Drugs, Showrooms and Financial Products: Competition and Regulation when Sellers Provide Expert Advice*

David Bardey† Denis Gromb‡ David Martimort§ Jerome Pouyet¶

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Abstract. We consider a market in which sellers can exert an information-gathering effort to advise buyers about which of two goods best fits their needs. Sellers may steer buyers towards the higher margin good. We show that for sellers to collect and reveal information, profits on both goods must be sufficiently close to each other, *i.e.*, lie within an *implementability cone*, which competition or regulation may ensure. Instruments to do so vary with the context. Controlling market power while improving the quality of advice is more difficult when sellers have private information on the profitability of the goods.

Keywords. Mis-Selling, Expertise, Retailing, Competition, Regulation, Asymmetric Information.

JEL codes. D82; I11; L13; L15; L51; G24.

1. Introduction

Motivation. In many instances, customers rely on sellers for expert advice on the goods or services they purchase: pharmacists advise clients on which non-subscription drugs to use, and sell them those drugs; retailers for high-tech products often also educate their customers; private and corporate bankers advise clients on investment opportunities which they then provide for a fee. Such situations are prone to conflicts of interest as the seller may bias his advice towards more profitable goods and services. And indeed cases highlighting such conflicts of interest surface regularly. As they do, the twin questions of

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†Universidad de Los Andes and visiting fellow at Toulouse School of Economics. E-mail: d.bardey@uniandes.edu.co.

‡HEC-Paris. E-mail: gromb@hec.fr.

§Paris School of Economics (EHESS). E-mail: david.martimort@parisschoolofeconomics.eu.

¶Paris School of Economics (CNRS & ENS). E-mail: jerome.pouyet@ens.fr.

competition and regulation arise. Commentators often invoke the lack of competition as a factor allowing sellers to slack on the provision of advice. But others blame competition for lowering sellers' incentives to offer advice, and argue for regulation.

The French sector for pharmaceutical drugs provides a striking illustration of these issues. The sector has long been protected by high entry barriers giving pharmacists local market power. Its critics point to excessive margins and allocative distortions. Another frequent complaint regards the opacity surrounding the relationships between pharmacists and drug-makers. Recently, the highly publicized *Mediator* scandal has put the spotlight on the risk of hidden influence and manipulations.[1] In response, a *"sunshine act"* (21/05/2013) was passed aiming at greater transparency.[2] Yet commissions and commercial relationships remain largely outside the act's scope, leaving much incomplete information on the margins of pharmacists. On the other hand, proponents of this market structure argue that beyond pure retailing services, pharmacists also offer advice (*i.e.*, checking prescriptions, detecting potential interaction between drugs, etc.) and that competition would lead to lower quality of advice.

This paper's objective is thus to study how competition and regulation affect sellers' provision of expert advice to buyers.

Main Elements of the Model. We consider a market for experience goods (Nelson, 1970) in which a buyer seeks to purchase one of two goods, A and B, from a seller. The buyer's needs can be of one of two types, A or B, and he enjoys a surplus only from the good fitting his needs. Buyer and seller have the same prior about the buyer's needs.

To this, we add two elements.

First, the seller can, at a private cost, observe a noisy signal of the buyer's needs. If he does, which we assume to be socially optimal, he is in a position to offer valuable advice to make a match more likely. However, because information collection (a binary decision), is costly and non-observable, moral hazard is at play. The seller's willingness to advise the buyer depends on his incentives.

Second, we assume that one of the goods, say good A, may or may not have a lower production cost, and that only the seller knows whether it does. With such informational asymmetry, a seller with a low cost for good A may thus be tempted to push this good to enjoy higher profits, which reduces his incentive to collect information.

Unregulated Monopoly. We begin by studying the seller's incentives to provide advice, and show that they depend on whether the profits for both goods are similar enough: profits must lay within an *implementatibilty cone* which we characterize. In particular, our assumption that providing advice is socially optimal means that the social surpluses for both goods lay within the cone.

We start with the case of an unregulated monopolist seller. A monopolist capturing only a fraction of the social surplus may favor good A for its higher margins *a priori*. That is, monopoly profits may lie outside the *implementability cone*. Two allocative

[1] See http://www.thelancet.com/pdfs/journals/lancet/PIIS0140-6736(11)60334-6.pdf.
[2] See http://www.nature.com/nm/journal/v17/n2/nm0211-144a/metrics/blogs.

distortions arise: prices exceed marginal costs and advice quality is too low. Thus we study the extent to which competition and regulation can curb both distortions.

Competition. Next, we study competition's effect on the seller's incentives to provide advice. We consider several models of competition.

We begin with *ex post* competition in which the buyer can seek advice from one seller but *in fine* purchase from another. This model fits the case of a brick-and-mortar retailer facing online rivals. If competition affects only the good for which the brick-and-mortar seller's margin is the highest, it erodes his highest profits, thus bringing the profits for both goods closer to each other. Technically, when both profits remain high enough, they may enter the *implementability cone*. In that case, competition promotes advising.

We then consider the case of *ex ante* competition, in which buyers commit to a seller before receiving advice. Two sellers sit at the extremes of a Hotelling segment and, before any advice is given, each buyer picks one seller based on posted prices and transportation costs. We determine a necessary and sufficient condition for a symmetric equilibrium to exhibit information collection. It requires that competition be neither too strong nor too weak. For high transportation costs, competition is too weak to correct the sellers' bias. For low transportation costs, competition erodes profits, which destroys incentives. We also show that the condition fails to hold for simple buyer preferences, i.e., competition reduces sellers' incentives to provide advice.

Regulation. In our Hotelling model, competition may fail to induce information gathering because sellers lack instruments to both extract the buyers' surplus and preserve their own incentives: unit prices play both roles. Instead a regulator may both regulate prices to curb market power and redistribute part of the surplus gain so obtained to the seller to preserve his incentives to collect information.

When the seller's cost structure is common knowledge setting prices equal to marginal costs maximizes welfare but also means the seller cannot recoup the cost of information gathering through higher sales revenues. Fees are thus needed. The cheapest way to solve the moral hazard issue is to set symmetric fees so that the seller's profits lay at the extremal point of the *implementability cone*. Yet, information gathering has lower social value than under complete information because the fees needed for incentives purposes also imply a costly *liability rent* for the seller.

Such regulation is infeasible if the seller has private information on costs. The seller's implied information rent biases him towards pushing good A. Regulation must thus compensate a low-cost seller for that rent. To make mimicking a high-cost seller unattractive, the optimal regulation combines two tools. First, it sets good A's price above marginal cost if the seller reports a high cost. This depresses demand, thereby discouraging a low-cost seller from reporting a high cost. The wedge between price and marginal cost implies that, unlike under complete information, part of a high-cost seller's profits arise from sales, and so fixed fees diminish. Such combination puts a high-cost seller's profits at the extremal point of the *implementability cone*. Second, optimal regulation induces a low-cost seller to reveal information with higher fees while prices remain equal to marginal costs. In other words, the low-cost seller's profits lay strictly inside the cone and the implementation costs are higher. Asymmetric information makes gathering information even less socially attractive due to the combination of *liability* and *information rents.*

Buyer-Seller Dynamics. Absent regulation, buyers can somewhat replicate the missing fee payments for information gathering by being themselves more active, that is, by making the probability of dropping a seller dependent on whether his advice was correct or not. Such retrospective rules help control moral hazard and adverse selection. They are akin to, but imperfect substitutes for, the optimal regulation's fees.

When the seller's cost is common knowledge, the optimal retrospective rule is to switch with some probability if a low-cost seller's recommendation of good A proves incorrect. This brings the seller's intertemporal profits inside the implementability cone. When the seller has private information on his costs, buyers also use this threat as a screening device and switch more often with high-cost sellers to induce information revelation from low-cost types.

The models of buyer-seller dynamics and of *ex post* competition have similar effects and results. In both cases, buyers exert pressure on sellers to create discipline. In the buyer-seller dynamic model, buyers tend to punish sellers when a bad outcome is consistent with a conflict of interest, or as a threat to induce information revelation. In the *ex post* competition model, the buyers' ability to switch to a rival also disciplines sellers. In both cases, intermediate intensities of competition make it more likely that profits are inside the cone. This discipline effect is absent in the *ex ante* competition model. In such a context, buyers are more passive and competition erodes profits symmetrically, making it less likely they lay inside the cone.

Paper Organization. Section 2 reviews the related literature. Section 3 presents the model. Section 4 characterizes the *implementability cone*, and studies the unregulated monopolist case. Section 5 studies *ex ante* and *ex post* competition. Section 6 studies the optimal regulation. Section 7 studies buyer-seller dynamics. Section 8 showcases three applications: the market for pharmaceutical drugs and the patient-doctor relationship, competition between brick-and-mortar and online retailers, and the market for financial advice. All proofs are in the Appendix.

2. Related Literature

Our paper builds on several branches of the literature.

Credence Goods. That sellers might know more about the quality of the product they sell or about the buyers' needs than buyers themselves is the central tenet of a large literature starting with Nelson (1970) and Darby and Karni (1973).[3] In Pitchik and Schotter (1987) and Fong (2005), information is gathered at no cost, and the key moral hazard problem at the core of our analysis is absent. Emons (1997, 2001) studies how information can be credibly conveyed and priced by a monopolist when effort in gathering information is verifiable. Wolinsky (1993), Board (2009) and Levin *et al.* (2009) consider various competitive environments that differ in terms of the kind of information provided. Those papers also differ from ours because they take as given the information structure and do not analyze the seller's incentives to acquire information in the first place. Bouckaert and Degryse (2000) and Emons (2000) analyze competition between experts and non-experts while Pesendorfer and Wolinsky (2003) and Dulleck and Kerschbamer (2009) analyze similar asymmetric competition when seller's effort is non-verifiable.

[3] See Dulleck and Kerschbamer (2006) for a survey of the theory.

Incentives for Mis-Selling. Inderst and Ottaviani (2009, 2012) analyze incentives to collect information in a market context but focus on the agency problems arising when selling is delegated to an agent. Inderst and Ottaviani (2012) consider the strategic choice of a contract between a seller and the advisor when the latter can recommend alternative products to buyers. Inderst and Ottaviani (2009) stress that such delegation is prone to a multitasking problem. Indeed, the agent must both find new clients and advise them on the suitability of his products. This leads to perverse incentives with the agent willing to mis-sell to customers. How much mis-selling is tolerated by the seller depends on, among other factors, the seller's ability to incentivize the agent through commissions contingent on customer satisfaction or to commit to *ex post* penalties for mis-selling. Our analysis differs along several important lines. First, we do not model agency problems between sellers and their sales agents but instead focus on agency problems between sellers and their customers or regulators. Second, and in full generality, we allow for incentive contracts contingent on the seller's information on the buyer's needs and observe that truthful advice derives from the seller's incentives to gather information. By contrast, Inderst and Ottaviani (2009, 2012) restrict the space of contracts to non-contingent ones and, as a result, have to assume that information revelation occurs in a subsequent cheap talk stage. Third, in our setup, the seller has private information about the profitability of the good he sells. This additional source of private information is a further source of rent and implies that the buyer is biased even in regulated environments.

Inderst and Ottaviani (2013) focus on refund or cancellation policies when buyers vary in their sophistication. The commitment provided by the cancellation policy implies an alignment between the seller's and the buyers' interests, provided that buyers are sufficiently rational to understand how the cancellation policy affects the seller's incentives. We do confirm that the degree of sophistication of buyers matters for disciplining sellers. Indeed, our analysis demonstrates that rational consumers who adopt retrospective rules to terminate relationships with sellers achieve most of the gains of an optimal regulation.

Several recent contributions are motivated by issues relevant to the financial services industry and focus notably on the provision of nonverifiable information to customers. Bolton, Freixas, and Shapiro (2007) analyze how incentives for information provision depend on competition among specialized financial intermediaries and show that competition leads to credible information disclosure. Garicano and Santos (2004) study efficiency in matching clients with agents in a context with private information about a client's value and moral hazard in effort provision. Although they view trade as being mediated by trust and address different issues, Gennaioli *et al.* (2015) argue, as we do, that financial advice is a service that shares many aspects with medicine, part of the agency problem being that advice maybe self-serving.

Delegated Expertise. To the extent that the seller's information-gathering decision and his signal are non-observable, our paper builds on the literature on delegated expertise initiated by Lambert (1986) and Demski and Sappington (1987) and further developed by Gromb and Martimort (2007) and Malcomson (2009) among others. (See also Chade and Kovrijnykh, 2016, and Zambrano, 2015.) A key difference with this literature is that we embed the expertise relationship into a market context so as to link incentives to gather information to the market structure.

3. The Model

PREFERENCES AND INFORMATION. A risk-neutral buyer considers purchasing good A or B from a risk-neutral seller. The buyer's needs can be $\theta = A$ or $\theta = B$, and he only values the good matching his needs. Specifically, for $\{i, j\} = \{A, B\}$, a type-i buyer derives no surplus from good j but has a net surplus $S(p_i)$ from and demand $D(p_i) = -S'(p_i)$ for good i sold at price p_i, with $S(\cdot)$ non-increasing and convex and thus $D(\cdot)$ non-increasing.

The common prior is that both types of needs are equally likely. However, the seller can collect information on the buyer's needs and advise him on which good to purchase. Specifically, by incurring a private cost $\psi > 0$, the seller observes a signal $\sigma \in \{A, B\}$ which is informative about the buyer's needs and has precision ε defined as

$$\varepsilon \equiv \Pr(\sigma = A \mid \theta = A) = \Pr(\sigma = B \mid \theta = B) \in (\frac{1}{2}, 1).$$

We assume that the seller's information-collection decision and the signal's realization are unobservable. This creates the potential for moral hazard.

Finally, we assume that the two goods have different marginal costs. While good B's cost is $c_B = c$, good A's cost c_A can be either $\overline{c}_A = c$ or $\underline{c}_A = c - \Delta c$, with $\Delta c > 0$. Moreover, the seller knows the value of c_A but the buyer only has a prior $\nu \equiv \Pr(c_A = \underline{c}_A)$.

The cost and information differences between goods may stem from their different nature. For instance, good A may be less common or more specific than good B. The costs may be production costs, opportunity costs of shelf or storage space, *etc.*

On the one hand, the seller can learn about the buyer's needs, thereby making a match, and hence a sale, more likely. On the other hand, if the seller has a low cost (*i.e.*, $c_A = \underline{c}_A$) and remains uninformed, he is biased towards selling good A which has a higher expected margin *a priori*. In what follows, we analyze the seller's incentives to collect and reveal information in different competition and regulation contexts.

ADDITIONAL NOTATIONS. The overall surplus when good $i = A, B$ with cost c_i is sold at price p_i is

$$W(c_i, p_i) = S(p_i) + (p_i - c_i)D(p_i)$$

which is maximized when price equals marginal cost (*i.e.*, $p_i = c_i$). Therefore the first-best surplus in a sale of good i is

$$W^*(c_i) \equiv W(c_i, c_i).$$

The monopoly price and profit in a sale of good $i = A, B$ are defined as

$$p^m(c_i) = c_i - \frac{D(p^m(c_i))}{D'(p^m(c_i))} \quad \text{and} \quad \pi^m(c_i) \equiv (p^m(c_i) - c_i)D(p^m(c_i)).$$

RUNNING EXAMPLES. In what follows, we adopt two interpretations of the surplus and demand functions. First, demand $D(p_i)$ may correspond to a quantity the buyer purchases if the match is good and the price is p_i. Demand $D(\cdot)$ is then a standard downward-sloping demand. Second, the buyer may demand a single unit of the good, his valuation v being private information and drawn as per c.d.f. $F(\cdot)$ with density $f(\cdot)$ over $[0, \overline{v}]$. The buyer purchases one unit if $v \geq p_i$ so that (expected) demand for good i is $D(p_i) = 1 - F(p_i)$, and the corresponding (expected) net surplus is then $S(p_i) = \int_{p_i}^{\overline{v}} (v - p_i) f(v) dv$.

We develop two running examples using these two interpretations. For multi-unit demand with constant elasticity $\eta > 1$ and normalization $D(p_i) = p_i^{-\eta}$, we have

$$W^*(c_i) = \frac{c_i^{1-\eta}}{\eta - 1}, \quad p^m(c_i) = \frac{\eta c_i}{\eta - 1}, \quad \pi^m(c_i) = \left(\frac{\eta}{\eta - 1}\right)^{-\eta} W^*(c_i).$$

For single-unit demand with the buyer's valuation drawn as per an exponential distribution with mean $1/\eta$, *i.e.*, $F(v) = 1 - \exp(-\eta v)$, expected demand given price p_i is $D(p_i) = \exp(-\eta p_i)$ while expected surplus is $S(p_i) = 1/\eta \exp(-\eta p_i)$ so that

$$W^*(c_i) = \frac{1}{\eta} \exp(-\eta c_i), \quad p^m(c_i) = c_i + \frac{1}{\eta}, \quad \pi^m(c_i) = \frac{1}{e} W^*(c_i).$$

According to the context, we will refer to either of those two interpretations of the surplus and demand functions (and their respective running examples).

Full Information Social Optimum. As a benchmark, consider the case in which information collection is contractible and both signal σ and cost c_A are observable.

