The paradox of pledgeability

Jason Roderick Donaldson\textsuperscript{a,b}, Denis Gromb\textsuperscript{c,*}, Giorgia Piacentino\textsuperscript{b,d}

\textsuperscript{a}Olin Business School, Washington University in St. Louis, 1 Brookings Dr, St. Louis, MO 63130, USA
\textsuperscript{b}Center for Economic Policy and Research, UK
\textsuperscript{c}HEC Paris, 1 Rue de la Libération, 78351 Jouy-en-Josas, France
\textsuperscript{d}Columbia Business School, Columbia University, 3022 Broadway, New York, NY 10027, USA

February 9, 2019

Abstract

We develop a model in which collateral serves to protect creditors from the claims of other

\textsuperscript{*}Corresponding author. E-mail address: gromb@hec.edu (D. Gromb).
creditors. We find that, paradoxically, borrowers rely most on collateral when pledgeability is high. This is when taking on new debt is easy, which dilutes existing creditors. Creditors thus require collateral for protection against possible dilution by collateralized debt. There is a collateral rat race. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment. There is a collateral overhang. Our results suggest that policies aimed at increasing the supply of collateral can backfire, triggering an inefficient collateral rat race. Likewise, upholding the absolute priority of secured debt can exacerbate the rat race.

*JEL classification:* G21, G32, G33, G38

*Keywords:* Collateral, Secured debt, Absolute priority rule, Pledgeability
1. Introduction

Collateral matters. In much of the finance literature, collateral matters because it mitigates enforcement frictions between borrowers and creditors; i.e., “collateral pledging makes up for a lack of pledgeable cash” (Tirole, 2006 p. 169). Collateral also plays another role, emphasized in the law literature. Collateral matters because it mitigates enforcement frictions among creditors, i.e., “a secured transaction [is] the protection ... against the claims of competing creditors” (Kronman and Jackson, 1979, p. 1143). These two roles of collateral correspond to the two components of property rights that accrue to secured creditors upon default: the right of access (a creditor’s right to seize collateral) and the right of exclusion (a creditor’s right to stop other creditors from seizing collateral) (e.g., Hart 1995, Segal and Whinston, 2012).

In this paper, we present a finance model based (solely) on the latter role. We find that borrowers rely on collateral when pledgeability is high, not low. That is, collateral does not make up for a lack of pledgeable cash. High pledgeability makes taking on new debt easier, which dilutes existing creditors. This leads existing creditors to require collateral for protection against possible dilution by collateralized debt. There is a collateral rat race. But, collateralized borrowing has a cost. It encumbers assets, constraining future borrowing and investment. There is a collateral overhang. Further, greater availability of collateral can have adverse effects, triggering an inefficient collateral rat race. Hence, policies aiming to increase the supply of collateral, such as expanding the set of assets that can be used as collateral, can backfire. Likewise, upholding the absolute priority of secured debt could facilitate dilution instead of protecting against it, triggering the rat race.

In our model, a borrower, B, has two riskless projects, Project 0 and Project 1, to finance sequentially. B finances Project 0 by borrowing from one creditor, $C_0$. After Project 0 is

---


under way, B can finance Project 1 by borrowing from another creditor, C₁. Project 0’s net present value (NPV) is positive, but Project 1’s NPV, revealed after Project 0 is under way, can be positive or negative.

We assume that B’s borrowing capacity is constrained by two frictions. First, pledgeability is limited. The total repayment from B to his creditors is limited to a fraction \( \theta \) of his projects’ final value, representing, for example, the liquidation value of the assets employed in the projects. Second, contracts are non exclusive in that when B takes on debt to C₀, he cannot commit not to take on new debt to C₁. \(^3\) We focus on the case in which B is close enough to default that this new debt can significantly dilute existing debt. Our model thus applies best to a borrower in relatively poor health or even distress.

We assume that collateral mitigates the nonexclusivity friction (but, for now, that it does not affect pledgeability). By borrowing secured, B ring fences his project(s) as collateral. Collateral is protected from the claims of other creditors. If B borrows via secured (i.e., collateralized) debt, the secured creditor has an exclusive claim over the collateral securing the debt. And we assume that this collateralization cannot be state-contingent. A secured creditor always has an exclusive claim (it can never be diluted), whereas an unsecured creditor always has a non-exclusive claim (it can always be diluted). In that respect, collateral is only a coarse solution to the nonexclusivity problem.

We derive four main results. Our first (Proposition 1) is that, paradoxically, if pledgeability \( \theta \) is sufficiently high, B perhaps is not able to borrow from C₀ unsecured. To see why, suppose B finances Project 0 by borrowing from C₀ via unsecured debt. Because unsecured contracts are nonexclusive, B can approach C₁ to finance Project 1. If B collateralizes both projects to borrow from C₁, then C₁ has an exclusive claim to them and, hence, priority over C₀, i.e., the new secured debt dilutes the existing unsecured debt. As a result, C₀ may not lend unsecured in the first place. However, this dilution occurs only if C₁ is willing to finance Project 1, \(^3\) This assumption rules out covenants by which a borrower commits not to borrow from new creditors in the future. In practice, such covenants have limited power to prevent secured borrowing (see Section 5).
i.e., only if the value of B’s pledgeable payoff exceeds his funding needs. In summary, high pledgeability relaxes B’s borrowing constraint with C₁ but tightens his borrowing constraint with C₀. Contrary to common intuition, high pledgeability undermines unsecured credit.

Our second main result (Proposition 2) is that, anticipating dilution with new secured debt to C₁, the initial creditor C₀ will only lend secured. There is a collateral rat race, by which collateralization is required as protection against future collateralization. In some circumstances, a mix of collateralized and uncollateralized debt is optimal. Hence, our model also casts light on the coexistence of these two types of debt in borrowers’ capital structure.

Our third main result (Proposition 3) is that if B borrows secured from C₀, he could be unable to fund Project 1, even when it has positive NPV. This is because collateralizing Project 0 uses up collateral, making it difficult for B to borrow to finance Project 1. In other words, collateralization encumbers B’s assets; i.e., it limits B’s ability to use them to obtain funding. This collateral overhang problem resonates with practitioners’ intuition that “encumbered assets are generally not available to obtain ... liquidity” (Deloitte 2014).

Next, we enrich our model by supposing that only a fraction of a project can be collateralized. Some assets can be pledgeable, i.e., they can be seized in the future, but not collateralizable, i.e., they are hard to assign property rights to. For example, they perhaps do not even exist at inception but are built or acquired in the course of the project. Our fourth main result (Proposition 4) is that, while higher collateralizability can relax borrowing constraints, it can also tighten them. This collateral damage arises because increasing collateralizability makes taking on new debt from C₁ easier, diluting C₀, which can trigger a collateral rat race.

We also explore three extensions. (1) We consider B’s choice between assets of different collateralizability and show that increasing collateralizability can distort it toward less productive, but more collateralizable, assets. (2) We allow collateral to mitigate enforcement problems between borrowers and creditors, increasing pledgeability, as in most of the finance literature, as well as establishing exclusivity, as in our baseline model. We show that the classical role dominates for low pledgeability, when borrowers need collateral to get projects off the ground,
but the new role we focus on dominates for high pledgeability, when creditors need collateral for protection against dilution. (3) We show that if B’s projects are so valuable that he is unconstrained, collateral plays no role.

Our analysis can speak to current financial policies. Some policies have aimed to increase the supply of collateral, deemed insufficient. For example, several countries expanded the set of movable or floating assets qualifying as collateral. Some central banks also committed to lend against illiquid financial securities at set rates and haircuts. Our results suggest such policies could backfire, because increasing the supply of collateral can increase the need for collateral, by triggering a rat race. Borrowers could switch from unsecured to secured borrowing, thereby encumbering collateral assets. Borrowers could also distort their asset base toward collateralizable assets. Increasing the supply of collateral could increase the demand for collateral instead of satisfying it.

The absolute priority rule describes the order of payment in bankruptcy, i.e., secured creditors get paid first and unsecured creditors get paid next. Our analysis suggests that upholding the absolute priority of secured debt as such can lead to inefficient investment. This lends support to arguments in the law literature against this absolute priority rule (see Bjerre, 1999; Lubben, 2016). Lawyers argue that “[c]urrent law forces onto borrowers the power to defeat unsecured lenders by issuing secured debt” (Bjerre, 1999, p. 308).

