THE PARADOX OF PLEDGEABILITY*

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Abstract

We develop a model in which collateral serves to protect creditors from the claims of competing creditors. We find that borrowers rely most on collateral when cash flow pledgeability is high, because this is when it is easy to take on new debt, diluting existing creditors. Creditors thus require collateral for protection against being diluted. This causes a collateral rat race that results in all borrowing being collateralized. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment, i.e. there is a collateral overhang. Our results suggest that the absolute priority rule, by which secured creditors are senior to unsecured creditors, may have an adverse effect—it may trigger the collateral rat race.

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1 Introduction

Collateral matters.[1] By pledging collateral, a borrower mitigates enforcement frictions in financial contracts and thus loosens his financial constraints. In other words, "collateral pledging makes up for a lack of pledgeable cash" (Tirole (2006), p. 169). This suggests that collateral should matter most when cash flow pledgeability is low. Yet, some of the world's most developed debt markets rely heavily on collateral. Notably, upwards of five trillion dollars of securities are pledged as collateral in US interbank markets,[2] where strong creditor rights, effective legal enforcement, intense regulatory supervision, and developed record-keeping technologies ensure that cash flow pledgeability is high. Why does collateral matter in these markets?

To address this question, we develop a model in which collateral plays a different role than it typically plays in the finance literature. This literature has focused on one role of collateral: mitigating enforcement problems between borrowers and creditors. But collateral plays a second role: mitigating enforcement problems among creditors. These two roles of collateral correspond to the two components of property rights, the "right of access," i.e. the right to seize collateral, and the "right of exclusion," i.e. the right to stop others from seizing collateral (see, e.g., Hart (1995) or Segal and Whinston (2012)). In this paper, we focus on this second role of collateral, which is also emphasized by practitioners and lawyers. For instance, Kronman and Jackson (1979) define "a secured transaction [as] the protection...against the claims of competing creditors" (p. 1143).[3]

We find that borrowers rely most on collateral when cash flow pledgeability is high, because this is when it is easy to take on new debt, diluting existing creditors. Creditors thus require collateral for protection against being diluted. This causes a collateral rat race that results in all borrowing being collateralized. But collateralized borrowing has a cost: it encumbers assets, constraining future borrowing and investment, i.e. there is a collateral overhang. Our results suggest that the absolute priority rule, by which secured creditors are senior to unsecured creditors, may have an adverse effect—it may trigger the collateral rat race.

Model preview. In the model, a borrower, B, has two riskless projects, Project 0 and Project 1, to finance sequentially. B finances Project 0 by borrowing from one creditor, C_0,

[1]See, e.g., Benmelech and Bergman (2009, 2011), Rampini and Viswanathan (2013), and Rampini, Sufi, and Viswanathan (2014) for empirical evidence on the importance of collateral for borrowing.

[2]See Homquist and Gallin (2014).

[3]This view of collateral is also in line with Parlour and Rajan's (2001) view that "collateral can be interpreted as a commitment on the part of a consumer to accept only one contract" (p. 1322). Empirical support for our assumption that collateral mitigates the friction of non-exclusive contracting is in Degryse, Ioannidou, and von Schedvin (2016).

and, after Project 0 is underway, B can finance Project 1 by borrowing from another creditor, C_1. Project 0's NPV is positive, but Project 1's NPV, which is revealed after Project 0 is underway, may be positive or negative. Thus, it is efficient for B always to undertake Project 0 and to undertake Project 1 only in the event that its NPV is positive.

The amount B can borrow is constrained by two frictions. First, cash flow *pledgeability* is limited. Specifically, the total repayment from B to his creditors cannot exceed a fixed fraction θ of the projects' cash flows (e.g. due to imperfect legal enforcement). Second, contracts are *non-exclusive* in that when B takes on debt to C_0, he cannot commit not to dilute this debt with new debt to C_1.[4] However, collateral mitigates this friction by establishing priority in bankruptcy. To finance a project, B can borrow via either secured (i.e. "collateralized") debt or unsecured debt.[5] If B borrows via secured debt, the secured creditor has an exclusive claim over the project's pledgeable cash flows. Thus, by borrowing collateralized, B "fences off" a project from the claims of competing creditors. This ring-fencing involves a cost $(1-\mu)$, where we refer to μ as the project's collateralizability. If instead B borrows via unsecured debt, the creditor still has a claim on B's pledgeable cash flow, but it is effectively junior to any new secured debt that B takes on. To be clear, we assume that collateralization mitigates the non-exclusivity friction but does not affect the limited pledgeability friction (except in the extension in Subsection 6.2).