Absent information, expected surplus is (weakly) maximized by the buyer purchasing good A as its cost is (weakly) lower. In that case, expected surplus is estimated based on the prior about good A being a good match, *i.e.*, with probability 1/2. Therefore, information collection is socially optimal for a given level of cost c_A if and only if

$$\sum_{\{i,j\}=\{A,B\}} \Pr(\theta = i) \left(\Pr(\sigma = i \mid \theta = i) W^*(c_i) + \Pr(\sigma = j \mid \theta = i) 0\right) - \psi$$
$$\geq \Pr(\theta = A) W^*(c_A) + \Pr(\theta = B) 0.$$

which simplifies to

$$\frac{\varepsilon}{2} W^*(c_B) - \frac{(1 - \varepsilon)}{2} W^*(c_A) \geq \psi. \tag{3.1}$$

The intuition is simple. The left-hand side's first term is information's social benefit: when the buyer's need is B (which has probability 1/2), information allows a match with probability ε, which yields surplus $W^*(c_B)$. Its second term captures information's social cost: when the buyer's need is A (which has probability 1/2), information, because it is noisy, may yield a mismatch with probability $(1 - \varepsilon)$, which destroys surplus $W^*(c_A)$.

Note that $W^*(\cdot)$ being non-increasing, the condition is tighter when the cost of good A is lower, *i.e.*, it is tighter for $c_A = c - \Delta c$ than for $c_A = c$. This simply reflects that information's social cost increases with surplus $W^*(c_A)$ foregone due to a noisy signal.

In what follows, we assume that information gathering is socially valuable even when good A's cost is low. It is then *a fortiori* socially valuable when the cost is high.

Assumption 1. *Information collection is socially optimal irrespective of good A's cost:*

$$\frac{\varepsilon}{2} W^*(c) - \frac{(1 - \varepsilon)}{2} W^*(c - \Delta c) \geq \psi.$$

4. Profits and information gathering

In this section, we start by characterizing the set of seller's profits for goods A and B that are compatible with information gathering and truthful advice (Section 4.1). We then use this analysis to study the case of an unregulated monopoly (Section 4.2).

4.1. The Cone of Implementable Profits

We first determine the seller's incentive compatibility condition. The seller's profit is zero unless his advice $\hat{\sigma}$ matches the buyer's needs θ, in which case it is denoted $\pi_\theta(c_A)$.[4]

The seller collects and reveals information under two conditions. First, his expected payoff from doing so must exceed that from remaining uninformed and recommending whichever of good A or B yields more profit *a priori*.[5] This condition is written as

$$\frac{\varepsilon}{2}\pi_A(c_A) + \frac{\varepsilon}{2}\pi_B(c_A) - \psi \geq \max\left\{\frac{\pi_A(c_A)}{2}, \frac{\pi_B(c_A)}{2}\right\} \quad \forall c_A \in \{\underline{c}_A, \bar{c}_A\}. \tag{4.1}$$

The second condition is that conditional on having acquired information, the seller must prefer reporting it truthfully, which can be written as

$$\frac{\varepsilon}{2}\pi_i(c_A) > \frac{(1-\varepsilon)}{2}\pi_j(c_A) \quad \forall\{i,j\} = \{A,B\} \quad \forall c_A \in \{\underline{c}_A, \bar{c}_A\}.$$

Note however that this condition is implied by condition (4.1), which can be rewritten as

$$\frac{\varepsilon}{2}\pi_i(c_A) \geq \frac{(1-\varepsilon)}{2}\pi_j(c_A) + \psi \quad \forall\{i,j\} = \{A,B\} \quad \forall c_A \in \{\underline{c}_A, \bar{c}_A\}. \tag{4.2}$$

Indeed, the seller would not collect a signal if this never affected his advice. Given this, we can now describe the set of profit levels ensuring information gathering and truthful advice, which is a cone in the seller's profit space (see Figure 1).

Lemma 1. *The set of profits inducing information gathering and revealing is given by*

$$\Gamma = \{(\pi_A(c_A), \pi_B(c_A)) \text{ s.t. } \pi_A(c_A) = \pi^* + (1-\varepsilon)x + \varepsilon y; \pi_B(c_A) = \pi^* + \varepsilon x + (1-\varepsilon)y; x \geq 0; y \geq 0\}$$

which is a positive cone with extremal point

$$\pi_A^*(c_A) = \pi_B^*(c_A) = \pi^* = \frac{2\psi}{2\varepsilon - 1}. \tag{4.3}$$

4.2. Unregulated Monopolist

We now study the case of a monopoly seller charging fixed prices per unit of good.[6]

Game Form. The sequence of events is as follows. First, the seller observes his cost c_A

[4] In what follows, we make the dependence of all variables on random variable c_A explicit.

[5] Randomized strategies between those two options are weakly dominated.

[6] With two-part tariffs, the seller would capture the full surplus and thus offer socially optimal advice.

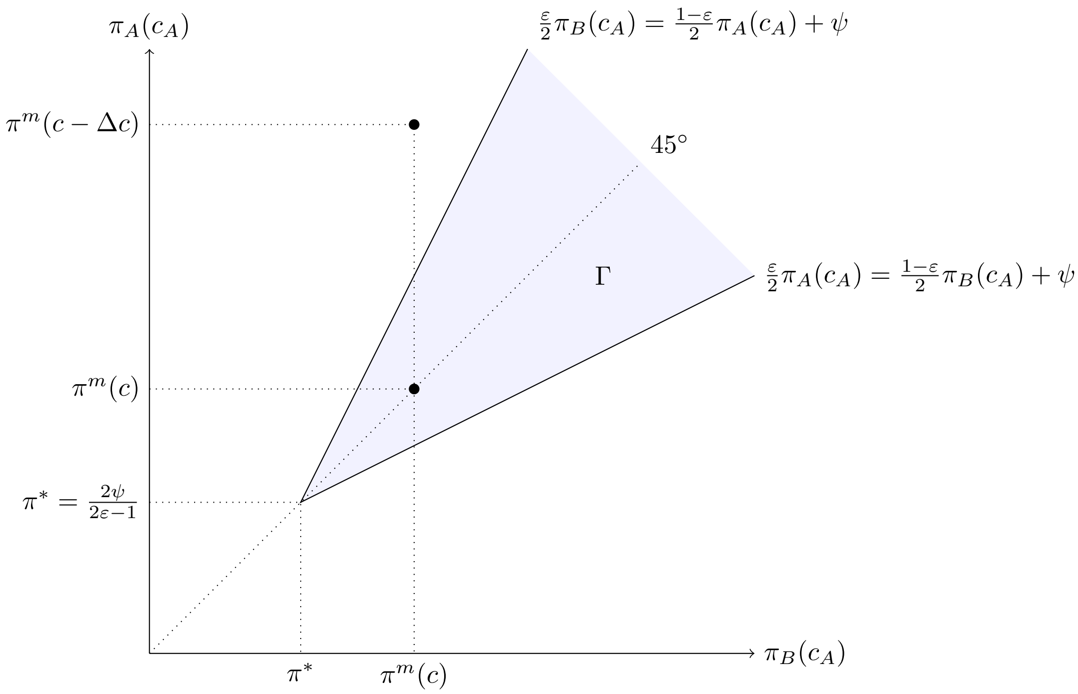

Figure 1 – The set of profits Γ inducing information gathering and truthful advice is a cone.

and chooses prices p_A^m and p_B^m.[7] Second, he chooses whether to collect information and if so, observes signal σ privately. Finally, the seller recommends good A or B and demand is expressed if the buyer's need matches the good.

Information collection is optimal for the seller given his cost c_A if

$$\frac{\varepsilon}{2}\pi^m(c_B) - \frac{(1-\varepsilon)}{2}\pi^m(c_A) \geq \psi.$$

The condition can be understood by replacing social surplus with monopoly profits in information value condition (3.1). Again, it is tighter for a low-cost than for a high-cost seller because information's private cost, *i.e.*, the foregone profit $\pi^m(c_A)$ due to an inaccurate signal, decreases with cost c_A.

From now on, to focus on the relevant cases, we assume the following condition holds.

Assumption 2. *Only a high-cost seller collects information and reports it truthfully, i.e.,*

$$\frac{(2\varepsilon-1)}{2}\pi^m(c) \geq \psi \geq \frac{\varepsilon}{2}\pi^m(c) - \frac{(1-\varepsilon)}{2}\pi^m(c-\Delta c).$$

The first inequality means that a high-cost seller gathers (and reveals) information. The second one means that a low-cost seller remains uninformed and pushes good A. Assumption 2 ensures that a low-cost seller's profits $(\pi^m(c-\Delta c), \pi^m(c))$ lie outside cone

[7] Proposition 1 holds even if the seller commits to prices before learning his cost (Mylovanov and Tröger, 2012). This highlights the robustness of the low-cost seller's incentives to push good A *a priori*.

Γ, whereas those of the high-cost seller, $(\pi^m(c), \pi^m(c))$, lie within the cone (Figure 1).[8]

Outcome. Finding the perfect Bayesian equilibria of this sequential game of incomplete information is simplified by noting that the seller's cost does not affect the buyer's preferences. Hence, the seller has no incentive to hide his cost which can thus be assumed common knowledge. The only issue is whether the seller's advice is informed or not.

Proposition 1. *Assume the seller is an unregulated monopoly. Under Assumption 2, the unique (perfect Bayesian) equilibrium outcome is as follows.*

- *The seller charges monopoly prices for both goods: $p_A = p^m(c_A)$ and $p_B = p^m(c_B)$.*
- *A high-cost seller collects information and offers truthful advice.*
- *A low-cost seller remains uninformed and recommends good A.*

The outcome departs from social optimality in two ways: prices are above marginal costs, and a low-cost seller does not offer truthful advice.

5. Competition

This section studies how competition affects the seller's incentives to provide advice. The analysis requires specifying the nature of competition.

5.1. Ex Ante Competition

We model *ex ante* competition as follows. Two identical sellers, located at the extremes of segment $[0, 1]$ compete for buyers uniformly distributed over $[0, 1]$. To simplify, good A's cost is the same for both sellers. Each buyer's transportation cost to a seller is t per unit of distance. As per our first interpretation of the demand function that was stressed above, the buyer's valuation v for one unit of good is drawn on $[0, \overline{v}]$ as per c.d.f $F(\cdot)$.

First, each buyer chooses a seller, and then learns his valuation v. Second, the seller decides whether to gather information. This timing rules out the possibility that sellers free ride on each other's advice (Section 5.2 studies free-riding).

We look for a symmetric equilibrium of this Hotelling game such that both sellers gather information, charge the same prices (p_A, p_B), and each serves half the market.

The analysis is made easier by using profits, not prices, as the optimization variables. Define price $P(\pi, c)$ as the solution to

$$\pi = (P(\pi, c) - c)(1 - F(P(\pi, c))),$$

[8] Note that Assumptions 1 and 2 are mutually compatible. Say Assumption 1 holds as an equality, *i.e.*, collecting information is socially neutral when $c_A = c - \Delta c$. Since the seller gets a fraction $k < 1$ of the overall surplus, he finds it optimal to remain uninformed and push good A. Assumption 1 also implies that collecting information has social value when $c_A = c$. Hence, fraction k can be set close enough to 1 so that a high-cost seller opts to gather information. Thus, the condition $(2\varepsilon - 1)kW^*(c) \geq 2\psi \geq \varepsilon k W^*(c) - (1 - \varepsilon)kW^*(c - \Delta c)$ is satisfied and Assumptions 1 and 2 both hold. In the constant elasticity of demand case, we have $k = (1 - 1/\eta)^\eta \in (0, 1)$. This holds even if demand is arbitrarily inelastic (*i.e.*, η arbitrarily large) and the seller gets an arbitrarily large fraction of the overall surplus. Similarly, in the exponential demand case, we have $k = 1/e < 1$.

which exists provided $\pi \leq \pi^m(c)$. For instance, $P(\pi_A, c_A)$ is the price implying profit π_A when the seller produces good A at cost c_A.

When the seller located at 0 sets prices $(\tilde{p}_A, \tilde{p}_B)$ giving profits $(\tilde{\pi}_A, \tilde{\pi}_B)$ (while the seller located at 1 charges prices (p_A, p_B) giving profits (π_A, π_B)), he attracts a fraction of buyers given by

$$\mathcal{D}(\tilde{\pi}_A, \tilde{\pi}_B, \pi_A, \pi_B) = \frac{1}{2} + \frac{1}{2t}\Big[\Big(\frac{\varepsilon}{2}S(P(\tilde{\pi}_A, c_A)) + \frac{\varepsilon}{2}S(P(\tilde{\pi}_B, c))\Big) - \Big(\frac{\varepsilon}{2}S(P(\pi_A, c_A)) + \frac{\varepsilon}{2}S(P(\pi_B, c))\Big)\Big]$$

with $S(p) = \int_p^{\overline{v}} (v - p)\, f(v)dv$. The right-hand side depends on the difference in the buyer's net surplus when he purchases from the seller located at 0 rather than from his rival located at 1. Hence, the expected profit of the seller located at 0 when gathering and revealing information is

$$\Big(\frac{\varepsilon}{2}\tilde{\pi}_A + \frac{\varepsilon}{2}\tilde{\pi}_B - \psi\Big)\, \mathcal{D}(\tilde{\pi}_A, \tilde{\pi}_B, \pi_A, \pi_B). \tag{5.1}$$

This expression highlights an important complementarity between the demands for both goods. The prices of both goods determine each buyer's choice of a seller.

Price-cost margins for both goods are thus linked. Writing the first-order conditions ensuring that (π_A, π_B) forms a symmetric equilibrium, and rearranging leads to

$$\frac{\varepsilon\pi_A + \varepsilon\pi_B - 2\psi}{t} + 1 = \frac{(P(\pi_A, c_A) - c_A)f(P(\pi_A, c_A))}{1 - F(P(\pi_A, c_A))} = \frac{(P(\pi_B, c) - c)f(P(\pi_B, c))}{1 - F(P(\pi_B, c))}.$$

Thus for transportation cost $t \in (0, +\infty)$, the equilibrium profits lie on the (upward sloping) locus $\pi_B = \Phi(\pi_A, c_A)$ defined implicitly by

$$(P(\pi_A, c_A) - c_A)\frac{f(P(\pi_A, c_A))}{1 - F(P(\pi_A, c_A))} = (P(\Phi(\pi_A, c_A), c) - c)\frac{f(P(\Phi(\pi_A, c_A), c))}{1 - F(P(\Phi(\pi_A, c_A), c))}. \tag{5.2}$$

In the high-cost seller case ($c_A = c$), locus (5.2) is the 45 degree-line. As t decreases, a high-cost seller makes the same margins on both goods and profits decrease. In the perfect competition limit ($t = 0$), he chooses a profit level for each good such that the margin is fully eroded, *i.e.*, $\pi^{pc}(c) = \psi/\varepsilon$.

The low-cost seller case is more involved. At the monopoly limit ($t = +\infty$), the seller gets monopoly profits $\pi^m(c - \Delta c)$ and $\pi^m(c)$ on goods A and B. As per Assumption 2, the seller remains uninformed, *i.e.*, monopoly profits lie outside the cone. Instead, under perfect competition (t goes to 0), the seller sets price levels that erode his margin, *i.e.*, $(\pi^{pc}(c - \Delta c), \pi^{pc}(c))$ is such that $\varepsilon/2\, \pi^{pc}(c - \Delta c) + \varepsilon/2\, \pi^{pc}(c) = \psi$. There again, profits are too small to motivate information gathering (see Figure 2).

From our discussion, competition might induce information gathering only for intermediate transportation costs, and we provide a sufficient condition for this to be true.

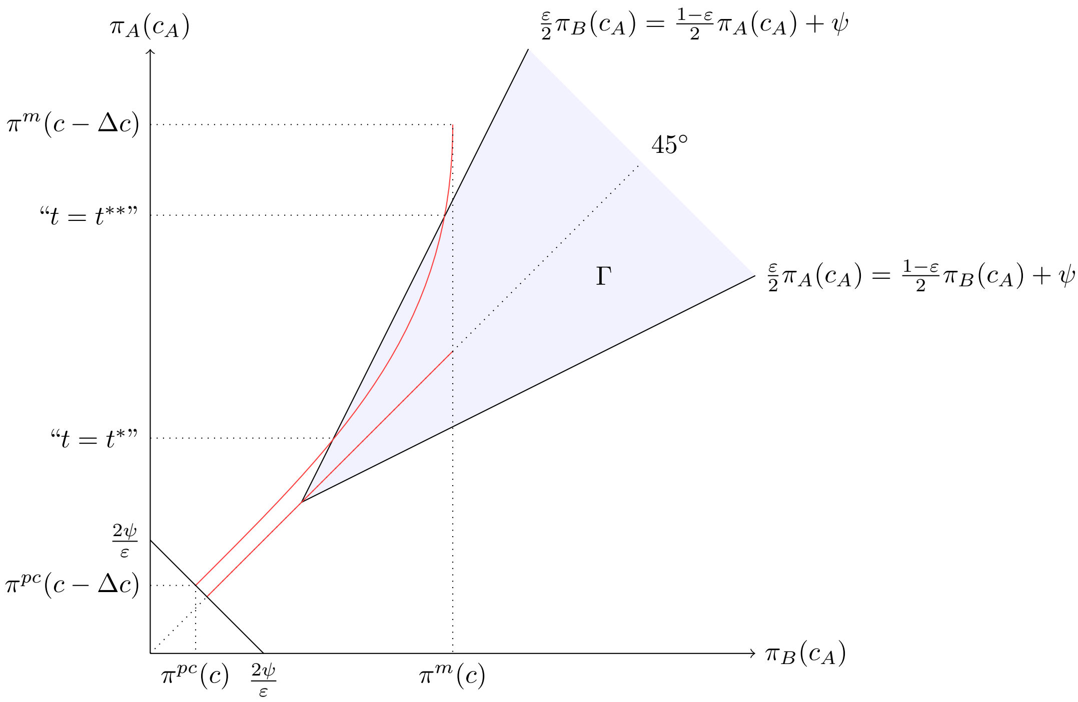

Figure 2 – The locus of the low-cost seller goes through the implementability cone. The locus of the high-cost seller coincides with the 45°-line.