We model a new role of collateral: to establish exclusivity. This is likely its main role when nonexclusivity is the first-order friction. For example, nonexclusivity is especially relevant for financial firms that can have numerous creditors. So is collateral. In fact, upward of $5 trillion of securities are pledged as collateral in interbank markets. Pledgeability is ostensibly high in these markets, due to strong creditor rights, effective legal enforcement, intense regulatory supervision, and developed record-keeping technologies. Hence, the reliance on collateral is not easily explained by the classical theory; i.e., collateral matters even when it is not

4 Relatedly, Bolton and Oehmke (2015) argue that the priority for derivatives, such as safe harbors for derivatives collateral, can increase a borrower’s total cost of funding and decrease overall efficiency.
necessary to make up for a lack of pledgeable cash.

Generally, our model applies to borrowers for which dilution of existing debt is a first-order concern, namely, those relatively likely to default, due to poor financial health, or even distress (cf. Section 4.10). For example, in practice, firms in distress can collateralize assets to meet their operating costs, thereby using dilution to gamble for resurrection. Likewise, households collateralize assets to get emergency liquidity, e.g., by remorgaging. Borrowers can also de facto collateralize assets via trade credit or leasing.

Our focus on how collateral establishes exclusivity contrasts with the finance literature. Typically, two assets exist whose different intrinsic properties determine which one serves as collateral. For example, a pledgeable asset (e.g., physical capital) could be used as collateral to finance a divertable asset (e.g., human capital). In contrast, our model has a single asset (cash), and it is the borrower’s choice of debt instrument that determines whether it serves as collateral. This can cast light on cases in which whether an asset is used as collateral seems not to depend on its intrinsic properties. For example, investors use securities as collateral to borrow cash in the repo market and use cash as collateral to borrow securities in the securities lending market. They do not use a pledgeable asset as collateral to borrow a divertable asset. We suggest, they do so to establish exclusivity.

Our distinction between pledgeability and collateralizability offers a new perspective on which assets can be good collateral, i.e., those to which property rights can be assigned ex ante, which need not coincide with those that can be seized ex post. For example, the collateralizable part of projects would include fixed assets (e.g., real estate), but not necessarily movable assets (e.g., inventories). The pledgeable part of projects could represent the tangible assets deployed, not all of which need be collateralizable. This distinction also has an interpretation in terms of legal and technological development. Strengthening creditor rights or improving collection and monitoring technologies should make assets more pledgeable, but not necessarily more collateralizable. For this, strengthening property rights or improving collateral registration technologies is required.
To our knowledge, our model is the first to focus on the role collateral can play in mitigating nonexclusivity, arguably its role legally. Ayotte and Bolton (2011) also adopt a legal definition of a secured creditor’s property rights, but they focus on when these rights should be enforced given that other creditors can be unaware of them. Bolton and Oehmke (2015) also analyze the priority of some claims over others, but they focus on when derivatives should be senior to other claims.\footnote{Some finance papers study nonexclusive contracts but not collateral, e.g., Acharya and Bisin (2014), Admati, DeMarzo, Hellwig, and Pfleiderer (2018), Attar, Casamatta, Chassagnon, and Décamps (2017, forthcoming), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Bizer and DeMarzo (1992), DeMarzo and He (2018), Kahn and Mookherjee (1998), Leitner (2012), and Parlour and Rajan (2001).}

Some of our results have connections to finance papers that do not study collateral. Our paradox of pledgeability (higher pledgeability undermines a borrower’s ability to commit to future borrowing decisions) is a liabilities-side analog of Myers and Rajan’s (1998) asset-side paradox of liquidity (higher asset liquidity undermines a borrower’s ability to commit to future investment decisions by making liquidating assets for expropriation more attractive).\footnote{Donaldson and Micheletti (2018) argue that higher pledgeability can also paradoxically foster systemic risk, by leading borrowers to favor non-resaleable over resaleable debt instruments (e.g., repos over bonds).} In our model, the borrower always wants to dilute but cannot when pledgeability is low because creditors will not lend. In Myers and Rajan (1998), the borrower does not always want to liquidate. In our collateral rat race result, collateral plays a similar role to short maturity in the Brunnermeier and Oehmke (2013) maturity rat race.

Our finding that partial collateralization can achieve the efficient state-contingent outcome with dilution in some states but not in others relates to theories in which some device (cash, security, covenants, priority) makes debt and investment more state contingent. Notably, in Hart and Moore (1995), senior long-term debt prevents an empire-building manager from making bad investments but prevents him from pursuing good one, too. One key difference with our paper comes from who designs the claims. In their model, investors design claims to induce the manager to undertake only the projects that benefit them. In our model, the borrower designs claims to preserve sufficient flexibility to undertake efficient projects that do
not benefit investors. Another is that Hart and Moore’s mechanism is not renegotiation proof (ours is). In Holmström and Tirole (1998), a firm can implement state-contingent liquidity insurance if it can take on new senior debt in some states. Unlike in our model, existing creditors welcome such dilution, as it rescues the assets backing their claims. In Acharya, Almeida, and Campello (2007), a borrower can hold cash to access liquidity when he would otherwise be constrained. Borrowing to hold cash transfers liquidity across states. In their model, holding cash is akin to preserving the option to dilute in ours. Somewhat similarly to how using cash to pay down debt can be inefficient in their model, over-collateralization can be inefficient in ours, giving too much protection against dilution and inducing the collateral overhang.

The collateral overhang problem bears some similarity to the Myers (1977) debt overhang problem, as debt in place prevents a borrower from funding positive NPV projects. In the debt overhang problem, a borrower will not raise capital because this would subsidize existing debt. In the collateral overhang problem, he cannot raise capital because existing debt is collateralized to prevent this. Further, the collateral overhang problem arises even when the debt-overhang problem does not, i.e., when existing debt is riskless or can be renegotiated.

The paper proceeds as follows. Section 2 presents a numerical example. Section 3 presents the model. Section 4 contains the main results. Section 5 discusses the contracting environment. Section 6 includes empirical implications. Section 7 concludes. The Appendix contains all proofs.

2. Numerical example

A borrower B has two projects, Project 0 and Project 1. Project 0 has positive NPV. It costs 200 at Date 0 and pays off 600 at Date 2. Project 1 has negative NPV. It costs 500 at Date 1 and pays off 400 at Date 2. B is penniless and finances Project 0 from one creditor, C₀, and Project 1 from another creditor, C₁, via secured or unsecured debt, with secured
debt paid first. Last, pledgeability is limited. B’s total repayment is at most a fraction $\theta$ of
his total cash flow $600 + 400$.

First, we show that with pledgeability, $\theta = 2/5$, B does the efficient thing. He invests
in Project 0, but not in Project 1. To see why, say B borrows unsecured from $C_0$. Now, at
Date 1, B wants to do Project 1 despite its negative NPV. By borrowing secured from $C_1$,
he dilutes $C_0$. This transfers the cost of the investment to $C_0$, so B can capture (roughly)
Project 1’s PV, not its NPV. Because secured debt is paid first, $C_1$ is willing to lend, even to
finance the negative NPV project, provided the total pledgeable cash flow from both projects
covers the cost of Project 1. But, with $\theta = 2/5$, this is not satisfied:

$$\frac{2}{5} \times (600 + 400) = 400 < 500. \quad (1)$$

B is too constrained to fund Project 1 at Date 1, even with secured debt. Hence, he does
not dilute $C_0$. Paradoxically, due to low pledgeability, he can fund Project 0 at Date 0, even
with unsecured debt.

Now, we show that with higher pledgeability, $\theta = 1/2$, B can no longer borrow unsecured
from $C_0$. To see why, observe that now the total pledgeable cash flow from both projects
can cover the cost of Project 1:

$$\frac{1}{2} \times (600 + 400) = 500. \quad (2)$$

Here, because secured debt gets paid first, $C_1$ is willing to lend. $C_0$ gets paid second and,
hence, is diluted. In fact, it gets paid nothing. B’s total pledgeable cash flow minus his
repayment to $C_1$ is zero:

$$\frac{1}{2} \times (600 + 400) - 500 = 0. \quad (3)$$

Thus, $C_0$ requires its debt to be secured as protection against dilution with secured debt to
$C_1$. There is a collateral rat race.
Now suppose B finances Project 0 secured. The debt is riskless and, hence, has face value 200. B cannot do Project 1 and, hence, does only positive NPV projects. But what if Project 1 were unexpectedly good, with payoff 550? Could B finance it? Because C₀, the secured creditor, is paid first, C₁ is willing to finance Project 1 as long as the pledgeable cash flow net the repayment to C₁ covers its cost. But, with \( \theta = \frac{1}{2} \), this is not satisfied:

\[
\frac{1}{2} \times (600 + 550) - 200 = 375 < 500. \tag{4}
\]

By collateralizing its debt to C₀, B has encumbered its assets and cannot pledge enough to C₁ to finance a positive NPV project. There is a collateral overhang.

3. Model

In this section, we set up the model.