Our view that collateral mitigates the non-exclusivity friction by establishing priority among creditors is in line with the law literature. Indeed, legally, "[t]he absolute priority rule describes the basic order of payment in bankruptcy. Secured creditors get paid first, unsecured creditors get paid next" (Lubben (2016), p. 581). Lawyers have also observed that using collateral to establish priority may serve to dilute existing creditors and alternatively to protect them against dilution: Listokin (2008) says that "[l]ate-arriving secured creditors can leapfrog earlier unsecured creditors, redistributing value to the benefit of the issuer and the secured creditor but to the detriment of unsecured creditors" (p. 1039), whereas Schwartz (1997) points out that borrowers can also use collateral to "protect lenders against dilution by issuing secured debt" (p. 1397).

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[4]Note that this assumption rules out covenants by which a borrower contractually commits to one creditor not to borrow from new creditors in the future. As we discuss in detail in Subsection 6.1, such covenants sometimes do mitigate the non-exclusive-contracting friction in reality. However, their effectiveness is limited in circumstances in which the borrower can use collateral to borrow secured from new creditors. As Bolton and Oehmke (2015) put it:

> an important question is whether the firm can commit ex ante not to collateralize...ex post, for example via covenants that restrict such collateralization.... Under current U.S. bankruptcy law this is difficult: If a breach of such a covenant is discovered in bankruptcy, the collateral has already left the firm and...cannot be recovered by lenders (p. 2368).

[5]In Subsection 6.5, we allow for more general borrowing instruments and show that our main results are robust.

Results preview. Our two main results are that (i) if pledgeability θ is sufficiently high, B may be able to borrow from C_0 only via secured debt and that, as a result, (ii) if B borrows via secured debt and collateralization is costly ($\mu < 1$), B may not undertake positive NPV projects due to a "collateral overhang" problem.

To see why B can borrow from C_0 only via secured debt for high θ, suppose B finances Project 0 by borrowing from C_0 via *unsecured* debt. Because unsecured contracts are non-exclusive, B can borrow from another creditor, C_1, to finance Project 1. If B collateralizes his projects to borrow from C_1, then C_1 is prioritized over C_0—the new secured debt dilutes the existing unsecured debt. As a result, C_0 may not lend to B via unsecured debt in the first place. However, this dilution occurs only if B is not too constrained to borrow from C_1—i.e. if the pledgeable fraction θ of B's cash flow exceeds the cost of investment. In summary, if pledgeability is sufficiently high, then B dilutes C_0's unsecured debt with new secured debt to C_1 and, in anticipation, C_0 may not lend unsecured, but only with collateral. I.e., for high θ, there is a *collateral rat race*, by which collateralization is required to protect against future collateralization. Hence, contrary to common intuition in the finance literature, high cash flow pledgeability undermines unsecured credit.

If B borrows from C_0 via secured debt, he must pay the cost of collateralizing Project 0. This cost "uses up" pledgeable cash flow, constraining B's remaining debt capacity. This makes it difficult for B to borrow to finance Project 1. Hence, collateralization effectively encumbers B's assets, in the sense that it limits B's ability to use them to invest in Project 1, even if it is valuable. We call this a collateral overhang problem. Our model thus reflects practitioners' intuition that "asset encumbrance not only poses risks to unsecured creditors...but also has wider...implications since encumbered assets are generally not available to obtain...liquidity" (Deloitte Blogs (2014)).

Whenever θ is high, secured debt and unsecured debt can coexist and their interaction can lead to investment inefficiencies; there may be underinvestment as described above or, for other parameters, there may be over-investment. In particular, if the probability that Project 1 has positive NPV is sufficiently high, then B may borrow from C_0 via unsecured debt. In this case, B can "reuse" pledgeable cash flow to borrow from C_1 via secured debt. This leads to over-investment, since it subsidizes B's investment in Project 1, giving him the incentive to invest in it, even if it has negative NPV.