Proposition 2. *Assume that*

$$\max_{\pi\in[0,\pi^m(c-\Delta c)]} \frac{\varepsilon}{2}\Phi(\pi, c - \Delta c) - \frac{(1-\varepsilon)}{2}\pi > \psi. \tag{5.3}$$

Two thresholds t^* and t^{**} exist (with $t^{**} > t^* > 0$) such that the sellers gather and reveal information in a symmetric equilibrium if and only if $t \in [t^*, t^{**}]$.

Condition (5.3) ensures that the equilibrium locus $\pi_B = \Phi(\pi_A, c - \Delta c)$ enters the implementability cone. When condition (5.3) holds, Hotelling competition reduces profits on both goods but more so for good A. Thus profits for both goods become more similar, increasing the seller's incentives to gather information (see Figure 2).

Intuitively, for condition (5.3) to hold, $\Phi(\pi_A, c - \Delta c)$ must be sufficiently flat around $\pi^m(c_A)$, which requires that the density function be of a much smaller magnitude at the high monopoly price $p^m(c)$ for good B than at the low monopoly price for good A. If so, competition impacts more the profits on good A than on good B. Our running examples provide counterexamples for which condition (5.3) does not hold.

Running Examples. In the exponential case and with $c_A = \underline{c}_A$, locus (5.2) is given by

$$P(\pi_A, c_A) - c_A = P(\Phi(\pi_A, c_A), c) - c,$$

where $\pi_A = (P(\pi_A, c_A) - c_A)\exp(-\eta P(\pi_A, c_A))$, *i.e.*, margins are identical for both goods. Simplifying yields

$$\pi_B = \Phi(\pi_A, c - \Delta c) = \pi_A \exp(-\eta \Delta c).$$

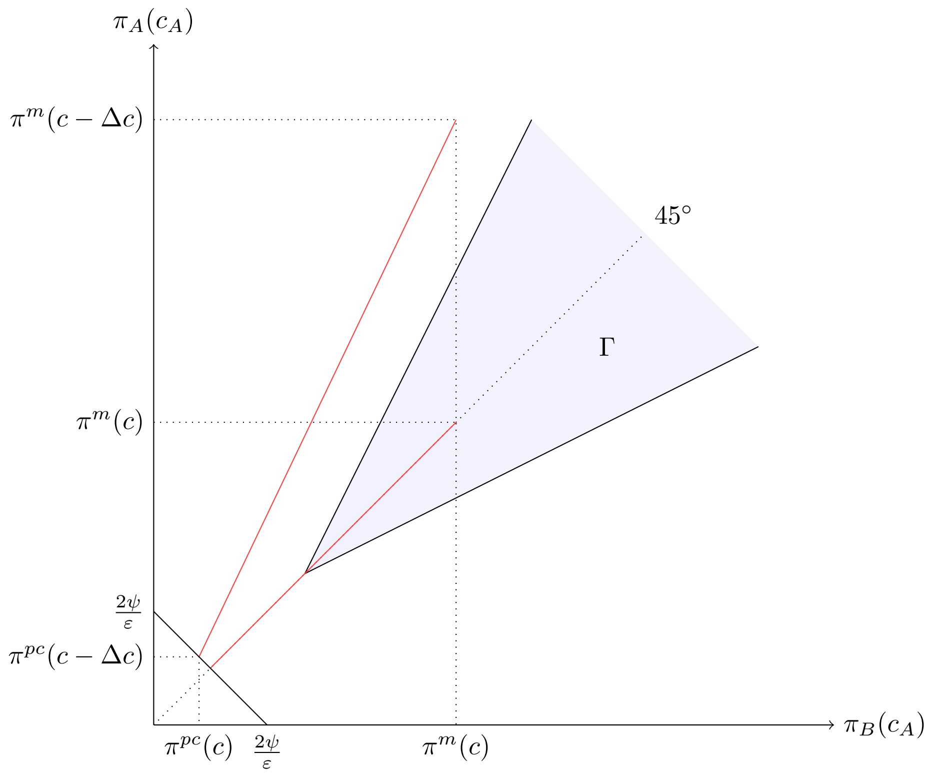

Figure 3 – Exponential distribution and constant elasticity demand: The locus of the low-cost seller is a straight line and thus never crosses the implementability cone Γ under Assumption 2. The locus of the high-cost seller coincides with the 45°-line.

This locus is a straight line going through the origin and starting from the monopoly profits. Under Assumption 2, this line does not enter the implementability cone.

In the multi-unit demand case, the locus of equilibrium profits is given by

$$(P(\pi_A, c_A) - c_A)\frac{D'(P(\pi_A, c_A))}{D(P(\pi_A, c_A))} = (P(\Phi(\pi_A, c_A), c) - c)\frac{D'(P(\Phi(\pi_A, c_A), c))}{D(P(\Phi(\pi_A, c_A), c))}. \tag{5.4}$$

In the case of demand with constant elasticity, this implies that margins are equal across goods. Then, equilibrium profits lie on a straight line going through the origin

$$\pi_B = \Phi(\pi_A, c - \Delta c) = \pi_A \left(\frac{c - \Delta c}{c}\right)^{\eta - 1}.$$

This shows that under Assumption 2, the straight line lies outside the cone (Figure 3).

These two examples reveal that *ex ante* competition may reduce the inefficiency coming from the traditional price distortion caused by market power but may fail to solve the information gathering issue. Typically, in both examples, competition erodes the profits on good A and B too symmetrically for them to enter the implementability cone.

5.2. Ex Post Competition in the Market for Good A

Consider now the case of *ex post* competition in the market for good A. We assume that *after* receiving the seller's advice, the buyer can switch (or not) to a rival. This form of competition causing free riding in the provision of advice may tighten the incentive constraint for information gathering. It may be particularly relevant for brick-and-mortar retailers facing competition from online retailers.

We stress the role competition plays in modifying the distribution of profits, pushing them towards being more symmetric which might thus facilitate information gathering. To simplify, we do not consider competition in the market for good B. In that case, $\pi^m(c)$ stands for the seller's long-run profit. Instead, good A may be a more novel product in a market with potential entry. More importantly, under this assumption, competition has the most chance to imply a balanced distribution of profits across goods.

To simplify, we adopt the single-unit demand interpretation. A buyer can get advice from one seller and then possibly shop from another. There are n potential rivals, each of which sets a price $\tilde{p}_j$ (for $1 \leq j \leq n$) drawn randomly and independently from $[c - \Delta c, +\infty)$ as per the same c.d.f. $H(\cdot)$. To capture the free-riding issue, the realizations of prices are observed after the seller's pricing and advising decisions, but prior to the buyer's purchasing decision. Hence, more intense competition corresponds to a larger n and a lower (residual) demand $D(p, n)$ for each price p.[9,10]

Following a recommendation for good A, the buyer purchases it from the seller at price p_A rather than from one of the rivals if and only if he values the good more than p_A and if the rivals' prices exceed p_A, *i.e.*, if and only if $\min\{v, \min_j \tilde{p}_j\} \geq p_A$. Hence for the seller, the buyer's expected demand is $D(p, n) \equiv (1 - F(p))(1 - H(p))^n$.

Given cost c_A, the seller's maximum profit and optimal price if he recommends good A and good A is a good match are thus

$$\pi_A^c(c_A, n) = \max_{p \geq 0} (p - c_A)D(p, n)$$

$$p_A^c(c_A, n) = c_A + \frac{1}{\frac{f(p_A^c(c_A,n))}{1-F(p_A^c(c_A,n))} + n\frac{h(p_A^c(c_A,n))}{1-H(p_A^c(c_A,n))}}.$$

As competition intensifies (*i.e.*, n increases), price $p_A^c(c_A, n)$ decreases towards marginal cost and the seller's profit conditional on recommending good A goes to zero. For good B, the seller's profit and prices remain unchanged if he recommends good B and good B is a good match

$$\pi_B^c(c, n) = \pi^m(c), \quad \text{and} \quad p_B^c(c, n) = p^m(c).$$

We can now summarize *ex post* competition's effect on the seller's provision of advice.

Proposition 3. *There exists n^*, n^{**} and n^{***} with $\max\{n^*, n^{***}\} \leq n^{**}$ such that*

- *A low-cost seller collects and reveals his information if and only if $n \in [n^*, n^{**}]$.*
- *Else he remains uninformed and recommends good A if $n < n^*$ or good B if $n > n^{**}$.*
- *A high-cost seller collects information and reveals it if $n \leq n^{***}$ and remains uninformed and recommends good B otherwise.*

[9] We make the dependence of demand, profits and prices of good A on n explicit in what follows.

[10] An alternative interpretation is that rival j sets price $p_j = c - \Delta c$ but that purchasing from him involves a switching cost drawn from $[0, +\infty)$ as per c.d.f. $H(\cdot)$.

To induce information gathering by both seller types, competition should not be too intense. Else, the profit for good A would vanish and incentives to gather information and recommend that good would disappear for both seller types. Yet, some degree of competition is needed to bring a low-cost seller's profits on goods A and B closer to each other (*i.e.*, within the cone) so that he does not remain uninformed and push good A.

Remarkably, competition can promote truthful advising even if rivals free ride on advice provision. In a more complete model, the signal's precision would be endogenous. In that case, the possible profit loss that arises from this demand leakage following informed advice might reduce this precision which introduces an extra cost of competition.

5.3. Comparing Ex Ante and Ex Post Competition

The difference between the models of competition of Sections 5.1 and 5.2 is that, for *ex post* (resp. *ex ante*) competition, the buyer can (resp. cannot) purchase from another seller after having received advice from the seller he initially visited.

The two models make opposite assumptions about switching costs. Under *ex ante* competition, switching costs are so high that buyers are captive of the seller they pick initially. Under *ex post* competition, there are no such costs. In both settings, competition, if strong enough, erodes profits too much to induce information gathering. Nevertheless, it is interesting that, despite the possibility of free riding, *ex post* competition is not necessarily less conducive to information gathering. Indeed, with *ex ante* competition, the fact that buyers are captive *ex post* induces a complementarity between the demands for both goods. This complementarity implies that *ex ante* competition, although it shifts downwards the profits for both goods, is also likely to keep these profits more alike. Instead, *ex post* competition may erode profits differently depending on the recommendation. Such an asymmetry improves incentives for information gathering by bringing profits for both goods closer to each other so that they now lie in the implementability cone. *Ex post* competition may thus improve information gathering.

Despite those differences, both scenarios have in common that the intensity of competition (via transportation costs for *ex ante* competition and the number of competitors for *ex post* competition) affects sellers' profit on both goods in the same way. Only when competition intensity is moderate can profits be large enough and sufficiently close to each other to ensure information gathering and truthful advice. In the *ex ante* scenario, this outcome does not arise in our running examples because competition erodes profits symmetrically, thereby maintaining the asymmetry that prevents information gathering under monopoly. However, as long as competition erodes profits on the high-margin good A more than those on the low-margin good B, more intense competition facilitates information gathering. The decision to promote competition in sectors in which customers rely on sellers for advice should thus depend on how, and in which directions, competition affects the structure of profits on the different goods on sale.

6. Regulation

We now characterize the regulation maximizing the buyer's expected surplus. It relies on an incentive contract to counter the low-cost seller's bias towards pushing good A.

Contracts. We adopt a normative approach and consider the largest possible contract set. From the *Revelation Principle*, we can focus on *direct*, *truthful* and *obedient* mechanisms (Myerson, 1982).[11] In direct mechanisms, the seller makes reports $\hat{c}_A$ and $\hat{\sigma}$ on cost c_A and signal σ. They specify report-contingent prices p for both goods, report-contingent fixed payments T for selling each good, and report-contingent fixed payments $T - R$ in case of a mismatch. A contract is thus a six-tuple

$$C = \{(p_{\hat{\sigma}}(\hat{c}_A), T_{\hat{\sigma}}(\hat{c}_A), R_{\hat{\sigma}}(\hat{c}_A))\}_{\hat{c}_A \in \{\underline{c}_A, \overline{c}_A\}, \hat{\sigma} \in \{A,B\}}.$$

The contract must induce truthful reporting (*i.e.*, $\hat{c}_A = c_A$ and $\hat{\sigma} = \sigma$) and information gathering and be obedient.

Timing. The game unfolds as follows. The seller observes cost $c_A \in \{\underline{c}_A, \overline{c}_A\}$. An incentive contract C that maximizes the buyer's expected surplus is designed. The seller makes a report $\hat{c}_A$ about c_A. The seller chooses whether to observe signal $\sigma \in \{A, B\}$ at cost ψ. If the advice matches the buyer's needs (*i.e.*, if $\hat{\sigma} = \theta$), the buyer purchases $D(p_{\hat{\sigma}})$ units of the good, the seller incurs cost $c_{\hat{\sigma}} D(p_{\hat{\sigma}})$ and receives revenue $p_{\hat{\sigma}} D(p_{\hat{\sigma}})$. Else, demand and cost are zero, and the seller incurs a penalty $R_{\hat{\sigma}}$. To simplify, we assume that the seller has no gain after incorrect advice: $R_\sigma(\hat{c}_A) = T_\sigma(\hat{c}_A)$.

In the case of a contract between an upstream producer and a seller, fixed payments T may represent fixed fees the former pays the latter, and penalty R a pay-back transfer. In that case, assuming $T_\sigma \geq 0$ and $R_\sigma = T_\sigma$ is akin to assuming the seller has limited liability. Here, we take this feature of optimal contracts as given to save on notation.[12]

6.1. Pure Moral Hazard

To build intuition, consider first the case in which information gathering and signal are non-observable but c_A is common knowledge. (This amounts to assuming cost $\hat{c}_A = c_A$.) The problem is thus to inducing the seller to collect and report signal σ truthfully.

Constraint (4.1) suggests that selling either good must be rewarded and the cheapest way to do so is to make the seller indifferent between recommending good A or B based on his prior. In that case, the signal tilts the seller's decision towards truth-telling.

Different price-fee combinations ensure indifference but in the least-distortionary one, prices equal marginal costs to maximize overall surplus, while fixed fees induce information gathering and set profits at the extreme point of the cone. The seller must get some surplus to induce him to collect information and fixed fees are best to ensure he does.

These findings highlight an important *dichotomy between pricing and information gathering incentives* when costs are common knowledge. Prices determine overall surplus while fees provide incentives for gathering information and giving truthful advice.

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[11] Our environment now combines moral hazard and adverse selection and one must take some care in dealing with simultaneous deviations along both actions and reports. See Laffont and Martimort (2002, Chapter 7) for a detailed analysis of those mixed models.

[12] This payment structure is consistent with the *Principle of Delegated Expertise*: an optimal contract should reward experts only for recommendations confirmed by verifiable outcomes (Gromb and Martimort, 2007). Inderst and Ottaviani (2009) make a similar assumption on the payment structure.

PROPOSITION 4. *Suppose cost c_A is common knowledge so the only incentive problem is to induce information gathering and truthful advice. The optimal contract is as follows.*[13]

- *Both goods are priced at marginal cost:*

$$p_\sigma^{mh}(c_A) = c_\sigma, \quad \forall \sigma \in \{A, B\}, \quad \forall c_A \in \mathcal{C}_A. \tag{6.1}$$

- *Profits and fixed fees are constant across goods:*

$$\pi_\sigma^{mh}(c_A) = T_\sigma^{mh}(c_A) = \pi^* = \frac{2\psi}{2\varepsilon - 1}, \quad \forall \sigma \in \{A, B\}, \quad \forall c_A \in \mathcal{C}_A. \tag{6.2}$$

- *Information gathering is induced by the regulator when:*

$$\frac{\varepsilon}{2} W^*(c) - \frac{(1-\varepsilon)}{2} W^*(c - \Delta c) \geq \psi + \frac{\psi}{2\varepsilon - 1}\pi^*. \tag{6.3}$$

The seller is rewarded only for a good match. That he cannot be punished for a bad match is akin to a limited liability constraint. Hence the seller enjoys a *liability rent* $\psi / (2\varepsilon - 1)$ to gather information. Note that the lower the signal's precision, the larger the seller's liability rent and the fixed fees. Indeed, the noisier the mapping between information gathering and outcomes, the larger the rewards needed to induce information collection. Finally, agency costs tighten the condition ensuring information gathering. Compared to (3.1), the limited liability rent reduces the set of parameters for which information gathering is optimal. The buyer's net surplus is reduced to $W^*(c_A) - \pi^*$ for good A and $W^*(c) - \pi^*$ for good B, which may lie outside the implementability cone.