3.1. Players and projects

There is one good, called cash, that is the input of production, the output of production, and the consumption good. A borrower B lives for three dates, \( t \in \{0, 1, 2\} \), and consumes at Date 2. B has no cash but has access to two investment projects, Project 0 at Date 0 and Project 1 at Date 1. Both projects are riskless and pay off at Date 2, but the payoff of Project 1 is revealed only at Date 1. Project 0 costs \( I_0 \) at Date 0 and pays off \( X_0 \) at Date 2. At Date 1, there are two states, \( s \in \{L, H\} \), with \( p := P[s = H] \). In state \( s \), Project 1 costs \( I_1^s \) at Date 1 and pays off \( X_1^s \) at Date 2. Everyone is risk-neutral, and there is no discounting or asymmetric information.

B can fund his projects by borrowing \( I_0 \) at Date 0 and \( I_1^s \) in state \( s \) at Date 1 from competitive credit markets. We assume that B makes a take-it-or-leave-it offer to borrow from creditor \( C_t \) at Date \( t \in \{0, 1\} \).
3.2. Pledgeability and collateralizability

B promises to repay his creditor(s) under two frictions. First, pledgeability is limited in that B can divert a fraction \((1 - \theta)\) of the projects’ payoffs, leaving only the pledgeable fraction \(\theta\) for his creditors. \(\theta\) is the fraction of a project’s final value that creditors can seize. For example, \(\theta X\) could represent the value of the assets used in a project and \((1 - \theta)X\) could represent its terminal cash flow.

Second, contracts are nonexclusive in that if B borrows from \(C_0\) at Date 0, he cannot commit not to borrow from \(C_1\) at Date 1, potentially diluting \(C_0\)’s initial claim. (This rules out covenants that prevent future borrowing. See Section 5.)

The role of collateral in our model is to mitigate the second friction. If B chooses to collateralize (or secure) a fraction \(\sigma\) of a project with payoff \(X\), a creditor gets the exclusive right to that fraction of the project’s pledgeable payoff, i.e., absolute priority over \(\sigma \theta X\) (see also Kiyotaki and Moore 2000, 2001). However, we assume that \(\sigma\) does not depend on the state. In that sense, collateral is only a coarse solution to the nonexclusivity problem (see Section 5).

3.3. Borrowing instruments

At Date \(t\), B borrows the cost of Project \(t\) from \(C_t\) against the promise to repay the fixed amount \(F_t\) at Date 2. This promise can be secured, i.e. collateralized, or unsecured. If B chooses to collateralize a fraction \(\sigma_0\) of Project 0 to \(C_0\), then \(C_0\) has priority over \(\sigma_0 \theta X_0\). This fraction of Project 0 cannot be collateralized again to \(C_1\). However, anything that B has not collateralized to \(C_0\) can be collateralized to \(C_1\). Thus, B can choose to give \(C_1\) a senior claim on (at most) the fraction \((1 - \sigma_0)\) of Project 0 and all of Project 1.

If there are multiple unsecured creditors, we assume that they are on equal footing if B defaults at Date 2. This is consistent with their pari passu legal treatment. We capture this by having unsecured creditors 50–50 Nash bargain at Date 2 over the residual value after the secured debt is paid (see Section 5).
Our results are not sensitive to the fine details of the contracting environment. None of them depends on whether Date 2 repayments can be state-contingent, and only the collateral damage results depend on the priority rule among unsecured creditors (cf. Section 4.7 and Section 4.8). Instead, our main results rely only on the assumptions that B cannot commit not to collateralize in the future, secured debt is treated as senior, and the fraction $\sigma_0$ of Project 0 B collateralizes cannot depend on the Date 1 state. These assumptions reflect real-world constraints that current law imposes on borrowers (see Section 5).

3.4. Payoffs

We now give the players’ terminal payoffs. Define the indicator variable $\mathbb{1}_t$ as

$$\mathbb{1}_t := \begin{cases} 1 & \text{if Project } t \text{ is undertaken}, \\ 0 & \text{otherwise}. \end{cases} \quad (5)$$

Thus, the total payoff from all projects undertaken is given by

$$X := \mathbb{1}_0 X_0 + \mathbb{1}_1 X_1. \quad (6)$$

B’s payoff is the sum of two terms: (1) the non-pledgeable part of the payoff from the project(s) and (2) the residual of the pledgeable part of the payoff after making repayments, i.e., B’s payoff is $(1 - \theta)X + \max \{\theta X - F_0 - F_1, 0\}$. If B does not default, i.e., $F_0 + F_1 \leq \theta X$, then creditor $C_t$ gets $F_t$. If B defaults, i.e., $F_0 + F_1 > \theta X$, then $C_0$ and $C_1$ divide $\theta X$ according to priority.

3.5. Assumptions

We impose several restrictions on parameters.

**Assumption 1.** Project 0’s pledgeable payoff in state $L$ alone is worth more than its
investment cost: \( I_0 < (1 - p)\theta X_0 \).

Thus, \( C_0 \) is willing to lend if it anticipates no dilution in state \( L \). This also implies that it is efficient to undertake Project 0, i.e. that \( I_0 < X_0 \).

**Assumption 2.** Project 1’s NPV is positive in state \( H \) but negative in state \( L \): \( I_1^H < X_1^H \) but \( I_1^L > X_1^L \).

This implies that it is efficient to undertake Project 1 in state \( H \) only.

**Assumption 3.** In both states, \( s \in \{L, H\} \), the combined pledgeable cash flow from Project 0 and Project 1 is less than the investment cost: \( \theta(X_0 + X_1^s) < I_0 + I_1^s \) for \( s \in \{L, H\} \).

This implies that the limited pledgeability friction is severe enough that it can prevent \( B \) from investing even when doing so would be efficient (i.e., in state \( H \)).

**Assumption 4.** Project 1’s non-pledgeable payoff is not too small: \( (1 - \theta)X_1^L > \theta X_0 - I_0 \).

This more technical condition ensures that the payoff of Project 1 is always large enough that \( B \) has the incentive to undertake it (Lemma 1).

**Assumption 5.** Project 1’s cost is not too large: \( I_1^H < \theta(X_0 + X_1^H) \).

This technical condition ensures that in state \( H \) the cost of Project 1 is not so large that it simply cannot be financed.

These assumptions serve to streamline the analysis. They ensure that nonexclusivity, the basic friction in our model, bites: \( B \) must dilute \( C_0 \) to invest in state \( H \) (Assumption 3) but also has incentive to dilute \( C_0 \) to invest in state \( L \) (Assumption 4). This is likely to be the case if \( B \) represents a weak borrower, who is financially constrained or even distressed (see Section 4.10).

4. **Results**

In this section, we analyze the model’s subgame perfect equilibrium outcomes.
4.1. Preliminaries

The following two lemmas are useful. They follow quickly from the assumptions.

**Lemma 1.** *B always borrows if feasible.*

Notably, this result implies that as long as \( C_1 \) is willing to lend at Date 1, \( B \) borrows to invest in Project 1, even in state \( L \). Borrowing from \( C_1 \) can dilute \( C_0 \)'s debt, subsidizing \( B \)'s investment, and Assumption 4 implies that subsidy is large enough that \( B \) always wants to do so.

**Lemma 2.** *If \( B \) can commit not to borrow from \( C_1 \) in state \( L \) at Date 1, \( B \) can borrow from \( C_0 \) at Date 0.*

This result follows from Assumption 1 that Project 0's pledgeable payoff in state \( L \) alone is more than \( I_0 \).

4.2. First best

In the first-best outcome, all positive NPV projects are undertaken.

**Lemma 3.** *The first-best outcome is to undertake Project 0 at Date 0 and Project 1 at Date 1 if and only if \( s = H \).*

4.3. Paradox of pledgeability

We find that \( B \) can (and does) borrow unsecured at Date 0 only if pledgeability is low.

**Proposition 1 (Paradox of pledgeability).** Define

\[
\theta^* := \frac{I_1^L}{X_0 + X_1^L}.
\]

If \( \theta < \theta^* \), \( C_0 \) lends unsecured and the first best is attained; i.e., \( B \) borrows (secured) from \( C_1 \) in state \( H \) and does not borrow in state \( L \).
If $\theta \geq \theta^*$, $C_0$ does not lend unsecured.

The intuition is as follows. If $C_0$ lends unsecured, $B$ can make $C_1$ senior by borrowing secured and promising him (up to) the whole pledgeable cash flow $\theta(X_0 + X_1^*)$. But $C_1$ is willing to lend only if $\theta(X_0 + X_1^*)$ exceeds the cost $I_1^*$ of Project 1. This condition always holds in state $H$ (Assumption 5) but holds in state $L$ only for $\theta \geq \theta^*$. Otherwise, for $\theta < \theta^*$, $B$ cannot get Project 1 off the ground in state $L$. He does not have enough pledgeable cash flow fund it, no matter how much he dilutes his existing debt. Because $C_0$ is not concerned with dilution in state $L$, it is willing to lend unsecured at Date 0 even though it will be diluted in state $H$ (Lemma 2). This makes it easier for $B$ to borrow from $C_1$ in state $H$ and invest efficiently.

