suppose it holds up to rank T. Denote by $(\Pi I^*_T, \Pi E^*_T)$ the unique payoff profile yielded by an optimal renegotiation-proof contract in the T-period model. Consider the $(T+1)$-model. If the first period is not financed in the optimal renegotiation-proof contract then we are back to a T-period model and the optimal renegotiation-proof continuation payoff profile is $(\Pi I^*_T, \Pi E^*_T)$. If the first period is financed in the optimal renegotiation-proof contract then the continuation contract after a default is a renegotiation-proof contract of the T-period model that maximizes a function of the form

$$\Pi I_T - \lambda_T \Pi E_T$$

Being renegotiation-proof it is such that

$$\Pi I_T \leq \Pi I_T^* \text{ and } \Pi E_T \geq \Pi E_T^*$$

Hence,

$$\Pi I_T = \Pi I_T^* \text{ and } \Pi E_T = \Pi E_T^*$$

G Proof of Result

We know that the optimal renegotiation-proof contract is either $\langle 0,1,0\rangle$ or $\langle 1,1,0\rangle$. In none of them the budget constraint is binding. In $\langle 0,1,0\rangle$, the investor's surplus is $-\delta F + (1-\theta)\delta^2(\hat{X} - F)$. In $\langle 1,1,0\rangle$, it is $-(1+\delta)F + (1-\theta^2)\delta^2(\hat{X} - F)$. The difference is $F + (1-\theta)[\delta^2 - (1+\theta)\delta^2](\hat{X} - F)$. Hence the result.

H Proof of Proposition ??

Denote by ΠI^* the supremum of the set of the investor's payoffs in all weakly renegotiation-proof contracts. ΠI^* is positive since the investor can always choose never to invest.

Lemma 11 *Suppose that* $\Pi I^* > 0$. *In any weakly renegotiation-proof contract, for all* t, $\beta(\mathbf{1}_t) \geq \frac{\Pi I - \delta \Pi I^*}{(1-\delta)\Pi I^*}$.

Proof. Let C be a weakly renegotiation-proof contract. Let t_n denote the n-th rank t such that $\beta(\mathbf{1}_t) < 1$. By Lemma 7, we know that t_1 is finite. Suppose $\Pi I(\mathbf{1}_{t_1}) < \Pi I$. Then $\Pi E(\mathbf{1}_{t_1}) \geq \Pi E$ otherwise C would strictly Pareto dominate $C(\mathbf{1}_{t_1})$. The proof of Proposition 7 directly applies and contradicts $\Pi I > 0$. Hence $\Pi I(\mathbf{1}_{t_1}) \geq \Pi I$ which, when developed rewrites:

$$\beta(\mathbf{1}_{t_1})[\Pi I(\mathbf{1}_{t_1}, F) - F] + (1 - \beta(\mathbf{1}_{t_1}))\delta \Pi I(\mathbf{1}_{t_1}, \emptyset) \geq \Pi I \tag{3}$$

ΠI^* being maximal, we have $\Pi I(\mathbf{1}_{t_1}, \emptyset) \leq \Pi I^*$.

Also $\Pi I(\mathbf{1}_{t_1}, F) - F \leq \Pi I^*$. This is (maybe) a bit less obvious. An investment having been made and the continuation contract being $C(\mathbf{1}_{t_1}, F)$, the investor makes a profit of

$\Pi I(\mathbf{1}_{t_1}, F)$. Initially, the investor always has the option to propose an original contract C' specifying that he will invest in the first project and that then continuation contract will be $C'(F) = C(\mathbf{1}_{t_1}, F)$. The contract C' will bring him a surplus $\Pi I' = \Pi I(\mathbf{1}_{t_1}, F) - F$ and be weakly renegotiation proof. Hence, ΠI^* being maximal, we have

$$\Pi I(\mathbf{1}_{t_1}, F) - F \leq \Pi I^*$$

Substituting in inequality 3 we get

$$\beta(\mathbf{1}_{t_1}).\Pi I^* + (1 - \beta(\mathbf{1}_{t_1}))\delta \Pi I^* \geq \Pi I$$

or $\beta(\mathbf{1}_t) \geq \frac{\Pi I - \delta \Pi I^*}{(1-\delta)\Pi I^*}$. ■

Note that now the only possibility for $\Pi I^* > 0$ is if ΠI^* is not reached. Otherwise the previous lemma would imply that $\beta(\mathbf{1}_{t_1}) = 1$, contradicting the definition of t_1. We can now prove the proposition by contradiction. Suppose that $\Pi I^* > 0$. Consider a weakly renegotiation-proof contract C yielding a payoff $\Pi I > 0$ to the investor. Since the entrepreneur can always report low profits and have the project refinanced with at least probability $\frac{\Pi I - \delta \Pi I^*}{(1-\delta)\Pi I^*}$, we have

$$\Pi E \geq \frac{\hat{X} - X^L}{1 - \delta . \frac{\Pi I - \delta \Pi I^*}{(1-\delta)\Pi I^*}}$$

But we can find weakly renegotiation-proof contracts with ΠI arbitrarily close to ΠI^*. Hence, we should be able to have at the same time $\Pi I > 0$ and ΠE arbitrarily close to $\frac{\hat{X} - X^L}{1-\delta}$ which, as already proved, is not possible.