6.2. Moral Hazard and Adverse Selection

We now turn to the scenario where the seller has private information about his cost for good A. While this information has no value in an unregulated context because it does not affect the buyer's utility, it has value in a regulation context. Indeed, manipulating information revelation on the cost structure to a regulator becomes a way for the seller to channel customers towards the informationally sensitive good that provides information rent. Private information impacts on incentives for information gathering.

To illustrate, we first consider the optimal contract under pure moral hazard (as in Section 6.1) and ask whether private information about cost induces advice manipulation.

Consider an uninformed low-cost seller. Based on his prior, he is tempted to report a high cost. Indeed, this does not change the fees for selling either good since condition (6.2) implies they are cost-independent, but brings the seller an extra gain

$$\frac{1}{2}\Delta c D(c).$$

This information rent equals the expected gain from selling $D(c)$ units of good A at a cost that is Δc below the high cost. The expectation is based on prior beliefs as the seller always recommends good A and thus remains uninformed.

[13] Superscript *mh* stands for *moral hazard* to stress this is the only incentive constraint considered.

The next step is thus to express the seller's information rent taking into account that information gathering and signals are non-verifiable. We define this rent as

$$\mathcal{U}(c_A) = \max_{\substack{\hat{c}_A \in \mathcal{C}_A \\ x \in [0,1] \\ (y_A, y_B) \in [0,1]^2 \\ y_A + y_B = 1}} x\left(\left(\frac{\varepsilon}{2}\sum_{\sigma\in\{A,B\}} (p_\sigma(\hat{c}_A) - c_\sigma) D(p_\sigma(\hat{c}_A)) + T_\sigma(\hat{c}_A)\right) - \psi\right)$$

$$+ (1-x)\left(\frac{1}{2}\sum_{\sigma\in\{A,B\}} y_\sigma((p_A(\hat{c}_A) - c_\sigma) D(p_\sigma(\hat{c}_A)) + T_\sigma(\hat{c}_A))\right)$$

where x is the probability of gathering information and y_σ the probability of recommending good σ while uninformed.

We now characterize conditions for both seller types to collect information and report it truthfully, *i.e.*, $x = 1$. Inducing a high-cost seller to collect information requires that equilibrium profits for that type lie in the cone, which can be written as

$$\mathcal{U}(\overline{c}_A) \geq \max\left\{\frac{\pi_A(\overline{c}_A)}{2}, \frac{\pi_B(\overline{c}_A)}{2}\right\}. \tag{6.4}$$

The left-hand side is the equilibrium payoff of a high-cost seller who reports his cost truthfully, gathers information and gives truthful advice. The right-hand side is the gain from remaining uninformed and making a recommendation based on prior beliefs.[14]

The key incentive problem now stems from a low-cost seller's possible "triple deviation": he can inflate his cost, remain uninformed, and manipulate his advice. This deviation moves the profits of the low-cost seller out of the implementability cone. The rest of the analysis consists in determining how regulation can adjust these profits to motivate information gathering. A low-cost seller's incentive constraint is

$$\mathcal{U}(\underline{c}_A) \geq \max\left\{\mathcal{U}(\overline{c}_A) + \frac{\varepsilon\Delta c}{2} D(p_A(\overline{c}_A)); \frac{\pi_B(\overline{c}_A)}{2}; \frac{\pi_A(\overline{c}_A)}{2} + \frac{\Delta c}{2} D(p_A(\overline{c}_A))\right\}. \tag{6.5}$$

The left-hand side is the equilibrium payoff of a low-cost seller who reports his cost truthfully, gathers information and gives truthful advice. The right-hand side's first term is the gain from inflating his cost, gathering information and reporting it truthfully. The second term is the gain from inflating his cost, remaining uninformed, and recommending good B. The third term is the the gain from inflating his cost, remaining uninformed, and recommending good A. This strategy would be the most attractive with a contract designed only to induce information gathering.

Intuitively, making pushing good A less attractive helps incentive compatibility. Doing so requires either reducing a high-cost seller's fixed fee for selling good A or increasing good A's price to lower demand and so reduce the information rent. This points to a trade-off between decreasing a low-cost seller's information rent and increasing a high-cost seller's liability rent. Indeed, reducing the fixed fee for selling good A might bias a high-cost seller towards good B. Avoiding such a bias requires increasing the reward for

[14] We omit a high-cost seller's option to report a low cost and check later that this constraint is slack.

good B and thus a high-cost seller's reward for gathering information. We show in the Appendix that this option is nevertheless never optimal.

A higher price and lower sales for good A if the seller reports a high cost has drawbacks too. Indeed, the seller evaluates the expected gain of inflating his cost based on his prior. Because a low-cost seller expects an information rent when remaining uninformed, price distortions on good A must be large enough. Thus decreasing the information rent requires large distortions, which is less attractive when a low cost is likelier.

We can now characterize the optimal contract in this environment.

PROPOSITION 5. *Assume c_A is private information and both information gathering and signals are non-observable. The optimal contract when both types gather information is as follows.*[15]

- *Both seller types charge prices equal to marginal cost for good B:*

$$p_B^{sb}(c_A) = c \quad \forall c_A \in \mathcal{C}. \tag{6.6}$$

- *A low-cost seller charges a price equal to marginal cost for good A while a high-cost seller charges a price above marginal cost:*

$$p_A^{sb}(\underline{c}_A) = c - \Delta c, \tag{6.7}$$

$$p_A^{sb}(\overline{c}_A) = \tilde{\overline{c}}_A \text{ where } \tilde{\overline{c}}_A = c + \frac{\nu}{(1-\nu)\varepsilon}\Delta c > c. \tag{6.8}$$

- *The high-cost seller makes the same profits on each good than when cost is common knowledge:*

$$\pi_A^{sb}(\overline{c}_A) = \pi_B^{sb}(\overline{c}_A) = \pi^*. \tag{6.9}$$

- *The low-cost seller's profits on each good can be chosen equal but greater than when cost is common knowledge:*

$$\pi_A^{sb}(\underline{c}_A) = \pi_B^{sb}(\underline{c}_A) = \pi^* + \frac{1}{2\varepsilon}\Delta c D(p_A^{sb}(\overline{c}_A)) > \pi^*. \tag{6.10}$$

An optimal contract must afford a low-cost seller an extra rent $\Delta c D(p_A(\overline{c}_A))/2$, which shifts profits inside the cone. Many profit pairs induce information gathering by that seller type. In one of them, profits on both goods are equal. Since the cheapest way to incentivize the seller is to give him positive profits only when his advice proves correct, this information rent can be distributed over all such events so that the seller's profit following any such advice must now exceed its complete information value π^* by an amount $\Delta c D(p_A(\overline{c}_A))/4$ divided by the probability that $\varepsilon/2$ that such advice is optimal.

PAYING SELLERS VIA FEES OR SALES REVENUES? To reduce the low-cost seller's information rent and bring the profits closer to the cone's extremal point, price distortions are needed for the high-cost seller. Indeed, increasing good A's price reduces demand and

[15] Superscript *sb* stands for *second best* to stress that all constraints are now taken into account.

thus the low-cost seller's information rent. It is as if the high-cost seller had a *virtual cost* $\tilde{\bar{c}}_A$. Because revenues from selling good A for a high-cost type are now positive, there is less need to pay this seller for those sales through a fee than when marginal cost pricing erodes profits as for good B

$$T_A^{sb}(\bar{c}_A) < T_B^{sb}(\bar{c}_A) = \pi^*.$$

Instead, marginal cost pricing on both goods for the low-cost seller implies no sales revenues and thus the information rent must materialize through fees

$$T_A^{sb}(\underline{c}_A) = T_B^{sb}(\underline{c}_A) = \pi^* + \frac{1}{2\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) > \pi^*.$$

Information Gathering. Now the cost of gathering information includes both the liability rent due to the non-verifiability of information gathering and the information rent due to private information about costs. This modifies the conditions for its optimality.

Proposition 6. *The optimal regulation requires that both a low-cost and a high-cost seller gather information when:*

$$\frac{\varepsilon}{2}W^*(c) - \frac{(1-\varepsilon)}{2}W^*(c-\Delta c) \geq \psi + \frac{\psi}{2\varepsilon-1} + \frac{1}{2}\Delta c D(p_A^{sb}(\bar{c}_A)) \tag{6.11}$$

and

$$\frac{(2\varepsilon-1)}{2}W^*(c) \geq \psi + \frac{\psi}{2\varepsilon-1} + \frac{\varepsilon}{2}\left(W^*(c) - W^*(\tilde{\bar{c}}_A)\right). \tag{6.12}$$

Condition (6.11) for a low-cost seller to gather information is tighter under asymmetric information due to the information rent needed on top of the limited liability rent. While a high-cost seller does not get an information rent, condition (6.12) for his information gathering is also tighter due to the allocative cost of replacing cost with virtual cost.

Like competition, regulation has a difficult time eliminating price distortions (induced by private information, not market power) while inducing information gathering. In particular, under adverse selection, a tension appears between the traditional information rent that induces price distortions and information gathering. Since experience goods are usually "experienced" through repeat purchases, the next section studies the possibility that buyer-seller dynamics *de facto* implement optimal regulation.

7. Buyer-Seller Dynamics

To analyze how a buyer can use retrospective rules to control the seller, an inherently dynamic issue, we consider an infinitely repeated trading relationship. There is no need here to look for a Nash equilibrium between sellers and our approach will be just based on the analysis of the seller's best response to the strategies of a forward-looking buyer who bases his future shopping decisions on advice quality.

We assume that the seller's cost c_A is time-invariant. The buyer's types θ_t in different periods t are i.i.d., *i.e.*, A or B with equal probability. Let δ denote the discount factor, common to both players.

In each period, the seller must learn which good is the best match with the buyer's preferences and he again incurs the corresponding disutility cost ψ. The seller must also choose the prices charged for both goods.

The buyer can switch to a rival seller. We denote by $\mathcal{S}_0$ his expected surplus with this outside opportunity. Instead, $\mathcal{S}(c_A)$ denotes the continuation value of pursuing the relationship with the current seller. The rival has similar characteristics and *a priori* the relationship should give the same expected surplus up to (unmodeled) switching costs for the buyer. We thus have $\mathcal{S}_0 = \mathbb{E}_{c_A}(\mathcal{S}(c_A)) - Z$, where Z is a switching cost. We assume that $\Delta\mathcal{S}(c_A) = \mathcal{S}(c_A) - \mathcal{S}_0 > 0$ for all c_A, *i.e.*, the buyer finds it costly of quitting and starting afresh elsewhere. We also assume that the following assumption holds:

ASSUMPTION 3.

$$\frac{\varepsilon}{2}(S(p^m(c)) + S(p^m(c-\Delta c))) \geq \frac{1}{2}S(p^m(c-\Delta c)).$$

This condition simply means that the buyer enjoys a greater expected surplus from a static relationship with a low-cost seller if this seller, who always charges monopoly prices, provides truthful information than if he systematically pushes good A.[16] Finally, when the buyer switches, the seller makes zero profit in the continuation.

The buyer can commit to probabilities of dropping the seller following a good or bad match.[17] Hence we denote by $\beta_\sigma(c_A)$ (resp. $\gamma_\sigma(c_A)$) the probability of continuing the relationship when the (truthful on the equilibrium path) advice following signal σ proves correct (resp. incorrect). The problem is stationary because we assume independent draws of the buyer's preferences over time. Accordingly, we thus describe a stationary equilibrium where the probabilities of continuing the relationship are kept constant.

While some of the contracting possibilities of the regulatory context of Section 6 are no longer available, some features found under regulation arise here too. First, the continuation payoff plays the role of the fee in a regulatory setting. Second, our assumption that the buyer adopts a retrospective rule to retain the seller or not resembles the commitment power given to the regulator. Although the control of the seller by retrospective buyers is an imperfect substitute for regulation, it exhibits similar patterns.

7.1. Moral Hazard

INCENTIVE CONSTRAINTS. Hereafter, the sole agency problem is to induce the seller to collect and reveal information each period. Let denote by $\mathcal{U}(c_A)$ the continuation value for the seller with cost c_A on the equilibrium path. It satisfies:

$$\mathcal{U}(c_A) = \max_{(p_A,p_B)} \frac{\varepsilon}{2}\left((p_A - c_A)D(p_A) + \delta\beta_A(c_A)\mathcal{U}(c_A)\right) + \frac{1-\varepsilon}{2}\delta\gamma_A(c_A)\mathcal{U}(c_A)$$

$$+ \frac{\varepsilon}{2}\left((p_B - c)D(p_B) + \delta\beta_B(c_A)\mathcal{U}(c_A)\right) + \frac{1-\varepsilon}{2}\delta\gamma_B(c_A)\mathcal{U}(c_A) - \psi$$

[16] Obviously, a similar condition always holds for a high-cost seller who is indifferent between recommending either good and always makes truthful recommendations when Assumption 2 holds.

[17] Assuming commitment to the switching probabilities on the side of the buyer could be viewed as extreme; but it gives its best chance to the threat of quitting as a disciplining device and, as such, certainly provides an upper bound on the benefits of competition.

Since the seller chooses monopoly prices for both goods, we have

$$\mathcal{U}(c_A) = \frac{\frac{\varepsilon}{2}\pi^m(c_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi}{1 - \delta\left(\frac{\varepsilon}{2}(\beta_A(c_A) + \beta_B(c_A)) + \frac{1-\varepsilon}{2}(\gamma_A(c_A) + \gamma_B(c_A))\right)}. \tag{7.1}$$

Incentives for information gathering require preventing several possible deviations

$$\mathcal{U}(c_A) \geq \max\left\{\frac{1}{2}\pi^m(c_A) + \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))\mathcal{U}(c_A); \frac{1}{2}\pi^m(c) + \frac{\delta}{2}(\beta_B(c_A) + \gamma_B(c_A))\mathcal{U}(c_A)\right\}.$$

The right-hand side stems for the seller's payoff for both goods following a one-shot deviation in which he does not gather information and gives uninformed advice, while following such one-shot deviation he sticks to gathering and revealing information in the continuation.[18] Taken at the stationary equilibrium, the condition can be written as

$$\mathcal{U}(c_A) \geq \max\left\{\frac{\frac{1}{2}\pi^m(c_A)}{1 - \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))}; \frac{\frac{1}{2}\pi^m(c)}{1 - \frac{\delta}{2}(\beta_B(c_A) + \gamma_B(c_A))}\right\}. \tag{7.2}$$

OPTIMAL RETROSPECTIVE RULES. We now analyze the buyer's retrospective rules.

PROPOSITION 7. *Suppose that Assumptions 2 and 3 both hold and that δ is sufficiently close to 1.*

- *Both seller types always gather and reveal information.*
- *The relationship with a high-cost seller is always continued:*

$$\beta_A^{mh}(\bar{c}) = \gamma_A^{mh}(\bar{c}_A) = \beta_B^{mh}(\bar{c}) = \gamma_B^{mh}(\bar{c}_A) = 1. \tag{7.3}$$

- *The relationship with a low-cost seller is always continued if he recommends good B or if he correctly recommends good A:*

$$\beta_B^{mh}(\underline{c}_A) = \gamma_B^{mh}(\underline{c}_A) = \beta_A^{mh}(\underline{c}_A) = 1. \tag{7.4}$$

- *The relationship is terminated with positive probability if the low-cost seller wrongly recommends good A:*

$$\gamma_A^{mh}(\underline{c}_A) \in [0, 1). \tag{7.5}$$

QUITTING AS AN INCENTIVE DEVICE. Though quitting is costly, the buyer uses this threat to induce information gathering. There is no problem in continuing with a high-cost seller. This type provides advice in a static relationship, and the buyer may just always come back to shop from him whatever his advice. The issue is with a low-cost seller who is biased in a one-shot relationship towards pushing good A. The most efficient way of curbing this bias is to cut the gap between the intertemporal profits following recommendations. This is best achieved by making continuation after a recommendation for good A less likely. The cheapest way is to reduce the probability of continuation when

[18] One-shot deviations are enough to characterize incentive compatibility in a stationary environment.

the low-cost seller's recommendation for good A proves incorrect. The threat of quitting is effective only when the future matters enough, hence the qualifier on δ.

BACK INTO THE CONE. To better understand the benefits of dynamics it is useful to return to the characterization of incentive compatible allocations through (7.1) and (7.2). We can rewrite these constraints as

$$\frac{\varepsilon}{2}\pi^m(c) - \frac{1-\varepsilon}{2}\kappa(c_A)\pi^m(c_A) \geq \psi$$

and

$$\frac{\varepsilon}{2}\pi^m(c_A) - \frac{1-\varepsilon}{2}\kappa(c_A)\pi^m(c) \geq \psi$$

where

$$\kappa(c_A) = \frac{1 - \frac{\delta}{1-\varepsilon}\left(\frac{\varepsilon}{2}\beta_B(c_A) + \frac{1-\varepsilon}{2}\gamma_B(c_A) + \frac{1-2\varepsilon}{2}\gamma_A(c_A)\right)}{1 - \frac{\delta}{2}(\beta_A(c_A) + \gamma_A(c_A))}.$$

The first (resp. second) constraint captures the incentives to deviate by remaining uninformed and recommending good A (resp. B).