Conversely, higher pledgeability allows $B$ to pledge more to $C_1$, making $C_1$ more willing to lend. Although $B$ will be unable to repay both creditors in full (Assumption 3), $C_1$ is willing to lend secured, because this new debt is senior to $B$’s existing debt to $C_0$. However, anticipating dilution, $C_0$ will not lend in the first place. Higher pledgeability makes borrowing at Date 1 easier and, hence, paradoxically, borrowing at Date 0 harder.

Our result does not rely on the assumption that the repayment $F_t$ is not contingent on the state $s$, as contingent contracts do not help $B$ commit not to dilute $C_0$ in state $L$. We spell this out in the Appendix after the proof of the proposition.

Very low pledgeability would prevent borrowing at Date 0. This is ruled out by Assumption 4 (see, however, Section 4.9). And, further, very high pledgeability can restore efficiency. This is ruled out by Assumption 3.

---

7 Optimal dilutable debt also appears in [Diamond (1993), Donaldson and Piacentino (2018), Hart and Moore (1995), and Stulz and Johnson (1985)].

8 This intuition depends on the fixed scale of projects. Our insight that $B$ needs collateral even when pledgeability is high still obtains with scaleable projects. In that case, $B$ relies on collateral to protect against dilution for low pledgeability, too. Thanks to Adriano Rampini for a discussion of this point.
4.4. **Collateral rat race**

We now show that collateralization at Date 0 can protect from dilution at Date 1. An appropriate level of collateralization can yield the first-best outcome.

**Proposition 2 (Collateral rat race).** Define

\[ I_1^* := I_1^L + \theta(X_1^H - X_1^L). \]  

If \( I_1^H < I_1^* \), B borrows (partially) secured from \( C_0 \) and the first best is attained; i.e., B borrows (secured) from \( C_1 \) in state \( H \) and does not borrow in state \( L \).

To commit not to invest in Project 1 in state \( L \), B secures enough of Project 0’s cash flow to \( C_0 \) at Date 0 that he cannot use his leftover pledgeable cash flow to fund Project 1, i.e., he sets \( \sigma_0 \) large enough that

\[ \theta((1 - \sigma_0)X_0 + X_1^L) < I_1^L. \]  

(9)

To maintain enough financial flexibility that he can invest in Project 1 in state \( H \), he should leave enough of Project 0’s cash flow unsecured at Date 0 so that he can use his leftover cash flow to fund it; i.e., he sets \( \sigma_0 \) small enough that

\[ \theta((1 - \sigma_0)X_0 + X_1^H) \geq I_1^H. \]  

(10)

A proportion of secured debt \( \sigma_0 \) exists that constrains B in state \( L \), but not in state \( H \), i.e., that satisfies conditions (9) and (10), only if the cost of Project 1 is not to large, \( I_1^H < I_1^* \). Otherwise, constraining B in state \( L \) implies constraining him in state \( H \), too.

Overall, this result implies that, even if B is unable to invest efficiently if he borrows unsecured at Date 0 (Proposition 1), he could still be able to if he uses the appropriate amount of collateral. In this case, B must use collateral at Date 0 to commit not to use it at
Date 1. There is a collateral rat race in which creditors require collateral today to protect against dilution with collateral in the future.

This resonates with legal scholars’ observation that collateral is necessary to “protect lenders against dilution [with] secured debt” (Schwartz 1997, p. 1397) given that “[l]ate-arriving secured creditors can leapfrog earlier unsecured creditors, redistributing value to the benefit of the issuer and the secured creditor but to the detriment of unsecured creditors” (Listokin 2008, p. 1039).

4.5. Collateral overhang

While collateral can help restore efficiency by protecting creditors against inefficient dilution, it can also create inefficiencies by preventing efficient borrowing.

**Proposition 3** (Collateral overhang). If \( I_1^H \geq I_1^* \), \( B \) borrows (partially) secured from \( C_0 \) at Date 0 and cannot borrow from \( C_1 \) at Date 1 in either state. Hence, there is inefficient underinvestment in state \( H \).

If funding needs are large enough in state \( H \) (\( I_1^H \geq I_1^* \)) collateralization can encumber assets, leading to a collateral overhang problem. To prevent investment in state \( L \), the proportion of secured debt must be so large that it prevents investment in state \( H \), too; i.e., no \( \sigma_0 \) satisfies both conditions \([9]\) and \([10]\) at once. As a result, the risk of future collateralization can lead to inefficient preemptive collateralization. Further, ex interim renegotiation cannot resolve this inefficiency. Limited pledgeability implies that the pledgeable payoff is insufficient to compensate \( C_0 \)\(^9\).

**Corollary 1.** The equilibrium debt contract is renegotiation proof. That is, \( B \), \( C_0 \), and \( C_1 \) cannot renegotiate to undertake Project 1 in state \( H \) and thereby avoid the collateral overhang.

\(^9\) Relatedly, Bhattacharya and Faure-Grimaud (2001) show that when a firm’s investments are noncontractible, renegotiation between borrowers and creditors may not resolve the debt overhang problem.
This result underscores that B must dilute C₀’s debt in state \( H \) to borrow and invest in Project 1. It follows that the collateral overhang result is not reliant on our assumption that debt matures at Date 2, i.e., short-term debt does not help. Indeed, if C₀ lends to B via short-term debt, B has no debt in place at Date 1. And, without diluting debt in place, B cannot borrow enough to invest in Project 1 in state \( H \). We show this formally in the Appendix after the proof of the corollary.

Our results so far are closely in line with practitioners’ intuition that “asset encumbrance not only poses risks to unsecured creditors” (collateralization dilutes unsecured creditors) “but also has wider ... implications since encumbered assets are generally not available to obtain ... liquidity” (collateralization leads to the collateral overhang) (Deloitte, 2014).

This is also the case in the Myers (1977) debt overhang problem, but for a different reason. There, a borrower prefers not to raise capital because the benefits of a new investment go to existing creditors. In the collateral overhang problem, the incentives of the borrower and the creditor are reversed. The borrower would prefer to raise capital because the benefits of new investment go to him at the expense of the creditor, but debt is collateralized precisely to stop him from doing so. Further, the debt overhang problem does not arise if either debt in place can be renegotiated (so existing creditors can participate in financing the investment) or if it is riskless (so existing creditors get none of the benefits of new investments). The collateral overhang problem arises even though debt in place both can be renegotiated and is riskless.

4.6. **Equilibrium characterization**

The results so far imply the following equilibrium characterization.

**Corollary 2.** The equilibrium outcome is as follows.

If \( \theta < \theta^* \), the first best is attained. At Date 0, B borrows unsecured. At Date 1, B borrows secured in state \( H \) and does not borrow in state \( L \).
If $\theta \geq \theta^*$ and $I^H_1 < I^*_1$, the first best is attained. At Date 0, B borrows partially secured. At Date 1, B borrows secured in state $H$ and does not borrow in state $L$.

If $\theta \geq \theta^*$ and $I^H_1 \geq I^*_1$, the first best is not attained due to the collateral rat race and the collateral overhang. At Date 0, B borrows secured with face value $I_0$. At Date 1, B does not borrow in state $H$ or state $L$.

4.7. Collateralizability and collateral damage

So far, we have assumed that all pledgeable assets can serve as collateral. In reality, some assets may be pledgeable (they can be seized in the future) but not collateralizable (they are hard to assign property rights to). For instance, they may not even exist at inception, but can be built or acquired in the course of the project. Also, property rights on some existing assets, such as intellectual property, can be difficult to define legally. How do pledgeability and collateralizability interact?

To address this question, we extend the model by assuming that B can collateralize at most a fraction $\mu_t$ of Project $t$ at Date $t$, so B can divert $(1-\theta)X_t$ and collateralize $\theta \mu_t X_t$, but $\theta (1 - \mu_t)X_t$ is neither divertable nor collateralizable. Because different projects can employ different types of assets, $\mu_t$ depends on the project. We find that higher collateralizability can loosen borrowing constraints as well as tighten them.

Because B always borrows from $C_1$ if he can (Lemma 1), B borrows when he is unconstrained at Date 1, i.e., whenever

$$
\mu_1 \theta X^*_1 + \frac{1}{2} \left( (1 - \sigma_0) \theta X_0 + (1 - \mu_1) \theta X^*_1 \right) \geq I^*_1.
$$

Indeed, recall that $C_1$ is senior on the collateralized portion of Project 1, which is at most $\mu_1 \theta X^*_1$, and $C_0$ and $C_1$ are on equal footing for the uncollateralized portion of each project, i.e., $C_0$ and $C_1$ Nash bargain over $(1 - \sigma_0) \theta X_0$ and $(1 - \mu_1) \theta X^*_1$.

We show that high collateralizability $\mu_1$ can do damage: by making it easier for B to take
on new debt to \( C_1 \) at Date 1, diluting \( C_0 \), it can lead \( C_0 \) to require collateral as protection at Date 0. In other words, it can trigger a collateral rat race, leading to a collateral overhang.

**Proposition 4 (Collateral damage).** Define

\[
\mu^*_1 := \frac{2I^L_1 - (1 - \mu_0)\theta X_0}{\theta X^L_1} - 1
\]

and suppose \( p \) is not too large. If \( \mu_1 > \mu^*_1 \), \( B \) does not invest at Date 0 or Date 1.

Indeed, recall that \( B \) must not be able to dilute \( C_0 \) in state \( L \); i.e., condition (11) should hold in state \( L \) for no \( \sigma_0 \leq \mu_0 \). Since it is relaxed as \( \mu_1 \) increases, this is the case if and only if \( \mu_1 \leq \mu^*_1 \). Further, \( \mu^*_1 \) is increasing in \( \mu_0 \). That is, the more collateralizable Project 1 is, the more collateralizable Project 0 needs to be to offer \( C_0 \) protection.

### 4.8. Collateral supply and collateral shortage

Different assets can vary not only in productivity but also in collateral value. If so, the borrower’s choice of assets should trade off productivity against collateral value.

We assume \( B \) can do Project 0 with one of two types of assets, type I or type II, with different collateralizability. Say type I assets are perfectly collateralizable (\( \mu^I_0 = 1 \)), representing, e.g., a firm’s fixed assets or a bank’s financial securities eligible for general collateral (GC) repo. And suppose that type II assets are imperfectly collateralizable (\( \mu^II_0 < 1 \)), representing, e.g., a firm’s movable assets or a bank’s financial securities ineligible for GC repo. Finally, suppose \( X^II_0 > X^I_0 \), so that type II assets are the most productive.

We show that increasing collateralizability can distort \( B \)’s asset choice.

---

\(^{10}\) They could also represent assets that are costly to collateralize, due to costs of ex post monitoring (to ensure that collateral stays with the borrower), ex ante auditing (to ensure that collateral is unencumbered), registering securities in public records, warehousing securities with a custodian bank, or physical or legal ring-fencing.
Corollary 3. Let \( \sigma^*_0 \) denote \( C_0 \)'s demand for collateral; i.e., the smallest amount of collateral \( B \) can secure to \( C_0 \) so that \( C_1 \) prefers not to lend in state \( L \):

\[
\sigma^*_0(X_0) = \inf \left\{ \sigma_0 \mid \mu_1 \theta X_1^L + \frac{1}{2} \left( (1 - \sigma_0) \theta X_0 + (1 - \mu_1) \theta X_1^L \right) < I_1^L \right\}. \tag{13}
\]

\( C_0 \)'s demand for collateral \( \sigma^*_0(X_0) \) increases with Project 1’s collateralizability \( \mu_1 \). If \( \mu_1 \) is sufficiently high, \( B \) chooses low-productivity, high-collateralizability type I assets.

The more collateralizable Project 1 is, the more Project 0 needs to be collateralized to protect \( C_0 \) against dilution (as implied by Proposition 4). Thus, if Project 1’s collateralizability is high, \( B \) could be unable to borrow against the low-collateralizability type II assets at Date 0. He could be forced to choose low-productivity type I assets to raise finance [cf. Eq. (13)]. Thus, reforms increasing the effective supply of collateral at Date 1 (either increasing the proportion of assets used in Project 1 that can be used as collateral or decreasing the costs of collateralizing them) can increase the need for type I assets at Date 0 (they can increase the demand for collateral).

This result speaks to policies that aim to increase the supply of collateral. Recently, governments have been “manufacturing quality collateral,” because “there’s still not enough of the quality stuff to go around ... as quality collateral becomes impossible to find ... The crunch has further been heightened by the general trend towards collateralised lending and funding” [Financial Times, 2011]. For example, several countries recently expanded the set of movable assets that can be used as collateral. Further, some central banks committed to lend against illiquid securities at a set rate and haircut; e.g., the European Central Bank enacted its Long-term Refinancing Operation and the Reserve Bank of Australia its Committed Liquidity Facility. Moreover, in 2005, repo transactions backed by some assets

\(^{11}\)Several European countries recently allowed movable assets to be used as collateral (see Calomiris, Larrain, Liberti, and Sturgess, 2017; Campello and Larrain, 2016; Cerqueiro, Ongena, and Roszbach, 2016; and Thell, 2017) as did Zimbabwe, where cows, sheep, and goats used as collateral are now recorded in a central bank register [Financial Times, 2017].
became super senior in bankruptcy. And, yet, markets still perceived a shortage of collateral.

This resonates with the result above. In the context of our model, such increases in the supply of assets that can be used as collateral correspond to increases in collateralizability. For any portfolio of assets, the proportion $\mu$ that can be collateralized increases. These policies did not necessarily affect pledgeability. For example, repo borrowers found it easier to assign assets as collateral to specific repo creditors but did not find these assets any harder to divert. If the supply of assets that can be used as collateral at Date 1 increases, a higher proportion of assets must be collateralized at Date 0. This can lead borrowers to switch from unsecured to secured borrowing and to distort their asset base toward collateralizable assets. An increase in collateral supply can increase the demand for collateral, instead of satisfying it.

As Caballero (2006, p. 2) puts it, “[t]he world has a shortage of financial assets. Asset supply is having a hard time keeping up with the global demand for ... collateral.” (See also Di Maggio and Tahbaz-Salehi, 2015.)

4.9. The two roles of collateral

So far, we have abstracted from the collateral’s classical role in mitigating enforcement problems between borrowers and creditors to focus on its role in mitigating them among creditors. We now discuss an extension in which collateral plays both roles. To this end, we assume that collateralization not only establishes exclusivity, but also increases pledgeability. The fraction of a project that is pledgeable depends on whether debt is secured or not, with $\theta_c := c\theta$ if B borrows secured and $\theta_u := u\theta < c\theta$ if B borrows unsecured. We focus on the case in which B always has sufficient pledgeable cash flow to fund Project 0 via secured debt, $\theta_c X_0 > I_0$, and Assumption 5 holds with $\theta = \theta_c$, i.e., $I_1^H < \theta_c(X_0 + X_1^H)$.

Proposition 5  B’s unsecured borrowing capacity is hump-shaped in pledgeability, so increasing
$\theta$ helps for small $\theta$ but hurts for high $\theta$. Define

$$\theta_u^* := \frac{I_0}{uX_0}$$

and

$$\theta_c^* := \frac{I_1^L}{c(X_0 + X_1^L)}.$$  

If $\theta \in [\theta_u^*, \theta_c^*)$, $C_0$ lends unsecured and the first best is attained; i.e., $B$ borrows (secured) from $C_1$ in state $H$ and does not in state $L$. If $\theta < \theta_u^*$ or $\theta \geq \theta_c^*$, $C_0$ does not lend unsecured.

For $\theta < \theta_u^*$, $B$ cannot borrow unsecured from $C_0$ but must use collateral to increase his pledgeable payoff. For $\theta \geq \theta_c^*$, $B$ also cannot borrow unsecured from $C_0$ but must use collateral to protect $C_0$ from dilution.

4.10. Healthy versus constrained borrower

Our results apply when the risk of dilution is a concern for existing creditors, i.e., when new debt decreases the value of existing debt. This is more likely if (1) $B$ is in relatively poor health and (2) new debt finances investments that create little value for existing creditors. Here, we relax some of the assumptions in Section 3.5 to stress why (1) and (2) are necessary for the results.

In our baseline setup, we assume that Project 0’s payoff is so low that $B$ is close to default (Assumption 3), reflecting a borrower in relatively poor health, who is financially constrained or even distressed. We extend the model to allow $B$ to represent a healthy borrower as well. To this end, we assume that Project 0’s payoff can take two values: $X_0^H$ with probability $p_0$ and $X_0^L < X_0^H$ otherwise. Like Project 1’s payoff, it is revealed at Date 1.\(\text{[12]}\) (Our baseline model corresponds to $p_0 = 0$.)