Inserting the values found in (7.4) yields $\kappa(c) = 1 > \kappa(\underline{c}_A)$. In other words, while the dynamics do not control the high-cost seller's incentives, the threat of quitting is akin to lowering the stage-profit for good A which facilitates implementation. Much as in our model of *ex post* competition, such an asymmetry in the seller's forthcoming profits provides incentives to gather information.

7.2. *Moral Hazard and Adverse Selection*

INCENTIVE CONSTRAINTS. We now turn to the case where c_A is private information. The optimal quitting rule of Proposition 7 might no longer apply. A low-cost seller could mimic a high-cost seller by charging the same prices, which entails a short-run loss but ensures continuation. The buyer's quitting rule must also prevent such deviation. This means that this rule must satisfy the following truth-telling incentive constraint

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}(p^m(c) - \underline{c}_A)D(p^m(c)) + \frac{\delta}{2}(\beta_A(\bar{c}_A) + \gamma_A(\bar{c}_A))\mathcal{U}(\underline{c}_A). \tag{7.6}$$

The left-hand side is the low-cost seller's equilibrium payoff from collecting and revealing information.[19] The low-cost seller may always charge the same prices as a high-cost seller and recommend good A without collecting information. By doing so, the low-cost seller enjoys a short-run profit $\pi^m(c) + \Delta c D(p^m(c))$ when selling good A. Although this profit is lower than his short-run monopoly profit $\pi^m(\underline{c}_A)$, the low-cost type may benefit from the likelier continuation that pertains to a high-cost seller.

The quitting rule must also discourage the low-cost seller from mimicking a high-cost type, remaining uninformed and recommending good B

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}\pi^m(c) + \frac{\delta}{2}(\beta_B(\bar{c}_A) + \gamma_B(\bar{c}_A))\mathcal{U}(\underline{c}_A). \tag{7.7}$$

[19] Remember that the buyer commits to the quitting rule, so that the *Revelation Principle* (Myerson, 1982) applies and all cost information is revealed in one round.

Finally, it must also prevent a low-cost seller from mimicking a high-cost seller but acquiring information, in which case the incentive constraint writes as

$$\begin{aligned}(7.8)\quad \mathcal{U}(\underline{c}_A) \geq\ & \frac{\varepsilon}{2}(p^m(c)-\underline{c}_A)D(p^m(c))+\frac{\varepsilon}{2}\pi^m(c)-\psi \\ & + \delta\left(\frac{\varepsilon}{2}(\beta_A(\bar{c}_A)+\beta_B(\bar{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\bar{c}_A)+\gamma_B(\bar{c}_A))\right)\mathcal{U}(\underline{c}_A).\end{aligned}$$

Overall, the low-cost seller's incentive compatibility constraint becomes

$$\begin{aligned}(7.9)\quad \mathcal{U}(\underline{c}_A) \geq \max\Bigg\{ & \frac{\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))}{1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))};\frac{\frac{1}{2}\pi^m(c)}{1-\frac{\delta}{2}(\beta_B(\bar{c}_A)+\gamma_B(\bar{c}_A))}; \\ & \frac{\varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)-\psi}{1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\bar{c}_A)+\beta_B(\bar{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\bar{c}_A)+\gamma_B(\bar{c}_A))\right)}\Bigg\}.\end{aligned}$$

Private Information Matters. We first check whether private information on costs matters. To do so, we plug the rent profile and the continuation probabilities of Proposition 7 and check whether incentive constraint (7.9) holds. First, observe that, if moral hazard is the sole concern and Assumption 2 holds, the low-cost seller's payoff satisfies

$$(7.10)\qquad \mathcal{U}^{mh}(\underline{c}_A)=\frac{\frac{1}{2}\pi^m(\underline{c}_A)+\frac{1}{2}\pi^m(c)-\psi}{1-\delta+\frac{\delta}{2}(1-\varepsilon)(1-\gamma_A^{mh}(\underline{c}_A))}=\frac{\frac{1}{2}\pi^m(\underline{c}_A)}{1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A))}$$

where the first equality follows from writing $\mathcal{U}^{mh}(\underline{c}_A)$ on path and the second from noticing that (7.2) is binding for a low-cost seller when Assumption 2 holds.

The solution obtained under pure moral hazard fails to satisfy the truthtelling condition when the following condition holds.

Assumption 4.

$$\mathcal{U}^{mh}(\underline{c}_A) < \max\left\{\frac{\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))}{1-\delta};\frac{\varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)-\psi}{1-\delta}\right\}.$$

The right-hand side above is obtained by inserting the probabilities of continuation obtained from (7.3) into the right-hand side of (7.9). Henceforth, Assumption 4 ensures that private information on cost changes the buyer's behavior.

Optimal Quitting Rules. We can now summarize the main features of optimal rules.

Proposition 8. *Suppose that Assumptions 2, 3 and 4 hold and that δ is sufficiently close to 1:*

- *The low- and the high-cost seller both gather and reveal information.*
- *If the seller recommends good B, the relationship is continued:*

$$(7.11)\qquad \beta_B^{sb}(\underline{c}_A)=\gamma_B^{sb}(\underline{c}_A)=\beta_B^{sb}(\bar{c}_A)=\gamma_B^{sb}(\bar{c}_A)=1.$$

- *If the seller recommends good A, the relationship is continued if a seller correctly recommends good A and terminated with positive probability otherwise:*

$$\beta_A^{sb}(\underline{c}_A) = 1 \geq \gamma_A^{sb}(\underline{c}_A) \geq 0, \tag{7.12}$$

and

$$\beta_A^{sb}(\bar{c}_A) = 1 \geq \gamma_A^{sb}(\bar{c}_A) \geq 0. \tag{7.13}$$

QUITTING AS A SCREENING DEVICE. The buyer now wants to avoid that a low-cost seller unduly recommends good A without having collected information while charging the same price as a high-cost seller for that good and pocketing thereby some information rent. To avoid this possibility, the relationship should now be also terminated with some probability following a high price for and a recommendation for good A even if this is indeed the choice that would be made by a high-cost seller who has gathered information.

COMPARISON WITH THE OPTIMAL REGULATION. Both the regulator in Section 6 and the buyer in this section are concerned with the low-cost seller's incentives to mimic a high-cost seller, charge high prices and recommend good A. Yet, the buyer has no control on prices and fees are limited to be equilibrium continuation values. The only tool to reduce the low-cost seller's information rent is thus to stop the relationship. Relaxing the low-cost seller's incentive constraint requires at the same time to terminate more often the relationship if a high price is charged for good A and, maybe, to terminate this relationship less often in case good A is recommended and a low price is charged for that good although such distortion is necessary in a pure moral hazard environment.

8. ILLUSTRATIONS

This section illustrates our analysis with three examples where the provision of informational services is key to the retailing activity.

8.1. Health Care Sector

We beging with different markets and regulations of the health care sector.

DRUGS MARKETS AND PHARMACISTS. In most countries, the pharmaceutical sector is subject to price regulation, but also to strict constraints on competition. Some restrictions in drugs distribution such as constraints on ownership or on the number and locations of pharmacies are often justified by the fact that community pharmacists play a crucial role in detecting drug interactions and side-effects and facilitating appropriate medicines use. It is also legitimate to view entry barriers as emanating from reflecting political pressure by vested interests willing to protect market power.

In contrast with this folklore argument, our results suggest that competition and regulation may boost incentives for the provision of informational services. To illustrate how the lessons of our model shed new lights on actual practices, France is a good example. Recent regulation (*Arrêté* dated of November 28/2014) allows pharmacists to perceive a fee for their advising role where this role is broadly defined as checking prescriptions, making generic substitution whenever needed, ensuring patients' understanding, and detecting potential drugs interactions. Since prescribed drugs are usually subject to binding price cap regulations,[20] the gains that pharmacists may derive from private information on their margins is limited. The constant fee across drugs implemented by this regulation is perhaps best interpreted in light of the pure moral hazard problem of Section 6.1. Such a fee is indeed a way to pay for the limited liability rent needed so that pharmacists provide careful advice.

For non-prescription drugs, the situation is more complex. Because they are not subject to any price regulation, pharmacists may also enjoy more gains from private information on margins. This might also explain systematic biases in their recommendations. As pointed out in an Ecorys Study (2007) commissioned by the European Commission, entry barriers also induce high profit margins. The debate about the possible sources of the pharmacists' rents thus boils down to whether rents are justified by their expertise in providing information on therapeutic choices, or whether these rents just come from excessive market power and price-cost margin distortions (Philipsen and Faure, 2002).

Section 4 shows that this view is still incomplete in that excessive market power may also generate mis-selling. Our analysis in Section 5 suggests that competition can mitigate mis-selling since profits over both different drugs categories are adequately re-balanced and, as long as these profits are not too eroded (to cover the limited liability rent that is requested for incentive purposes).

In practice, facilitating competition can take two forms. On the one hand, it may go through increasing the number of shops authorized to sell non-prescription drugs by reducing barriers to entry. Additionally to diminishing price-cost margins on those drugs, Section 5.1 unveils how such *ex ante* competition may foster incentives and prevent mis-selling. This result seems in line with the Ecorys study.[21] On the other hand, competition in drugs market can also come from electronic commerce. For instance, in Germany, health insurers actively lobby for selling drugs online to lower expenditures. According to the French Competition Authority,[22] it seems that even though in Germany online drugs sales still have a low market share, online drugs platforms have introduced enough competition to lower price dispersion.[23] Section 4 highlights that, despite possible free-riding on advice, this type of competition may also work as a discipline device to reduce mis-selling because it erodes profits realized on high-margin drugs.

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[20]Dubois and Saethre (2016) provide evidence of those binding price constraints.

[21]Performing an analysis of variance and clustering Member States of the European Union into two groups according to the degree of regulation in drugs distribution, the Ecorys Study analyzes the consequences of regulation on variables of productivity and quality of services. While the efficiency of drugs distribution is unambiguously and negatively correlated with the degree of regulation, the sign of this correlation with quality appears to be more complex. More precisely, their results reveal that service variety, taken as a proxy for quality, is positively correlated with educational requirements and price/profit regulations. Differently, requirements on registration, licensing and obligatory membership of a professional organization exhibit a negative correlation with service variety.

[22]See http://www.oecd.org/officialdocuments/publicdisplaydocumentpdf/?cote=DAF/COMP/GF/WD(2014)44&docLanguage=Fr.

[23]A consumers study conducted in France reveals that in 2014 the price of non-prescription drugs, which represent about 20% of total sales in the pharmaceutical sector, varies from one to four. For the case of the U.S. drugs market, Sorensen (2000) provides empirical results that reveal that non-prescription drugs are characterized by a higher price dispersion than prescription drugs.

DOCTORS COMPETITION. Health economics has not come up with a unified view of doctors' competition and the doctor-patient relationship. Yet, three points are usually admitted. First, doctors may exert non-contractible effort affecting health outcomes. Second, except some special payment schemes applied to specific programs, health outcomes are not contractible either. Third, health outcomes may be observable by patients.[24]

We believe that the model with retrospective quitting rules in Section 5.1 fits well with doctors' competition when prices/fees are not regulated.[25] The time spent by doctors allows them to establish a precise diagnostic to choose the most suitable therapy. Such an effort is not verifiable. This effort increases the probability of good health outcome and patients may decide to follow the relationship with their doctor according to the observed outcome. Then, our set-up challenges Arrow (1963)'s view that transferring risk to health care providers better aligns incentives. Our results indeed reveal it may be optimal for a patient to continue the relationship with his doctor following bad outcomes. Our analysis shows that, only when the doctor may be biased towards a specific therapeutic choice, that termination is optimal with positive probability. In other words, a total risk transfer is not optimal even if doctors are risk neutral.

8.2. E-Commerce and "Showrooming"

Our analysis sheds light on the practice of "showrooming" according to which customers evaluate products in brick-and-mortar stores but buy online to get lower prices (see Van Baal and Dach, 2005, for instance and Bosman, 2011, for the case of books). Indeed, many product attributes are difficult to assess online and while Internet retailers tend to provide customer feedbacks or evaluations, the promotion effort of brick-and-mortars retailers in the form of personal interaction between customers and the sale force remains key to the buying decision.

That concern has led some manufacturers to find ways to counter showrooming, ranging from asking suppliers to create special products that would shield them from price comparisons (Zimmerman, 2012a), committing to match competitors' prices (Zimmerman, 2012b) to charging customers for trying products and reimbursing the charge only if they buy from the store (Bita, 2011).

Mehra *et al.* (2013) analyze how brick-and-mortars stores can counter customers' free-riding. Carlton and Chevalier (2001) focus on three categories of products and find evidence that manufacturers internalize free-riding by controlling the online distribution of their products. Klein (2015) and Miklòs-Thal and Shaffer (2015) analyze the role of vertical restraints in limiting free-riding.

Our analysis complements these in several ways. First, the firms' informational role is to provide the customer with the best product-fit by issuing a personalized but potentially

[24]The two last points are eloquently summarized in McGuire (2000) *"It may be infeasible to pay doctors on whether they are able to cure back pain because it is too costly to validate a patient's report. Nonetheless, the patient knows if his back still hurts. If the doctor is rewarded for doing a better job, because the patient is more likely to return or to recommend this doctor to friends, the doctor is encouraged to take unobserved actions to improve quality."*

[25]For instance it is the case in the so-called *Sector II* in France. The *Sector II* is the regime under which doctors can freely set their tariffs, usually above reimbursement levels of public coverage. Then, when they do not benefit from complementary health insurance coverage, patients face some out-of-pockets.

biased recommendation. Second, we empower customers with the possibility to terminate the relationship with the firm in case the recommendation turned out to be wrong.

The analysis in Section 5 is reminiscent of the showrooming issue: When customers can take a more competitive offer from retailers after having received a recommendation, competition can have a mixed impact on incentives to gather information and to make a good recommendation. If competition erodes mostly the profit of the good for which the brick-and-mortar retailer earns the highest margin, then profits may enter the implementability cone and competition promotes information collection. If competition, however, erodes profits somewhat symmetrically, then it may destroy the incentives to advise customers and becomes detrimental to welfare. Last, Section 7 suggests that loyalty programs which help customers keep track of their buying decisions may limit the bias in the firms' recommendations.

8.3. Financial Advising

The global financial crisis and its aftermath have shed a crude light onto the conflicts of interest arising between financial advisers and their advisees in virtually all areas of the finance industry, from credit rating agencies to investment advisors, and from retail mortgage financing to investment banking. Some even argue conflict of interest is inherent to the intermediation nature of investment banking where the financial advisor must have a view of both sides of the market (Fox, 2010). This has led to a call for tighter regulatory oversight and, in some cases, more intense competition.

Investment Advising. Large banks are facing increasing scrutiny over their sales practices. For instance, in 2015, JPMorgan Chase agreed to pay a $307m penalty for failing to disclose to its clients that it was steering them away from investment products offered by rivals and towards a more expensive share class of proprietary mutual funds, from which it generated more profits. More generally, private bankers and other investment advisors are often accused of pushing investment strategies with higher turnover, and thus higher fees, and higher switching costs (*e.g.*, exit fees) than optimal for their clients.[26]

Our analysis points to the intricate issues involved in the regulation of such conflicts of interest. Regulators may need to deal not only with the quality of advice directly but also account for the inherent lack of transparency, and thus the high degree of information asymmetry, regarding the margins financial advisors realize on different products or strategies. One interesting aspect is the advisor's alleged ability to build up switching costs as part of the products they sell their clients. Regulatory efforts to mitigate such switching costs may also indirectly impact the quality of advice provision through two channels. First, increased competition may reduce the rent on the bank's own investment products, thereby promoting information collection. Second, lower switching costs may make it easier and more credible for clients to follow dynamic strategies of the type highlighted in Section 7, again boosting the banks' incentives to provide quality advice.

[26] In a more unusual and colorful case, the Libyan Investment Authority (LIA) sued Goldman Sachs for $1.2bn to recover losses from nine "elephant trades" involving equity derivatives arranged in 2008 and which all expired worthless in 2011. The LIA alleged that Goldman exerted undue influence over its officials, who did not understand the trades, and earned about $222m from the trades. (Goldman Sachs was recently acquitted).

Credit Rating. Credit rating agencies were instrumental in the boom of the structured finance market in the years leading up to the financial crisis, and were accused of having employed excessively lax credit rating standards when that market collapsed so dramatically. The structure of the credit rating industry, including its oligopolistic nature and the fact issuers, not investors, pay for ratings, was blamed by many for the excesses of the credit bubble years, leading to calls for the emergence of new agencies to offer further competition to the handful of major incumbents.

Because rating agencies are remunerated by the issuers, not the investors who rely on their ratings for their investment decisions, the industry structure is probably best captured by the buyer-seller dynamics model of Section 7 which can reflect the reputation loss a rating agency may incur (Mathis, McAndrews and Rochet, 2009). Agencies are arguably biased towards higher ratings as they are more likely to be accepted by issuers (Faure-Grimaud, Peyrache and Quesada, 2009), and generate more fees in expectation, including from repeat business. Indeed, issuers can opt not to publish a given rating, and published ratings generate ongoing fees while the issue is outstanding. Our analysis contributes to this debate on whether competition can discipline rating agencies or whether more stringent regulation is required. It highlights conditions under which heightened competition between rating agencies can provide discipline, but also the limits of this mechanism. It also points to the challenges regulation might face when agencies, notably when rating agencies have better knowledge of the margins they enjoy from different issuers, notably through consulting services.

9. Conclusion

In many instances, customers rely on the expertise of sellers for advice about the goods or services they purchase from them. Such situations naturally give rise to conflicts of interest whereby seller may steer customers towards higher margin goods or services. Whether competition suffices to discipline expert-sellers' incentives is an issue of importance to understand a number of retailing practices.

This paper tackles this issue in a context with both moral hazard (the expert's decision to gather information is non-verifiable) and adverse selection (the expert has private information on his price-cost margins for different goods). Whatever the market structure or institutional context, the starting point of our analysis is the simple observation that information gathering incentives requires that the seller's profits on the different goods not be too different. Technically, the profits must lie within an implementability cone, else the expert would have incentives to remain uninformed and recommend the highest margin good. Absent competition, a monopoly might thus always push that good.

Monopoly comes not only with the usual price distortions, it also induces underprovision in informational services. Competition is then beneficial whenever it drives profits into the implementability cone, in which case it induces information gathering. Under such a scenario, competition not only erodes price-cost margins but also improves informational services.

Our analysis provides a mixed view of competition's impact. First, competition may erode profits so much as to discourage the provision of advice by sellers. Hence, only moderate competition can have a disciplining effect. Yet, this necessary condition is not sufficient without enough consumer rationality. Naive consumers who would pick a seller based on prices might be too aggressive as this leads sellers to cut price-cost margins, pushing their profits out of the implementability cone. More opportunistic buyers who may choose whether to retain or drop a given seller based on his recommendation may be better able to provide discipline; especially if sellers compete in high-cost margin good markets. The least naive consumers, because they might be involved repeatedly with sellers, should be able to use retrospective purchasing rules and buy again from a seller only if his advice proved correct. In such scenarios, consumers are *de facto* implementing (although imperfectly) what an optimal regulation does. Such long-run repeated relationships might be convenient descriptions of market contexts where switching costs play an important role such as in the physician-patient or customer-bank relationships.

A takeaway is that rather than a monopoly situation, buyers are always better off with moderate competition: at worst, price distortion is reduced, and at best so too is the underprovision of advice. For sectors with some degree of competition and characterized by entry barriers, such as pharmaceutical distribution, our analysis reveals that it would be crucial to develop empirical tests connecting the outcome, *i.e.* the quality of the matching between the customers and the products, with competition intensity. In particular, such empirical estimations should focus on "goods maturity" to predict when an increase of the competition intensity is likely to bring closer the sellers' profits on the different goods and then increases information gathering. The detailed analysis of specific markets may unveil new interesting features. They are left for future research.

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Appendix

Proof of Lemma 1. The constrained set defined by (4.1) can be further developed as a pair of constraints (4.2). These constraints define a positive cone Γ in the (π_A, π_B) space with an extremal point given by (4.3) and directions given by positive vectors $(\varepsilon, 1-\varepsilon)$ and $(1-\varepsilon, \varepsilon)$. $\square$

PROOF OF PROPOSITION 1. The buyer's preferences do not depend on the seller's private information c_A. Demand only depends on the price charged by the seller and not directly on the seller's cost. Thus beliefs (both on- and off-the equilibrium path) play no role in the buyer's behavior. For any specification of off-equilibrium beliefs, the best strategy for a seller with cost c_A is to charge the monopoly prices $p^m(c_A)$ for good A and $p^m(c)$ for good B, irrespective of the information he might have on the quality of the match.

Assumption 2 then ensures that the seller does gather information when his cost is high and does not do so when his cost is low. The low-cost seller always recommends good A while the high-cost seller makes a recommendation that reflects the signal he has learned when gathering information. This equilibrium allocation is unique and sustained by arbitrary beliefs following unexpected prices. □

PROOF OF PROPOSITION 2. Keeping the equilibrium prices (p_A, p_B) charged by his rival located at 1 on the segment as given (or alternatively, taking as fixed the profit levels (π_A, π_B) targeted on each good by this rival), seller located at 0 maximizes (5.1) subject to the feasibility constraints

$$\tilde{\pi}_A \leq \tilde{\pi}^m(c_A) \text{ and } \tilde{\pi}_B \leq \tilde{\pi}^m(c).$$

At a symmetric equilibrium with information gathering if any exists, the first-order conditions for optimality w.r.t each profit target give us

$$\text{(A.1)} \qquad \frac{\varepsilon}{2}\pi_A + \frac{\varepsilon}{2}\pi_B - \psi = t\left((P(\pi_A, c_A) - c_A)\frac{f(P(\pi_A, c_A))}{1 - F(P(\pi_A, c_A))} - 1\right)$$

and

$$\text{(A.2)} \qquad \frac{\varepsilon}{2}\pi_A + \frac{\varepsilon}{2}\pi_B - \psi = t\left((P(\pi_B, c) - c_A)\frac{f(P(\pi_B, c))}{1 - F(P(\pi_B, c))} - 1\right).$$

From this, we immediately deduce that equilibrium profits always lie on the locus $\pi_B = \Phi(\pi_A, c_A)$ implicitly defined by (5.2). Fixing a value of the transportation cost t, we can recover the values of the profit targets (π_A, π_B) from solving the system (A.1)-(A.2).

When t converges towards $+\infty$, the right-hand side above is finite only when $P(\pi_A, c_A)$ (resp. $P(\pi_B, c)$) itself converges towards $p^m(c_A)$ (resp. $p^m(c)$) and thus π_A (resp. π_B) converges towards $\pi^m(c_A)$ (resp. $\pi^m(c)$). When Assumption 2 holds, this means that (π_A, π_B) so obtained do not lie in the implementatibility cone Γ when t is large enough.

When t converges towards zero instead, the profit levels are obtained at the intersection of the loci (5.2) and the zero-profit condition

$$\text{(A.3)} \qquad \frac{\varepsilon}{2}\pi_A + \frac{\varepsilon}{2}\pi_B - \psi = 0.$$

Observe that, for $\pi_A = \pi_B = \pi^*$ this expression is worth $\varepsilon\pi^* - \psi = \frac{\psi}{2\varepsilon - 1} > 0$. Hence, again, profit levels on the zero-profit condition (A.3) do not belong to Δ when t is small enough.

Condition (5.3) then ensures that the locus $\pi_B = \Phi(\pi_A, c_A)$ enters the implementatibility cone Γ when $c_A = c - \Delta c$. From the results above, it requires that t belongs to an interval of the form $[t^*, t^{**}]$. □

PROOF OF PROPOSITION 3. For a given cost c_A, the seller is better off collecting information and giving truthful advice than remaining uninformed and recommending good A and good B respectively if

$$\text{(A.4)} \qquad \frac{\varepsilon}{2}\pi_A^c(c_A, n) + \frac{\varepsilon}{2}\pi^m(c) - \psi \geq \frac{1}{2}\pi_A^c(c_A, n)$$

and

$$\frac{\varepsilon}{2}\pi_A^c(c_A, n) + \frac{\varepsilon}{2}\pi^m(c) - \psi \geq \frac{1}{2}\pi^m(c). \tag{A.5}$$

As n decreases to 0, $\pi_A^c(c_A, n)$ increases to $\pi^m(c_A)$. This and Assumption 2 imply that for n small enough, conditions (A.4) and (A.5) hold for $c_A = c$ while only condition (A.4) holds for $c_A = c - \Delta c$.

As n goes to $+\infty$ instead, $\pi_A^c(c_A, n)$ goes to 0. This implies that for n large enough, condition (A.4) holds and condition (A.5) is violated irrespective of c_A.

These remarks imply the existence of n^* and n^{**} with $n^* < n^{**}$ such that conditions (A.4) and (A.5) bind for $c_A = c - \Delta c$ respectively, and that of n^{***} such that condition (A.5) binds for $c_A = c$. Condition (A.5) being tighter for higher values of c_A, we have $n^{***} < n^{**}$ □

PROOF OF PROPOSITION 4. An optimal contract maximizes the customer's expected net surplus

$$\frac{\varepsilon}{2}\sum_{\sigma\in\{A,B\}} S(p_\sigma(c_A)) - T_\sigma(c_A) = \frac{\varepsilon}{2}\sum_{\sigma\in\{A,B\}} W(c_\sigma, p_\sigma(c_A)) - \pi_\sigma(c_A)$$

subject to the information gathering moral hazard constraint (4.1).

Since the implementability cone Γ expressed as (4.1) does not depend on prices, overall surplus is obviously maximized with marginal cost pricing (6.1).

The maximum is obtained when $\frac{\varepsilon}{2}\sum_{\sigma\in\{A,B\}}\pi_\sigma(c_A)$ is minimized and, since Γ is a positive cone with directions $(\varepsilon, 1-\varepsilon)$ and $(1-\varepsilon, \varepsilon)$, this is achieved for the extremal point (4.3).

Inducing information gathering is thus valuable for the regulator when $(W^*(c_A)-\pi^*, W^*(c)-\pi^*)$ belongs to Γ. Developing this expression yields (6.3) and

$$\frac{(2\varepsilon-1)}{2}W^*(c) \geq \psi + \frac{\psi}{2\varepsilon-1}. \tag{A.6}$$

It is easy to verify that (6.3) implies (A.6) since $W^*(\cdot)$ is non-increasing. Hence, if it is optimal to induce information gathering by the low-cost seller, it is also so by a high-cost seller. □

PROOF OF PROPOSITION 5 AND PROPOSITION A.1. We consider contracts that induce information gathering from both types (Proposition 5). We turn later to the characterization of the conditions that ensure it is optimal to do so (Proposition A.1). First, we can write the regulator's objective under asymmetric information as

$$\mathbb{E}_{c_A}\left(\frac{\varepsilon}{2}\sum_{\sigma\in\Sigma} W(c_\sigma, p_\sigma(c_A)) - \psi - \mathcal{U}(c_A)\right). \tag{A.7}$$

Second, we develop the incentive constraints (6.4) and (6.5) respectively as

$$\mathcal{U}(\bar{c}_A) = \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi \geq \frac{1}{2}\pi_A(\bar{c}_A), \tag{A.8}$$

$$\mathcal{U}(\bar{c}_A) = \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi \geq \frac{1}{2}\pi_B(\bar{c}_A), \tag{A.9}$$

and

$$\mathcal{U}(\underline{c}_A) \geq \mathcal{U}(\bar{c}_A) + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A)), \tag{A.10}$$

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}\pi_B(\bar{c}_A), \tag{A.11}$$

$$\mathcal{U}(\underline{c}_A) \geq \frac{1}{2}\pi_A(\bar{c}_A) + \frac{\Delta c}{2}D(p_A(\bar{c}_A)). \tag{A.12}$$

Participation is ensured for both types when it is so for a high-cost seller

$$\mathcal{U}(\bar{c}_A) \geq 0. \tag{A.13}$$

An optimal contract maximizes (A.7) subject to the incentive constraints (A.8) to (A.12) and the participation constraint (A.13).[27]

BINDING CONSTRAINTS. Fixing prices, we must first minimize the expected rent left to the seller

$$\mathbb{E}_{c_A}(\mathcal{U}(c_A)) = \nu\mathcal{U}(\underline{c}_A) + (1-\nu)\left(\frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi\right). \tag{A.14}$$

We distinguish two cases depending on which of the constraints (A.8) to (A.12) are binding when minimizing (A.14). Of course, some of those constraints are necessarily binding.

Case 1. Constraints (A.8), (A.9) and (A.12) are binding. Consider first the case where (A.8) and (A.9) are both binding to minimize $\mathcal{U}(\bar{c}_A)$. It implies

$$\pi_A(\bar{c}_A) = \pi_B(\bar{c}_A) = \frac{2\psi}{2\varepsilon - 1} \text{ and } \mathcal{U}(\bar{c}_A) = \frac{\psi}{2\varepsilon - 1} \tag{A.15}$$

and thus (A.12) is more constraining than (A.11).

Finally, observe that the fact that (A.12) is more constraining than (A.10) amounts to

$$\frac{1}{2}\pi_A(\bar{c}_A) + \frac{1}{2}\Delta c D(p_A(\bar{c}_A)) \geq \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A))$$

or

$$2\psi + (1-\varepsilon)\Delta c D(p_A(\bar{c}_A)) \geq \varepsilon\pi_B(\bar{c}_A) - (1-\varepsilon)\pi_A(\bar{c}_A) = 2\psi$$

where the last equality follows when (A.8) and (A.9) are binding.

From (A.12) binding, it follows that

$$\mathcal{U}(\underline{c}_A) = \frac{\psi}{2\varepsilon - 1} + \frac{1}{2}\Delta c D(p_A(\bar{c}_A)) \tag{A.16}$$

which, altogether with (A.15), gives us the following expression of the seller's expected rent

$$\mathbb{E}_{c_A}\left(\mathcal{U}(c_A)\right) = \frac{\psi}{2\varepsilon - 1} + \frac{\nu}{2}\Delta c D(p_A(\bar{c}_A)). \tag{A.17}$$

Case 2. Constraints (A.9), (A.10) and (A.12) are binding. Consider now the case where (A.9) is binding, (A.8) slack and the right-hand side of (A.10) is weakly greater than the right-hand side of (A.12). This latter condition writes as

$$\mathcal{U}(\bar{c}_A) + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A)) \geq \pi_A(\bar{c}_A) + \frac{1}{2}\Delta c D(p_A(\bar{c}_A)).$$

or

$$\varepsilon\pi_B(\bar{c}_A) - (1-\varepsilon)\pi_A(\bar{c}_A) \geq 2\psi + (1-\varepsilon)\Delta c D(p_A(\bar{c}_A)). \tag{A.18}$$

This condition implies

$$\varepsilon\pi_B(\bar{c}_A) - (1-\varepsilon)\pi_A(\bar{c}_A) > 2\psi$$

[27]The incentive constraint of a high-cost seller and the participation constraint of a low-cost one can be shown to be satisfied.

which ensures that (A.8) holds.

Hence, the minimization of $\mathcal{U}(\bar{c}_A)$ subject to (A.9) and (A.18) implies that both constraints are binding. Thus, (A.10) and (A.12) are also both binding. This gives the following expressions of profits

$$\pi_A(\bar{c}_A) = \frac{2\psi}{2\varepsilon - 1} + \frac{(1-\varepsilon)^2}{2\varepsilon - 1}\Delta c D(p_A(\bar{c}_A)), \tag{A.19}$$

$$\pi_B(\bar{c}_A) = \frac{2\psi}{2\varepsilon - 1} + \frac{(1-\varepsilon)\varepsilon}{2\varepsilon - 1}\Delta c D(p_A(\bar{c}_A)). \tag{A.20}$$

These conditions imply

$$\pi_A(\bar{c}_A) < \pi_B(\bar{c}_A)$$

so that (A.8) holds.

Those formula thus also imply

$$\mathcal{U}(\bar{c}_A) = \frac{\psi}{2\varepsilon - 1} + \frac{(1-\varepsilon)\varepsilon}{2(2\varepsilon - 1)}\Delta c D(p_A(\bar{c}_A)), \tag{A.21}$$

$$\mathcal{U}(\underline{c}_A) = \frac{\psi}{2\varepsilon - 1} + \frac{\varepsilon^2}{2(2\varepsilon - 1)}\Delta c D(p_A(\bar{c}_A)). \tag{A.22}$$

It gives us the following expression of the seller's expected rent

$$\mathbb{E}_{c_A}(\mathcal{U}(c_A)) = \frac{\psi}{2\varepsilon - 1} + \frac{(\nu\varepsilon + (1-\nu)(1-\varepsilon))\varepsilon}{2(2\varepsilon - 1)}\Delta c D(p_A(\bar{c}_A)). \tag{A.23}$$

The comparison of (A.17) and (A.23) shows that the optimal contract that induces information gathering from both types is found in Case 1 (resp. Case 2) when

$$(\nu\varepsilon + (1-\nu)(1-\varepsilon))\varepsilon \geq \nu(2\varepsilon - 1) \Leftrightarrow \nu(1-\varepsilon) + (1-\nu)\varepsilon \geq 0$$

which is always true. Thus Case 1 is the only relevant one.

Prices. Taking into account the expression of the seller's rent so obtained above, prices must thus maximize

$$\mathbb{E}_{c_A}\left(\frac{\varepsilon}{2}W(c_A, p_A(c_A)) + \frac{\varepsilon}{2}W(c, p_B(c_A)) - \psi - \mathcal{U}(c_A)\right). \tag{A.24}$$

Inserting (A.17) into the above maximand and optimizing w.r.t. to prices $p_A(c_A)$ and $p_B(c_A)$ gives us (6.7) and (6.8).