If $X_0 = X_0^H$ and $X_0^H$ is high enough, $B$ invests in Project 1 only if it is efficient. $B$ is not

---

(12) So, now there are four states, one for each pair of types of Project 0 and Project 1. We continue to use $s$ to index the type of Project 1 as above, not to denote the aggregate state.
at risk of default and thus B borrowing from C₁ does not dilute C₀. Moreover, C₀ gets paid enough when B is healthy (X₀ = X₀ᴴ) that it will lend without collateral, even if it could be diluted when B is not (X₀ = X₀ᴸ).

**Lemma 4.** If X₀ᴴ is large enough, i.e.,

\[
X₀ᴴ > \frac{I₀ + p₀(I₁^s - X₁^s)}{θp₀}
\]

for s ∈ {L, H}, B undertakes Project 1 if and only if it has positive NPV and, moreover, he can always borrow unsecured from C₀.

So far, we have allowed for Project 1 to be efficient (X₁ᴴ) or inefficient (X₁ᴸ), but we assumed that it benefits the borrower and harms the existing creditor reflecting borrower-creditor conflicts of interest close to default. But new investments could also benefit both the borrower and creditor. Indeed, if there were a third type of Project 1, say HH, with pledgeable payoff greater than its cost θX₁ᴴᴴ > I₁ᴴᴴ, then undertaking Project 1 would benefit C₀, even if it were financed with secured debt to C₁.

**Lemma 5.** Suppose that B has (secured or unsecured) debt with face value F₀ to C₀ and θX₀ᴸ < F₀. If X₀ = X₀ᴸ and Project 1 is type HH, then B borrows secured from C₁ with face value F₁ = I₁ᴴᴴ and invests at Date 1. C₀ is better off than if B did not invest at Date 1. I.e. the new investment supports the existing debt.

---

13By Assumption 1 and Assumption 3, we focus on new investments that dilute the existing creditors—i.e. θX₁¹ < I₁¹.

14 Increases in creditor control rights decrease new borrowing (Chava and Roberts 2008; Roberts and Sufi 2009a, 2009b, 2012; Becher, Griffin, and Nini 2017). Moreover, while creditors loosen contract terms when a firm’s asset values increase, they keep them tight when it gets new investment opportunities, suggesting these can harm creditors (Roberts and Sufi 2009b). The textbook description of bankruptcy stresses that creditors’ underinvestment “problems ... usually arise because the goal of paying off the creditors conflicts with the goal of maintaining the business as a going concern ... [and] creditors ... may press for a liquidation” (Brealey, Myers, and Allen 2014, p. 853; see, e.g., Ayotte and Morrison 2009, Skeel 2004, Jenkins and Smith 2014).
Thus, even though collateralization can prevent the borrower from financing some efficient investments (Subsection 4.5), it does not prevent him from financing all of them; he can still finance those that are so good that they benefit creditors, too. This is consistent with evidence in Badoer, Dudley, and James (2018) and Rauh and Sufi (2010).

5. Discussion of contracting environment and covenants

The critical contracting assumptions are that (1) B cannot commit not to collateralize in the future, (2) secured debt is treated as senior, and (3) collateralization cannot be contingent on future events. Here, we argue that they reflect reality. We also discuss two other assumptions, which are less crucial for our results but also reflect practice: (4) unsecured creditors are on equal footing in default, and (5) B cannot borrow using floating charges.

5.1. Negative pledge covenants

We have assumed away unsecured debt with covenants restricting future secured borrowing. If perfectly enforced, such covenants could prevent dilution, hence limiting the need for collateral to establish priority. While such covenants exist, they are relatively ineffective in practice. This is because, whereas a secured creditor holds a claim against other creditors (via a property right), an unsecured creditor holds a claim against only the borrower [via a promissory right; cf. Ayotte and Bolton (2011) and Donaldson, Gromb, and Piacentino (2018)]. Thus, an unsecured creditor cannot recover collateral that has been seized by a secured creditor, even if the secured borrowing violated a covenant. Bjerre (1999) describes these legal restrictions.

Unfortunately, negative pledge covenants’ prohibition of such conduct [future secured borrowing] may be of little practical comfort, because as a general matter they are enforceable only against the borrower, and not against third parties who take security interests in violation of the covenant. Hence, when a borrower
breaches a negative pledge covenant, the negative pledgee generally has only a cause of action against a party whose assets are, by hypothesis, already encumbered (pp. 306–307).

Badoer, Dudley, and James (2018) find empirically that creditors require collateral to protect against dilution, because they deem covenants insufficient.

[We] provide empirical evidence that priority spreading [i.e., increased reliance on secured debt] occurs, in part, because security provides creditors with greater protection from dilution from other creditors than covenants that prioritize payments.

The effectiveness of negative pledge covenants in bankruptcy is especially limited for repo and derivatives liabilities, as these contracts are exempt from bankruptcy stays. Creditors can liquidate collateral without the approval of the bankruptcy court, making it difficult or impossible for any third party to enforce a claim to the collateral.

Negative pledge covenants can still be useful outside bankruptcy. Their violation constitutes a default, which borrowers can seek to avoid. However, this can be insufficient to prevent a borrower from taking on additional debt in general. For example, a borrower in financial distress is likely to default anyway and can therefore be willing to violate such covenants to gamble for resurrection by taking on new debt.

More generally, verifying that a solvent firm has violated a covenant can be difficult. Covenants are especially difficult to monitor or enforce for complex firms, notably banks, that can have thousands of counterparties. Banks effectively do not have to disclose their short-term borrowing.

There are no specific MD&A [managing discussion and analysis] requirements to disclose intra-period short-term borrowing amounts, except for [some] bank holding companies [that must] disclose on an annual basis the average, maximum month-end and period-end amounts of short-term borrowings (Ernst & Young 2010).

Rajan and Winton (1995) argue that they give creditors greater incentive to monitor, and Gârleanu and Zwiebel (2009) find that they help allocate decision rights efficiently given asymmetric information.
Banks may not be able to commit not to dilute existing debt with new debt for another reason. The very business of banking constitutes maturity and size transformation, which requires frequent short-term borrowing from many creditors. Covenants restricting a bank’s ability to borrow in the future could undermine its ability to engage in these banking activities (Bolton and Oehmke 2015). This nonexclusive contracting is especially important for banks and, thus, could be an important reason that interbank markets rely heavily on collateral.

5.2. **Secured debt is super senior**

The seniority of secured debt is a basic feature of US bankruptcy law. It reflects constraints on the ability to establish priority. In practice, unsecured debts cannot easily be prioritized temporally, because contracts can be backdated. Secured debts can be prioritized temporally by physically transferring collateral or by publicly registering a security interest in a property registry. These practical limits on enforcing priority reflect fundamental constraints to contracting, which Ayotte and Bolton (2011) explore. They conclude that collateralization “reduce[s] uncertainty and discovery costs of third parties who seek to acquire rights in the same property,” i.e., rights in the same collateral (p. 3403).

5.3. **State-contingent collateralization**

We have assumed away state-contingent collateralization. Were it possible, it could circumvent the inefficiencies arising in our analysis. Ex ante, at Date 0, B would commit to collateralize Project 0 to C₀ in state L but not in state H, thereby allowing B to take on new debt in state H but not in state L. Ex post, B would prefer to renege on his promise and collateralize Project 0 to C₁ in state L[16] Thus, contingent collateralization effectively requires the commitment not to collateralize in the future, which we have argued could be impossible. Furthermore, even bilateral contingent contracting can be difficult in reality for

---

[16] Analogously, C₀ could take extra collateral at Date 0 and commit to return (some of) it to B in state H. However, C₀ would prefer to renege and keep the collateral.
a number of reasons established in the literature.\textsuperscript{17}

An additional rationale for that collateralization not being state-contingent is that it often requires a physical transfer of assets between the borrower and the creditor. In legal parlance, the secured debt is possessory. In this case, state-contingent collateralization would require $C_0$ to be physically present at Date 1 to transfer possession, which could be costly or infeasible.\textsuperscript{18}

5.4. Priority among unsecured creditors

We have assumed unsecured creditors to be on equal footing or pari passu in case of default. This is consistent with their legal treatment. “[T]he ‘pari passu principle,’ provides that unsecured creditors rank equally with each other in right to payment, regardless of the temporal order in which they extend credit” (Bjerre 1999, p. 309). In practice, unsecured creditors are prioritized roughly according to the order in which they alert the court of a borrower’s default, i.e., the first to file or perfect is paid first (Picker, 1999). Our baseline assumption of 50–50 bargaining is akin to assuming that creditors are equally likely to win this race to alert the court, but our results also hold for general bargaining power.\textsuperscript{19}

5.5. Floating charges

Our analysis of limited collateralizability captures a borrower’s inability to collateralize all of the pledgeable assets he will have in the future. He could be able to collateralize

\textsuperscript{17}These include the absence of common knowledge of the state (Aghion, Fudenberg, Holden, Kunimoto, and Tercieux, 2012), limited cognition (Bolton and Faure-Grimaud, 2009; Tirole, 2009), fairness concerns (Hart and Moore, 2008), and the inability to commit not to renegotiate in complex environments (Hart and Moore, 1999; Segal, 1999).