Fixed fees. The profit levels for each good are both given by (A.15) if $\bar{c}_A$ realizes. From this and the existing distortion of $p_A^{sb}(\bar{c}_A)$ given in (6.8), we obtain

$$\frac{2\psi}{2\varepsilon - 1} - \frac{\nu}{(1-\nu)\varepsilon}D(p_A^{sb}(\bar{c}_A)) = T_A^{sb}(\bar{c}_A) < T_B^{sb}(\bar{c}_A) = \pi^*.$$

Profit levels for each good (an thus fixed fees since prices are then equal to marginal costs) remain indeterminate if $\underline{c}_A$ realizes. The sum of these fees is obtained from (A.16) as

$$\frac{\varepsilon}{2}\sum_{\sigma\in\{A,B\}} T_\sigma^{sb}(\underline{c}_A) = \frac{2\varepsilon\psi}{2\varepsilon - 1} + \frac{1}{2}\Delta c D(p_A^{sb}(\bar{c}_A)) \tag{A.25}$$

while information gathering in state $\underline{c}_A$ holds when

$$U^{sb}(\underline{c}_A) \geq \max\left\{\frac{T_A^{sb}(\underline{c}_A)}{2}, \frac{T_B^{sb}(\underline{c}_A)}{2}\right\}$$

which can be written as a pair of inequalities

$$\varepsilon T_A^{sb}(\underline{c}_A) - (1-\varepsilon)T_B^{sb}(\underline{c}_A) \geq 2\psi, \tag{A.26}$$

$$\varepsilon T_B^{sb}(\underline{c}_A) - (1-\varepsilon)T_A^{sb}(\underline{c}_A) \geq 2\psi. \tag{A.27}$$

It is straightforward to check that (A.25), (A.26) and (A.27) altogether define a non-empty set of fixed fees $(T_A^{sb}(\underline{c}_A), T_B^{sb}(\underline{c}_A))$ that can be used to implement the optimal contract. A particular case is to have equal fees and then

$$T_A^{sb}(\underline{c}_A) = T_B^{sb}(\underline{c}_A) = \frac{2\psi}{2\varepsilon - 1} + \frac{1}{2\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) > \pi^*. \tag{A.28}$$

PROFITS. From (A.15), we get (6.9). Because prices are equal to marginal costs on each good for a low-cost seller, (A.28) imply (6.10).

PAYOFF. When information is collected only by the high-cost seller, the expected consumer surplus becomes

$$\begin{aligned}\mathcal{W}_{11} = \nu\left(\frac{\varepsilon}{2}W^*(c-\Delta c) + \frac{\varepsilon}{2}W^*(c)\right)\\ + (1-\nu)\left(\frac{\varepsilon}{2}W(c, p_A^{sb}(\bar{c}_A)) + \frac{\varepsilon}{2}W^*(c)\right) - \psi - \frac{\psi}{2\varepsilon-1} - \frac{\nu}{2}\Delta c D(p_A^{sb}(\bar{c}_A)).\end{aligned}$$

INFORMATION GATHERING BY ONLY ONE TYPE. Suppose that the optimal contract requests that the high-cost seller never gathers information and that, in this case, good B is always sold. This possibility allows to save on the rent of the low-cost seller since the benefits of mimicking a high-cost type then disappear. The sole incentive constraint is that inducing information gathering for the low-cost seller

$$\mathcal{U}(\underline{c}_A) = \frac{\varepsilon}{2}\pi_A(\underline{c}_A) + \frac{\varepsilon}{2}\pi_B(\underline{c}_A) - \psi \geq \max\left\{\frac{1}{2}\pi_B(\underline{c}_A); \frac{1}{2}\pi_B(\underline{c}_A)\right\} \tag{A.29}$$

while the high-cost seller's participation constraint is

$$\mathcal{U}(\bar{c}_A) = \frac{1}{2}\pi_B(\bar{c}_A) \geq 0. \tag{A.30}$$

By an argument that replicates our findings in the case of pure moral hazard, we immediately obtain the following result.

PROPOSITION A.1. *Suppose also that c_A is private information and that both effort in information gathering and recommendations are non-observable. The optimal contract that induces information gathering by the low-cost seller only has the following properties.*

- *A low-cost seller charges price equal to marginal cost for both goods:*

$$p_A^{sb}(\underline{c}_A) = c - \Delta c \text{ and } p_B^{sb} = c. \tag{A.31}$$

- *The high-cost seller always sells good B at marginal cost:*

$$p_B^{sb}(\bar{c}_A) = c. \tag{A.32}$$

- *Fixed fees for the low-cost seller are:*

$$T_A^{sb}(\underline{c}_A) = T_B^{sb}(\underline{c}_A) = \pi^*. \tag{A.33}$$

Proof of Proposition A.1. An optimal contract that induces information gathering from the low-cost seller only is implemented at minimal cost when the high-cost seller always chooses good B when uninformed. It maximizes

$$\nu\left(\frac{\varepsilon}{2}W(\underline{c}_A, p_A(\underline{c}_A)) + \frac{\varepsilon}{2}W(c, p_B(\underline{c}_A)) - \psi - \mathcal{U}(\underline{c}_A)\right) + (1-\nu)\left(\frac{1}{2}W(c, p_B(\bar{c}_A)) - \mathcal{U}(\bar{c}_A)\right) \tag{A.34}$$

subject to the truthtelling (A.29) and participation (A.30) constraints. Those constraints are obviously binding. Optimizing w.r.t. prices gives (A.31) and (A.32). Finding the expressions of $T_A^{sb}(\underline{c}_A)$ and $T_B^{sb}(\underline{c}_A)$ in (A.33) is easily obtained. □

When information is collected only by the low-cost seller, the expected consumer surplus becomes

$$\mathcal{W}_{10} = \nu\left(\frac{\varepsilon}{2}W^*(c-\Delta c) + \frac{\varepsilon}{2}W^*(c) - \psi - \frac{\psi}{2\varepsilon - 1}\right) + (1-\nu)\frac{1}{2}W^*(c).$$

Suppose now that the optimal contract requests that the low-cost seller never gathers information and that, in this case, good A is always sold by that type while the high-cost seller gathers information. The incentive constraint of a low-cost seller willing to mimic a high-cost one is

$$\mathcal{U}(\underline{c}_A) = \frac{1}{2}\pi_A(\underline{c}_A) \geq \mathcal{U}(\bar{c}_A) + \frac{\varepsilon}{2}\Delta c D(p_A(\bar{c}_A)) \tag{A.35}$$

while the high-cost seller's participation constraint remains

$$\mathcal{U}(\bar{c}_A) = \frac{\varepsilon}{2}\pi_A(\bar{c}_A) + \frac{\varepsilon}{2}\pi_B(\bar{c}_A) - \psi \geq \max\left\{\frac{1}{2}\pi_B(\underline{c}_A); \frac{1}{2}\pi_B(\underline{c}_A)\right\}. \tag{A.36}$$

Proposition A.2. *Suppose also that c_A is private information and that both effort in information gathering and recommendations are non-observable. The optimal contract that induces information gathering by the high-cost seller only has the following properties.*

- *The low-cost seller only sells good A at price equal to marginal cost:*

$$p_A^{sb}(\underline{c}_A) = c - \Delta c. \tag{A.37}$$

- *The high-cost seller always sells good B at marginal cost and good A at price above marginal cost*

$$p_A^{sb}(\bar{c}_A) = c + \frac{\nu}{(1-\nu)\varepsilon}\Delta c, \text{ and } p_B^{sb}(\bar{c}_A) = c. \tag{A.38}$$

- *Profits for the high-cost seller are identical to those when the sole incentive problem comes from information gathering which gives the following expressions of fixed fees:*

$$T_A^{sb}(\bar{c}_A) + \frac{\nu}{(1-\nu)\varepsilon}\Delta c D(p_A^{sb}(\bar{c}_A)) = T_B^{sb}(\bar{c}_A) = \pi^*. \tag{A.39}$$

The fixed fee for the low-cost seller selling good A is:

$$T_A^{sb}(\underline{c}_A) = \frac{\varepsilon}{2}\Delta c D(p_A^{sb}(\bar{c}_A)). \tag{A.40}$$

PROOF OF PROPOSITION A.2. An optimal contract that induces information gathering from the high-cost seller only is implemented at minimal cost when it maximizes

$$\nu\left(\frac{1}{2}W(\underline{c}_A, p_A(\underline{c}_A)) - \mathcal{U}(\underline{c}_A)\right) + (1-\nu)\left(\frac{\varepsilon}{2}W(c, p_A(\bar{c}_A)) + \frac{\varepsilon}{2}W(c, p_B(\bar{c}_A)) - \psi - \mathcal{U}(\bar{c}_A)\right) \tag{A.41}$$

subject to the truthtelling (A.35) and participation (A.36) constraints. Those constraints are obviously binding. Optimizing w.r.t. prices gives (A.31) and (A.32). The proof for finding the expressions of $T_A^{sb}(\bar{c}_A)$ and $T_B^{sb}(\bar{c}_A)$ in (A.39) and (A.40) is then similar to that of Proposition A.1 although it takes into account that $p_A^{sb}(\bar{c}_A) > c$ so that profits net of fees on good A are positive. Condition (A.39) follows immediately from (A.35) binding and (A.37). □

When information is collected only by the low-cost seller, the expected consumer surplus becomes

$$\mathcal{W}_{01} = \nu\frac{1}{2}W^*(c - \Delta c) + (1-\nu)\left(\frac{\varepsilon}{2}W(c, p_A^{sb}(\bar{c}_A)) + \frac{\varepsilon}{2}W^*(c) - \psi - \frac{\psi}{2\varepsilon - 1}\right).$$

OPTIMALITY OF INFORMATION GATHERING. Information gathering by both types is optimal when:

$$\mathcal{W}_{11} \geq \max\{\mathcal{W}_{10}, \mathcal{W}_{01}\}.$$

Simplifying yields conditions (6.11) and

$$\frac{\varepsilon}{2}W(c, p_A^{sb}(\bar{c}_A)) - \frac{(1-\varepsilon)}{2}W^*(c) \geq \psi + \frac{\psi}{2\varepsilon - 1} + \frac{1}{2}\frac{\nu}{1-\nu}\Delta c D(p_A^{sb}(\bar{c}_A)).$$

Manipulating the left-hand side yields (6.12). □

PROOF OF PROPOSITION 7. To ease notations, let us define

$$K(c_A) = \frac{1-\varepsilon}{2}\pi^m(c_A) - \frac{\varepsilon}{2}\pi^m(c) + \psi.$$

Assumption 2 can be rewritten as

$$K(\bar{c}_A) < 0 < K(\underline{c}_A). \tag{A.42}$$

On top, observe that the following condition, that will be encountered in the analysis below,

$$\frac{\varepsilon}{2}\pi^m(\underline{c}_A) > \frac{2(1-\delta)}{\delta}K(\underline{c}_A) \tag{A.43}$$

holds when δ is close enough to 1.

Simplifying the set of incentive compatible constraints. We may rewrite (7.2) as a pair of constraints

$$\mathcal{U}(c_A) \geq \frac{\frac{1}{2}\pi^m(c)}{1-\frac{\delta}{2}(\beta_B(c_A)+\gamma_B(c_A))}, \tag{A.44}$$

and

$$\mathcal{U}(c_A) \geq \frac{\frac{1}{2}\pi^m(c_A)}{1-\frac{\delta}{2}(\beta_A(c_A)+\gamma_A(c_A))}. \tag{A.45}$$

We will neglect (A.44) and check that it is satisfied *ex post* once we have derived the solution to the so-called relaxed problem.

Together (7.1) and (A.45) imply that we may express the moral hazard incentive constraint when the seller has cost c_A in a more compact form as

$$\frac{\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi}{1-\delta\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)} \geq \frac{\frac{1}{2}\pi^m(c_A)}{1-\frac{\delta}{2}(\beta_A(c_A)+\gamma_A(c_A))}.$$

After manipulations, we obtain

$$\begin{aligned}\delta\Bigg(\frac{\pi^m(c_A)}{2}&\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)\\ &-\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(c_A)+\gamma_A(c_A))}{2}\Bigg) \geq K(c_A).\end{aligned} \tag{A.46}$$

Reciprocally, for a vector of probabilities of keeping the relationship $(\beta_A(c_A), \beta_B(c_A), \gamma_A(c_A), \gamma_B(c_B))$ that satisfies (A.46), we may recover the seller's continuation value $\mathcal{U}(c_A)$ from (7.1). Hence, (A.46) is both necessary and sufficient for characterizing the feasible set for problem (A.48).

Optimal probabilities of continuing the relationship. Proceeding as in the text, we may also define the continuation value for the customer $\mathcal{S}(c_A)$ in state c_A as the solution to the following problem:

$$\begin{aligned}\mathcal{S}(c_A) = \max_{\substack{(\beta_\sigma(c_A),\gamma_\sigma(c_A))_{\sigma\in\Sigma}\\ 0\leq\beta_\sigma(_A)\leq 1\\ 0\leq\gamma_\sigma(c_A)\leq 1}} &\frac{\varepsilon}{2}(S(p^m(c_A))+\delta(\mathcal{S}(c_A)-(1-\beta_A(c_A))\Delta\mathcal{S}(c_A)))\\ &+\frac{1-\varepsilon}{2}\delta(\mathcal{S}(c_A)-(1-\gamma_A(c_A))\Delta\mathcal{S}(c_A))\\ &+\frac{\varepsilon}{2}(S(p^m(c))+\delta(\mathcal{S}(c_A)-(1-\beta_B(c_A))\Delta\mathcal{S}(c_A))\\ &+\frac{1-\varepsilon}{2}\delta(\mathcal{S}(c_A)-(1-\gamma_B(c_A))\Delta\mathcal{S}(c_A))).\end{aligned} \tag{A.47}$$

subject to (7.1) and 7.2).

Isolating current period and continuation, we may rewrite this maximand as

(A.48)

$$(1-\delta)\mathcal{S}(c_A) = \max_{\substack{(\beta_\sigma,\gamma_\sigma)_{\sigma\in\Sigma}\\ 0\leq\beta_\sigma\leq 1\\ 0\leq\gamma_\sigma\leq 1}} \frac{\varepsilon}{2}(S(p^m(c_A)) + S(p^m(c)))$$
$$-\delta\left(\frac{\varepsilon}{2}(1-\beta_A(c_A)+1-\beta_B(c_A)) + \frac{1-\varepsilon}{2}(1-\gamma_A(c_A)+1-\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A)).$$

Up to some constants, we thus may write the corresponding Lagrangean as:

$$\delta\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A)) + \frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A))$$
$$+\lambda\Bigg(\delta\Bigg(\frac{\pi^m(c_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A)) + \frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)$$
$$-\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(c_A)+\gamma_A(c_A))}{2}\Bigg) - K(c_A)\Bigg)$$

where λ is the non-negative multiplier of (A.46). This Lagrangean is linear in the probabilities $(\beta_\sigma(c_A), \gamma_\sigma(c_A))$. Whether the optimal such probabilities are zero, one or interior depends on the sign of the coefficients of each of this linear expression. We now express these coefficients as

- for $\beta_A(c_A)$

$$\text{(A.49)}\qquad \frac{\delta}{2}\left(\varepsilon\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right) - \lambda\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\right);$$

- for $\beta_B(c_A)$

$$\text{(A.50)}\qquad \frac{\delta}{2}\varepsilon\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right);$$

- for $\gamma_A(c_A)$

$$\text{(A.51)}\qquad \frac{\delta}{2}\left((1-\varepsilon)\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right) - \lambda\left(\frac{\varepsilon}{2}(\pi^m(c_A)+\pi^m(c))-\psi\right)\right);$$

- for $\gamma_B(c_A)$

$$\text{(A.52)}\qquad \frac{\delta}{2}(1-\varepsilon)\left(\Delta\mathcal{S}(c_A)+\lambda\frac{\pi^m(c_A)}{2}\right).$$

Several facts immediately follow.

1. Suppose that (A.46) is slack so that moral hazard is not an issue. Making $\lambda = 0$ in the expressions (A.49) to (A.52) shows that all coefficients are positive so that all probabilities would be set to one. Inserting those values into (A.46) leads to $\delta K(c_A) > K(c_A)$; a contradiction when $c_A = \underline{c}_A$ since then $K(\underline{c}_A) > 0$ and a condition that holds when $c_A = c$ since then $K(c) < 0$ from (A.42). We deduce from this that, necessarily, (A.46) is binding if $c_A = \underline{c}_A$ and thus $\lambda > 0$ in that case.

Instead, (A.46) is slack if $c_A = c$, thus $\lambda > 0$ in that case and the solution is obtained as in (7.3).

2. Turning now to the case $c_A = \underline{c}_A$, we first observe that the expressions (A.50) and (A.52) are necessarily positive which implies (7.4).

3. Comparing the expressions in (A.49) and (A.51) and noting that $\varepsilon > \frac{1}{2}$, we have

$$\frac{\delta}{2}\left(\varepsilon\left(\Delta\mathcal{S}(\underline{c}_A) + \lambda\frac{\pi^m(\underline{c}_A)}{2}\right) - \lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi\right)\right)$$
$$> \frac{\delta}{2}\left((1-\varepsilon)\left(\Delta\mathcal{S}(\underline{c}_A) + \lambda\frac{\pi^m(\underline{c}_A)}{2}\right) - \lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi\right)\right).$$

Two cases must thus *a priori* be studied.