\textsuperscript{18}If $C_0$ is present at Date 1, it can lend secured with interest at Date 0 and commit to transfer $I^H_1$ to $B$ in state $H$ against the same collateral (a kind of state-contingent collateralization). This can achieve the efficient outcome but requires that $C_0$ can commit to state-contingent transfers that it would prefer to renege on ex post. Thanks to Martin Oehmke for pointing this out.

\textsuperscript{19}For an influence cost–based model endogenizing creditors’ bargaining positions in default, see Welch (1997).
his real estate, but not his inventory. In practice, unsecured creditors holding so-called floating charges have the option to crystalize them into secured claims. This arrangement approximates collateralizing movable assets such as inventories. Floating charges are an important component of corporate capital structure (Thorburn 2000). Including them would not substantively affect our analysis, because they do not establish priority on the same level as secured debt, which can dilute them. Mokal (2004) states:

As for the floating charge, however, the creation of subsequent fixed charges [i.e. secured debts] and the accumulation of new preferential claims can dilute the security, as can the debtor’s ability to alienate the collateral free of the charge. So the floating charge holder cannot even know exactly what assets it has security over, nor what value they have to it! (p. 13)

See Mokal (2003) for more on the law regarding this point.

6. Empirical content

6.1. Consistent evidence

In their study of nonexclusivity in credit, Degryse, Ioannidou, and von Schedvin (2016) analyze the effect of a borrower breaking up an exclusive relation with its existing creditor by borrowing from a new creditor. They find that the existing creditor becomes less willing to lend unsecured, but not less willing to lend secured. This finding is consistent with our paradox of pledgeability result: A borrower’s ability to borrow from a new creditor undermines his ability to borrow unsecured from existing creditors.

Abundant evidence exists that stronger creditor or collateral rights can increase lending and investment, just as higher pledgeability or collateralizability does in our model. This pattern could be the beneficial result of relaxing financial constraints, as in the received theory. It could also be the detrimental result of diluting existing debt. Hence, one should be careful about drawing welfare conclusions from these findings.

Vig (2013) analyzes a reform making it easier for secured creditors to seize assets in
default, which, in our model, corresponds to an increase in collateralizability. He finds that lending declined, in contrast to standard theories, but in line with our collateral damage result. Increasing collateralizability can make dilution easier and undermine unsecured credit.

Like us, Haselmann, Pistor, and Vig (2010) stress the distinction between collateralizability ($\mu$ in our model) and pledgeability ($\theta$ in our model). They find that $\mu$, captured by collateral law, matters more for credit supply than $\theta$, captured by bankruptcy law. This suggests that the nonexclusivity frictions, mitigated by collateral, could be as important as ex post limits to enforcement (pledgeability frictions), mitigated by bankruptcy.

6.2. Proxies

Our model generates predictions depending on pledgeability $\theta$ and collateralizability $\mu$. Empirical proxies for $\theta$ include the strength of bankruptcy law and the degree of asset tangibility. Proxies for $\mu$ include the strength of collateral or property law and the (inverse) frequency of asset turnover. Haselmann, Pistor, and Vig (2010) use collateral or property law and bankruptcy proxy to distinguish between property rights and creditor rights. Our model also yields predictions depending on the severity of the nonexclusivity friction. Proxies include credit market competition, the number of creditors a borrower has, the liberalness of bank branching regulation, and the degree of accounting opacity (which prevents covenants on new debts from being enforced).

6.3. New predictions

Our analysis suggests six, as yet untested, predictions specific to collateral establishing exclusivity.

**Prediction 1.** Firms with more pledgeable assets should have a larger fraction of secured debt.
Increasing pledgeability ($\theta$ in our model) can trigger a collateral rat race, making borrowers more reliant on secured debt (Proposition 2). For instance, firms with mostly tangible assets should have more secured debt, and a strengthening of bankruptcy law should lead to more debt collateralization.

**Prediction 2.** Firms with more collateralizable assets should have a larger fraction of secured debt.

Collateralizability ($\mu$ in our model) can also trigger a collateral rat race (Proposition 4). For instance, firms with low asset turnover should have more secured debt and a strengthening of collateral or property law should lead to more debt collateralization.

**Prediction 3.** A firm’s investment is hump-shaped in the pledgeability of its assets.

At low pledgeability levels, higher pledgeability loosens financial constraints allowing for more investment (its classical effect) but, at higher pledgeability levels, it tightens them, by making dilution easier (Proposition 5). For instance, among firms with mostly intangible assets, those with more tangible assets should invest more; whereas, among firms with more tangible assets, the reserves should hold. A strengthening of bankruptcy laws should increase investment when bankruptcy law is weak but reduce investment when it is strong.

**Prediction 4.** Investment is more likely to be decreasing in pledgeability when collateralizability is high.

For low $\theta$, borrowers are always constrained. For high $\theta$, they are constrained for low $\mu$ but not for high $\mu$ (Proposition 4). For instance, investment should be more likely to decrease with asset tangibility if asset turnover is low. And a strengthening of bankruptcy laws is more likely to lead to a decrease in investment when property or collateral law is strong.

**Prediction 5.** Investment is more likely to be increasing in pledgeability when the non-exclusivity friction is severe.
For borrowers in nonexclusive relation, high $\theta$ can undermine the ability to borrow due to the collateral overhang. For borrowers in exclusive relation, this paradoxical effect does not arise. For instance, investment is more likely to increase with asset tangibility among firms with multiple creditors and when banking competition is more intense. And a strengthening of bankruptcy law is more likely to lead to higher investment following bank branching deregulation.

**Prediction 6.** *Investment is more likely to be increasing in collateralizability when the nonexclusivity friction is severe.*

$\mu$ affects only nonexclusivity and thus does not help borrowers who are in exclusive relation already. For instance, investment is more likely to decrease with asset turnover among firms with multiple creditors and when banking competition is more intense. And a strengthening of property or collateral law is more likely to lead to higher investment following bank branching deregulation.

### 7. Conclusion

We have considered a model in which collateral serves to protect creditors against dilution with new secured debt. High pledgeability increases the risk of dilution, as it makes taking on new secured debt easier and thus, paradoxically, makes creditors less willing to lend unsecured. Collateralization is required to protect against future collateralization. There is a collateral rat race. This reliance on collateral leads to a collateral overhang problem, whereby collateralized assets are encumbered and cannot be used to raise liquidity. We find that increasing the supply of collateral can aggravate this problem, by triggering the collateral rat race. Likewise, so can upholding the absolute priority rule, by which secured creditors get paid first in bankruptcy.
Appendix A. Proofs

A.1. Proof of Lemma 1

To see that B always wants to borrow from C_0, observe that he gets zero if he does not. This is because if B does not invest in Project 0, he cannot invest in Project 1 either, because

\[ \theta X_s^0 < I_s^0 \]  

(A1)

for \( s \in \{L, H\} \), by Assumption 1 and Assumption 3.

To see that B always wants to borrow from C_1, suppose he has debt with face value \( F_0 \) to C_0. It must be that \( F_0 \geq I_0 \) by C_0’s participation constraint. Thus, if B does not borrow from C_1, he gets at most \( X_0 - I_0 \). If B borrows from C_1 in state H, his payoff increases by at least the value of Project 1, \( X_1^H - I_1^H > 0 \), and perhaps more if C_0’s payoff decreases. If B borrows from C_1 in state L, he gets at least his default payoff of \( (1 - \theta)(X_0 + X_1^*) \), which, by Assumption 4, exceeds \( X_0 - I_0 \).

\[ \Box \]

A.2. Proof of Lemma 2

If B does not borrow in state L, his pledgeable cash flow is \( \theta X_0 \) in state L. Thus, the expected cash flow that B can pledge to C_0 at Date 0 is at least \( P[H] \times 0 + P[L] \times \theta X_0 = (1 - p)\theta X_0 \). This is greater than \( I_0 \) by Assumption 1. Thus, B can pledge enough to C_0 to satisfy its participation constraint.

\[ \Box \]

A.3. Proof of Lemma 3

The result follows from Assumption 1 and Assumption 2.
A.4. Proof of Proposition 1

If C₀ lends unsecured, B can and does borrow secured from C₁ if and only if

$$\theta(X₀ + X₁^s) \geq I₁^s$$  \hspace{1cm} (A2)

or

$$\theta \geq \frac{I₁^s}{X₀ + X₁^s}.$$  \hspace{1cm} (A3)

This always holds for $s = H$ (Assumption 5), but not if $s = L$ if $\theta < \theta^*$. Thus, if $\theta < \theta^*$, there is no dilution in state $L$. Hence, C₀ lends (Lemma 2).