(a) Case 1. $\lambda > 0$ is such that the coefficient in (A.49) is zero while that in (A.51) is negative. In that case, we should have

$$\gamma_A^1(\underline{c}_A) = 0 \le \beta_A^1(\underline{c}_A) \le 1$$

and

$$\lambda = \frac{\varepsilon\Delta\mathcal{S}(\underline{c}_A)}{\frac{\varepsilon}{2}\pi^m(c) - \psi} > 0$$

where the denominator is positive since $\frac{\varepsilon}{2}\pi^m(c) - \psi > -K(c) > 0$.

(b) Case 2. $\lambda > 0$ is such that the coefficient in (A.49) is positive while that in (A.51) is zero. In that case, we should have

$$0 \le \gamma_A^{mh}(\underline{c}_A) \le \beta_A^{mh}(\underline{c}_A) = 1$$

and

$$\lambda = \frac{(1-\varepsilon)\Delta\mathcal{S}(\underline{c}_A)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi} > 0$$

where the denominator is again positive since $\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi > \frac{\varepsilon}{2}\pi^m(c) - \psi > -K(c) > 0$.

The comparison of Case 1 and Case 2 is straightforward. The customer's expected payoff is always greater in Case 2 since

$$\varepsilon\beta_A^1(\underline{c}_A) \le (1-\varepsilon)\gamma_A^{mh}(\underline{c}_A) + \varepsilon.$$

Inserting the expressions of $\beta_B^{mh}(\underline{c}_A)$ $\gamma_B^{mh}(\underline{c}_A)$ and $\beta_A^{mh}(\underline{c}_A)$ obtained from (7.4), we obtain that $\gamma_A^{mh}(\underline{c}_A)$, when interior, must solve

$$\delta\left(\frac{\pi^m(\underline{c}_A)}{2}\left(\frac{1-\varepsilon}{2}(\gamma_A^{mh}(\underline{c}_A) - 1) + 1\right) - \left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A) + \pi^m(c)) - \psi\right)\frac{(1 + \gamma_A^{mh}(\underline{c}_A))}{2}\right) = K(\underline{c}_A)$$

which gives

$$\gamma_A^{mh}(\underline{c}_A) = 1 - 2\frac{(1-\delta)K(\underline{c}_A)}{\delta\left(\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi\right)} \in [0,1). \tag{A.53}$$

CHECKING THE OMITTED CONSTRAINT (A.44). This condition clearly holds in the case $c_A = \bar{c}_A$. For $c_A = \underline{c}_A$, we can rewrite the seller's value as in (7.10) and we must thus check that

$$\frac{\frac{1}{2}\pi^m(\underline{c}_A)}{1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A))} \geq \frac{\frac{1}{2}\pi^m(c)}{1-\delta}. \tag{A.54}$$

Inserting the expression for $\gamma_A^{mh}(c_A)$ found in (A.53), we find:

$$1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A)) = (1-\delta)\frac{\frac{1-\varepsilon}{2}\pi^m(\underline{c}_A)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi}.$$

Inserting into (A.54) amounts to checking that

$$\frac{2\varepsilon-1}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi > 0$$

which obviously holds since Assumption 2 is satisfied.

CONDITIONS FOR INDUCING INFORMATION GATHERING. The fact that the high-cost seller gathers information is obvious. Had the consumer decided not to induce information gathering from the low-cost seller, he would simply choose to continue the relationship whatsoever. This leads to the following condition for inducing information gathering

$$\frac{1}{2}S(p^m(\underline{c}_A))+\delta\mathcal{S}(\underline{c}_A) \leq \frac{\varepsilon}{2}(S(p^m(\underline{c}_A))+S(p^m(c)))+\delta\mathcal{S}(\underline{c}_A)-\delta\Delta\mathcal{S}(\underline{c}_A)(1-\gamma_A^{mh}(\underline{c}_A))$$

which gives

$$\frac{\varepsilon}{2}S(p^m(c))-\frac{1-\varepsilon}{2}S(p^m(\underline{c}_A)) \geq \frac{(1-\varepsilon)(1-\delta)K(\underline{c}_A)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi}\Delta\mathcal{S}(\underline{c}_A) \tag{A.55}$$

where the right-hand side is obtained after some simplifications using the definition of $\gamma_A^{mh}(\underline{c}_A)$ given in (A.53). Since Assumption 3 holds, the left-hand side of (A.55) is positive. This condition thus holds for δ close enough to 1. □

PROOF OF PROPOSITION 8. The proof follows similar steps to that of Proposition 7.

SIMPLIFYING THE SET OF INCENTIVE COMPATIBLE CONSTRAINTS. First, we observe that (7.1) and (A.45) are still true for type $\bar{c}_A$. The moral hazard constraint for that type writes again as

$$\delta\left(\frac{\pi^m(c)}{2}\left(\frac{\varepsilon}{2}(\beta_A(\bar{c}_A)+\beta_B(\bar{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\bar{c}_A)+\gamma_B(\bar{c}_A))\right)\right.$$
$$\left.-(\varepsilon\pi^m(c)-\psi)\frac{(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))}{2}\right) \geq K(\bar{c}_A). \tag{A.56}$$

Following again (7.1) and (A.45), the moral hazard constraint for type $\underline{c}_A$ who reports truthfully his cost parameter must also be written as

$$\delta\left(\frac{\pi^m(\underline{c}_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)\right.$$
$$\left.-\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(\underline{c}_A)+\gamma_A(\underline{c}_A))}{2}\right) \geq K(\underline{c}_A). \tag{A.57}$$

We then assume that (7.6) is a more stringent constraint than (7.7) and (7.8) and we check *ex post* this assertion once we have derived the optimal contract. Using also (7.1) to express the value $\mathcal{U}(\underline{c}_A)$ on the equilibrium path, incentive compatibility becomes

$$\mathcal{U}(\underline{c}_A)=\frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi}{1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)}\geq\frac{\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))}{1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))}.$$

Or, after manipulations,

$$\text{(A.58)}\quad\left(1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))\right)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)$$
$$\geq\left(1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)\right)\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right).$$

Optimal probabilities of continuing the relationship. We can thus write the customer's problem under asymmetric information as:

(A.59)

$$(1-\delta)\mathbb{E}_{c_A}(\mathcal{S}(c_A))=\max_{\substack{(\beta_\sigma(c_A),\gamma_\sigma(c_A))_{\sigma\in\Sigma}\\0\leq\beta_\sigma(_A)\leq1\\0\leq\gamma_\sigma(c_A)\leq1}}\mathbb{E}_{c_A}\left(\frac{\varepsilon}{2}(S(p^m(c_A))+S(p^m(c)))\right.$$
$$\left.-\delta\left(\frac{\varepsilon}{2}(1-\beta_A(c_A)+1-\beta_B(c_A))+\frac{1-\varepsilon}{2}(1-\gamma_A(c_A)+1-\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A))\right)$$

subject to (A.56), (A.57) and (A.58).

As a preliminary remark, it is worth noticing that (A.56) already holds even when the relationship is not continued under any circumstances. Since the customer's payoff diminishes when the relationship is terminated, this means that (A.56) is always slack at the optimum. We thus denote by λ and μ the non-negative multipliers of the remaining constraints (A.57) and (A.58) respectively and we form the corresponding Lagrangean as

$$\mathbb{E}_{c_A}\left(\delta\left(\frac{\varepsilon}{2}(\beta_A(c_A)+\beta_B(c_A))+\frac{1-\varepsilon}{2}(\gamma_A(c_A)+\gamma_B(c_A))\right)\Delta\mathcal{S}(c_A))\right)$$
$$+\lambda\Bigg(\delta\Bigg(\frac{\pi^m(\underline{c}_A)}{2}\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)$$
$$-\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\frac{(\beta_A(\underline{c}_A)+\gamma_A(\underline{c}_A))}{2}\Bigg)-K(\underline{c}_A)\Bigg)$$
$$+\mu\Bigg(\left(1-\frac{\delta}{2}(\beta_A(\bar{c}_A)+\gamma_A(\bar{c}_A))\right)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)$$
$$-\left(1-\delta\left(\frac{\varepsilon}{2}(\beta_A(\underline{c}_A)+\beta_B(\underline{c}_A))+\frac{1-\varepsilon}{2}(\gamma_A(\underline{c}_A)+\gamma_B(\underline{c}_A))\right)\right)\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\Bigg).$$

This Lagrangean is again linear in the probabilities of continuing the relationship, we obtain the following expressions of the various coefficients:

- for $\beta_A(\underline{c}_A)$

$$\text{(A.60)}\quad \frac{\delta}{2}\left(\varepsilon\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right)\right.$$
$$\left.-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right);$$

- for $\beta_B(\underline{c}_A)$

$$\text{(A.61)}\quad \frac{\delta}{2}\varepsilon\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right);$$

- for $\gamma_A(\underline{c}_A)$

$$\text{(A.62)}\quad \frac{\delta}{2}\left((1-\varepsilon)\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}\right)+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right.$$
$$\left.-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right);$$

- for $\gamma_B(\underline{c}_A)$

$$\text{(A.63)}\quad \frac{\delta}{2}(1-\varepsilon)\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)\right);$$

- for $\beta_A(\bar{c}_A)$

$$\text{(A.64)}\quad \frac{\delta}{2}\left(\varepsilon(1-\nu)\Delta\mathcal{S}(\bar{c}_A)-\mu\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right);$$

- for $\beta_B(\bar{c}_A)$

$$\text{(A.65)}\quad \frac{\delta}{2}\varepsilon(1-\nu)\Delta\mathcal{S}(\bar{c}_A);$$

- for $\gamma_A(\bar{c}_A)$

$$\text{(A.66)}\quad \frac{\delta}{2}\left((1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\bar{c}_A)-\mu\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\right);$$

- for $\gamma_B(\bar{c}_A)$

$$\text{(A.67)}\quad \frac{\delta}{2}(1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\bar{c}_A).$$

Several facts immediately follow.

1. The coefficients in (A.61), (A.63), (A.65) and (A.67) are all positive. Thus, we have necessarily (7.11).
2. Suppose that (A.57) is slack so that moral hazard is not an issue for type $\underline{c}_A$. Making $\lambda = 0$ in the expressions (A.60) to (A.63) shows that all coefficients are positive so that all probabilities would be set to one. Inserting those values into (A.57) leads to $\delta K(\underline{c}_A) > K(\underline{c}_A)$; a contradiction. We deduce from this that (A.57) is necessarily binding and thus $\lambda > 0$.

3. Comparing the expressions in (A.60) and (A.62) and noting that $\varepsilon > \frac{1}{2}$, we have:

$$\frac{\delta}{2}\Bigg(\varepsilon\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}\right)+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)$$
$$-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\Bigg)$$
$$>\frac{\delta}{2}\Bigg((1-\varepsilon)\left(\nu\Delta\mathcal{S}(\underline{c}_A)+\lambda\frac{\pi^m(\underline{c}_A)}{2}\right)+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)$$
$$-\lambda\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\Bigg).$$

From there, we could proceed as in the proof of Proposition 7, and recognize that the customer's expected surplus is maximized when the coefficient in (A.64) is positive while that in (A.66) is zero. It follows that the optimal probabilities of continuing satisfy

$$0\le\gamma_A^{sb}(\underline{c}_A)\le\beta_A^{sb}(\underline{c}_A)=1$$

i.e., (7.12) while the multiplier of (A.57) is

$$\lambda=\frac{(1-\varepsilon)\Delta(1-\nu)\mathcal{S}(\underline{c}_A)+\mu\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)}{\frac{2\varepsilon-1}{2}\pi^m(\underline{c}_A)+\frac{\varepsilon}{2}\pi^m(c)-\psi}>0.$$

4. Suppose that $\mu=0$ at the solution. Then, it should be that

$$\gamma_A^{sb}(\bar{c}_A)\le\beta_A^{sb}(\bar{c}_A)=1\quad\text{and}\quad\gamma_A^{sb}(\underline{c}_A)=\gamma_A^{mh}(\underline{c}_A).$$

Inserting those values into (A.58), this constraint would be slack when

$$(1-\delta)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi\right)\ge\left(1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))\right)\left(\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right).$$

But this condition is violated by the first condition that follows from Assumption 4 which ensures that in fact $\mu>0$.

Two cases must thus *a priori* be studied.

(a) Case 1. $\mu>0$ is such that the coefficient in (A.64) is zero while that in (A.66) is negative. In that case, we must have

$$\gamma_A^1(\bar{c}_A)=0\le\beta_A^1(\bar{c}_A)\le 1$$

and

$$\mu=\frac{\varepsilon(1-\nu)\Delta\mathcal{S}(\bar{c}_A)}{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c))-\psi}.$$

(b) Case 2. $\mu>0$ is such that the coefficient in (A.64) is positive while that in (A.66) is zero. In that case, we should have

$$0 \le \gamma_A^{sb}(\bar{c}_A) \le \beta_A^{sb}(\bar{c}_A) = 1,$$

i.e., (7.13) and

$$\mu = \frac{(1-\varepsilon)(1-\nu)\Delta\mathcal{S}(\bar{c}_A)}{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c)) - \psi}.$$

Because $\varepsilon > 1/2$, the coefficient in (A.64) is always greater than that in (A.66). The comparison of CASE 1 and CASE 2 is straightforward. The customer's expected payoff is always greater in CASE 2 since

$$\varepsilon\beta_A^1(\bar{c}_A) \le (1-\varepsilon)\gamma_A^{sb}(\bar{c}_A) + \varepsilon.$$

5. Inserting the expressions of $\beta_B^{sb}(\bar{c}_A)$ $\gamma_B^{sb}(\underline{c}_A)$ and $\beta_A^{sb}(\underline{c}_A)$ obtained respectively from (7.11) and (7.13), we obtain that $\gamma_A^{as}(\bar{c}_A)$, when interior, must solve

$$\text{(A.68)} \quad \left(1 - \frac{\delta}{2}(1+\gamma_A^{sb}(\bar{c}_A))\right)\left(\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c)) - \psi\right) = \left(1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))\right)\left(\frac{1}{2}\pi^m(c) + \frac{\Delta c}{2}D(p^m(c))\right).$$

The existence of such $\gamma_A^{sb}(\bar{c}_A) < 1$ then follows from Assumption 4. Hence, the second item in (7.12) follows.

CHECKING THE OMITTED CONSTRAINTS (7.7) AND (7.8). We need to check that

$$\mathcal{U}^{as}(\underline{c}_A) = \frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c)) - \psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))} = \frac{\frac{1}{2}\pi^m(\underline{c}_A)}{1-\frac{\delta}{2}(1+\gamma_A^{mh}(\underline{c}_A))}$$
$$\ge \max\left\{\frac{\frac{1}{2}\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))}{1-\frac{\delta}{2}(1+\gamma_A^{sb}(\bar{c}_A))}; \frac{\frac{1}{2}\pi^m(c)}{1-\delta}; \frac{\varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{sb}(\bar{c}_A))}\right\}.$$

By (A.68) the left-hand side is equal to the first term in the maximand on the right-hand side. The second and third terms in this maximand are dominated by the first one when

$$\text{(A.69)} \quad \frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c)) - \psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))} \ge \max\left\{\frac{\frac{1}{2}\pi^m(c)}{1-\delta}; \frac{\varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right)-\psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{sb}(\bar{c}_A))}\right\}$$

When δ goes to 1, (A.53) shows that $\gamma_A^{mh}(\underline{c}_A)$ goes also to 1 from below. Then, the first inequality in (A.69) follows from the fact that, in the limit, we have

$$\frac{\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c)) - \psi}{1-\delta+\delta\frac{1-\varepsilon}{2}(1-\gamma_A^{mh}(\underline{c}_A))} > \frac{\frac{1}{2}\pi^m(c)}{1-\delta}$$

which itself follows from

$$\frac{\varepsilon}{2}\pi^m(\underline{c}_A) + \frac{\varepsilon}{2}\pi^m(c) - \psi \ge \frac{1}{2}\pi^m(c),$$

which is implied by Assumption 2.

When δ goes to 1, (A.68) shows that $\gamma_A^{sb}(\bar{c}_A)$ goes also to 1 from below. Then, the second inequality in (A.69) follows from when, again in the limit, we have

$$\frac{\varepsilon}{2}(\pi^m(\underline{c}_A)+\pi^m(c)) - \psi \ge \varepsilon\left(\pi^m(c)+\frac{\Delta c}{2}D(p^m(c))\right) - \psi$$

which itself follows from

$$\pi^m(\underline{c}_A) > \pi^m(c) + \Delta c D(p^m(c)).$$

Those arguments show that, when δ is close enough to 1, the omitted constraints (7.7) and (7.8) are satisfied.

Conditions for inducing information gathering. When δ is close enough to 1, inducing partial information gathering is costly because the customer incurs the cost of switching, while continuing the relationship with the optimal probabilities $\gamma_A^{sb}(\bar{c}_A)$ and $\gamma_A^{mh}(\underline{c}_A)$ found above to incentivize the seller approximates the full information outcomes since these probabilities are very close to 1. Hence, Assumption 3 again ensures that inducing information gathering by both types is optimal.

$\square$