Now, if $\theta \geq \theta^*$, inequality (A3) is also satisfied for $s = L$. B will borrow secured from C₁ in both states $s \in \{L, H\}$. C₀ does not lend unsecured, because, by Assumption 3, with both projects, pledgeable cash flow is insufficient to repay both creditors.

The argument above for $\theta \geq \theta^*$ does not rely on our assumption that the repayment $F_t$ does not depend on the state $s$. To see why, note that in the first best, C₁ must be guaranteed $I₁^H$ in state $H$ to lend, which implies (by Assumption 3) that C₀ will get less than $I₀$ in state $H$. Thus, B must repay C₀ more than $I₀$ in state $L$. However, for $\theta \geq \theta^*$, he will not, because he will dilute C₀ with secured debt to C₁ in state $L$ as well. \hfill \Box

A.5. Proof of Proposition 2

The first best is attained if and only if inequalities (9) and (10) both hold, i.e.

$$\frac{I₁^H - \theta X₁^H}{\theta X₀} \leq 1 - \sigma₀ < \frac{I₁^L - \theta X₁^L}{\theta X₀}.$$  \hspace{1cm} (A4)

The left-most term is always less than one by Assumption 5 and the right-most term is always greater than zero by Eq. (A1). There exists $\sigma₀ \in [0, 1]$ satisfying the condition as long as the left-most term is less than the right-most term, which amounts to $I₁^H < I₁^L$. \hfill \Box
A.6. Proof of Proposition 3

B always prefers to borrow from C₀ than not to (Lemma 2). To do so, he must commit not to invest in state L, i.e., set σ₀ so that inequality (9) holds. But, for \( I₁^H > I₁^∗ \), this implies that inequality (10) is violated (cf. proof of Proposition 2), i.e., B does not invest in state H.

A.7. Proof of Corollary 1

Because the equilibrium is efficient for \( I₁^H \leq I₁^∗ \) (Proposition 2), we need to check for renegotiation proofness only for \( I₁^H > I₁^∗ \). For renegotiation to be feasible, B, C₀, and C₁ must all be weakly better off. Thus, C₀ and C₁’s joint payoff must weakly increase after renegotiation. Absent renegotiation, C₀’s payoff is \( I₀ \geq σ₀θX₀ \) and C₁’s payoff is zero. If renegotiation yields investment in state H, C₀ and C₁’s joint payoff is at most the total pledgeable cash flow \( \theta(X₀ + X₁^H) \) net of the investment cost. Renegotiation is feasible if and only if, for \( θ \geq θ^∗ \),

\[
θ(X₀ + X₁^H) - I₁^H \geq σ₀θX₀.
\]  

(A5)

But, this implies that inequality (9) is violated. Thus, renegotiation is infeasible.

**Short-term debt.** We have assumed that B cannot borrow from C₀ via one-period debt and roll over. We now show this is without loss of generality under renegotiation-proofness.

To consider short-term debt, we need to specify the sequence of moves at Date 1 and what happens if B defaults at Date 1. We assume that short-term debt matures after B has had the opportunity to borrow from C₁ and invest in Project 1, without loss of generality. We assume that C₀ gets the right to liquidate B’s projects, but that their liquidation value is zero. Alternatively, B and C₀ can renegotiate.

**Proposition 6** Renegotiation-proof short-term debt does not improve on the implementation of long-term contracts.

---

20If the debt matured earlier, B could not repay it because his projects do not payoff until Date 2.
Proof: B has no cash flows at Date 1, so $C_0$ has zero recovery value in liquidation. Thus, $C_0$ always prefers a rescheduling to Date 2 to liquidation at Date 1, and hence B has incentive to dilute $C_0$’s unsecured debt, even if it is short term. (Also, short-term secured debt leads to the same collateral overhang as long-term secured debt. It prevents B from borrowing from $C_1$ in state $H$ when dilution is efficient.)

A.8. Proof of Corollary

This follows from Proposition 1, Proposition 2, and Proposition 3.

A.9. Proof of Proposition

B borrows from $C_1$ in state $L$ whenever

$$\mu_1 \theta X^L_1 + \frac{1}{2} (\theta X^L_0 + (1 - \mu_1) \theta X^L_1) \geq I^L_1, \quad (A6)$$

where the expression is given by Eq. (11) with $s = L$ and $\sigma_0 = \mu_0$, i.e., substituting for the maximum amount of collateralization of Project 0. The inequality reduces to $\mu_1 \geq \mu^*_1$. $C_0$ gets repaid less than $I_0$ in state $L$ by Assumption 3. This implies that $C_0$ does not lend if state $L$ is sufficiently likely; i.e., if $p$ is not too large B cannot borrow at Date 1 either by Eq. (A1).

A.10. Proof of Corollary

From Eq. (13), we have

$$\sigma^*_0(X_0) = 1 - \frac{2I^L_1 - \theta (1 + \mu_1) X^L_1}{\theta X_0}, \quad (A7)$$

which is increasing in $\mu_1$.

B can fund Project 0 if he uses type I assets but not if he uses type II assets if and only
if $\sigma_0^*(X^L_0) \leq \mu_0^I$ and $\sigma_0^*(X^{II}_0) > \mu_0^{II}$. Rearranging, and using $\mu_0^I = 1$, this amounts to

$$\frac{2I^L_1 - (1 - \mu_0^{II})\theta X^{II}_0}{\theta X^L_1} < 1 + \mu_1 \leq \frac{2I^L_1}{\theta X^L_1}. \quad (A8)$$

Creditors being competitive, B captures all the surplus and uses the high-productivity type II assets if funding them is feasible, which, as per inequality (A8), is the case only if $\mu_1$ is sufficiently low. If $\mu_1$ is sufficiently high, B must choose type I assets. (The left-most term is always smaller than the right-most term, which is greater than one by Assumption 2. So, a $\mu_1$ exists for which type I assets can be financed but financing type II assets cannot.)

A.11. Proof of Proposition 5

B can finance Project 0 only if his pledgeable cash flow exceeds $I_0$. B can borrow from $C_0$ via unsecured debt if Project 0’s unsecured pledgeable cash flows are sufficient to cover the investment, i.e.,

$$\theta_u X_0 \geq I_0 \quad (A9)$$

and B cannot borrow secured from $C_1$ in state $L$, i.e.

$$\theta_c (X_0 + X^L_1) < I^L_1 \quad (A10)$$

These conditions both hold if and only if $\theta \in [\theta_u^*, \theta_c^*]$, in which case the first best is attained.

A.12. Proof of Lemma 4

For $X_0 = X^H_0$, B can borrow $I^*_1$ risk free and leave his debt to $C_0$ risk free if his pledgeable cash flow exceeds the sum of the face values, i.e.,

$$\theta(X^H_0 + X^*_1) \geq F_0 + I^*_1 \quad (A11)$$
If $C_0$ is repaid in full for $X_0 = X_0^H$, it will lend unsecured with face value $F_0 \leq I_0/p_0$ (even if it is repaid nothing when $X_0 = X_0^L$). The condition above is implied by

$$\theta(X_0^H + X_1^s) \geq \frac{I_0}{p_0} + I_1^s. \quad \text{(A12)}$$

Given risk-free borrowing at Date 1, B’s Date 1 investment strategy is not distorted. \qed

A.13. Proof of Lemma 5

If $X_0 = X_0^L$ and B does not do Project 1, B defaults on its debt to $C_0$ for sure (because $F_0 > \theta X_0^L$). He gets $(1 - \theta)X_0^L$.

If B does Project 1 instead, he can finance it secured from $C_1$ with face value $I_1^{HH}$ (because $\theta X_1^{HH} > I_1^{HH}$). B does not default on his debt to $C_1$ and may or may not default on his debt to $C_0$. Hence, he gets $\max\{(1 - \theta)(X_0^L + X_1^{HH}), X_0^L + X_1^{HH} - F_0 - I_1^{HH}\}$. This is greater than $(1 - \theta)X_0^L$. Hence, B does the project. $C_0$’s payoff is thus

$$\min\{F_0, \theta(X_0^L + X_1^{HH}) - I_1^{HH}\}. \quad \text{(A13)}$$

Because $F_0 > \theta X_0^L$ and $\theta X_1^{HH} - I_1^{HH} > 0$, this exceeds $\theta X_0^L$ and, thus, $C_0$ is better off if B does the project. \qed
References