Whenever θ is low, in contrast, B borrows only via unsecured debt and there is no investment inefficiency. In this case, B can finance Project 0 by borrowing from C_0 via unsecured debt and can finance Project 1 by borrowing from C_1 via junior unsecured debt exactly when it has positive NPV. Hence, increasing pledgeability may decrease efficiency.

Policy. Our model casts light on the ongoing policy debate about the supply of collateral in financial markets. Recently, central banks have been "manufacturing quality collateral" because "there's still not enough of the quality stuff to go around...as quality collateral becomes impossible to find.... The crunch has further been heightened by the general trend towards collateralised lending and funding" (Kaminska (2011)).[6] Our analysis suggests that expanding the supply of collateral may backfire by making creditors less willing to lend unsecured, thus tightening credit constraints. The reason is that when collateral supply is high, it is easy to borrow via secured debt. This makes it easy for a borrower to dilute unsecured creditors by taking on new secured debt, which triggers the collateral rat race.

Moreover, the inefficiencies in our model are the result of the way courts enforce priority. Specifically, "[c]urrent law forces onto borrowers the power to defeat unsecured lenders by issuing secured debt, even when borrowers would prefer to give up that power in order to protect their unsecured lenders from the corresponding threat" (Bjerre (1999), p. 308). Indeed, our analysis suggests that upholding the absolute priority of secured debt can lead to inefficient investment. Thus, we suggest that a policy maker should remove the absolute priority of secured debt (see also Bjerre (1999) and Lubben (2016)).

Financial collateral. Interbank markets motivate our focus on the role of collateral in mitigating the non-exclusivity friction. When we extend the model to incorporate the role of collateral in mitigating the limited-pledgeability friction as well in Subsection 6.2, we find that this classical role of collateral dominates when pledgeability is low, but that the new role we focus on dominates when pledgeability is high. This is consistent with the pervasive use of collateral in interbank markets, such as the repo market. This is not easily explained by the classical theory—i.e. that pledging collateral makes up for a lack of pledgeable cash—for two reasons. (i) In interbank markets, pledging collateral may not be necessary to make up for a lack of pledgeable cash. In fact, in the securities lending market, cash itself *is* the collateral—borrowers pledge cash to borrow securities. Further, even in the repo market, the securities used as collateral are typically so liquid that they are referred to as "cash equivalents." (ii) Relatedly, in the repo market, borrowers often buy securities "on margin"—i.e. a borrower uses a small amount of initial capital as a down payment to buy assets on credit, using the assets themselves as collateral. In this case, the borrowed assets coincide with the collateralized assets. This is the case in our model, but typically not in models in which collateral makes up for a lack of pledgeable cash. In these models, a borrower typically posts a "tangible" or "illiquid" asset as collateral to borrow cash.

Related literature. Our paper makes three main contributions relative to the literature.

[6] One way for a central bank to manufacture collateral from illiquid securities is to commit to lend against the securities at a specified rate and haircut, as the European Central Bank did with its Long-term Refinancing Operation and the Reserve Bank of Australia did with its Committed Liquidity Facility.

First, we provide an explanation for the pervasive use of collateral in high pledgeability environments, such as US interbank markets, which we argue is a challenge for received theories. Second, we provide a formal analysis of the role of collateral in mitigating conflicts of interest among creditors, which has not yet been explored in the corporate finance literature. Third, we show that the ability to provide exclusivity selectively can be a friction. This gives a new perspective on the problem of sequential borrowing with non-exclusive contracts explored in Admati, DeMarzo, Hellwig, and Pfleiderer (2013), Bizer and DeMarzo (1992), Brunnermeier and Oehmke (2013), DeMarzo and He (2016), and Kahn and Mookherjee (1998).

Our paper is also related to papers that argue that decreasing credit market frictions can have perverse effects. Myers and Rajan (1998) argue that increasing asset liquidity can decrease efficiency by reducing a borrower's ability to commit to future investment decisions. We argue that increasing cash flow pledgeability can decrease efficiency because it reduces a borrower's ability to commit to future borrowing decisions. Donaldson and Micheler (2016) suggest that increasing cash flow pledgeability can increase systemic risk, because it leads borrowers to favor non-resaleable, over resaleable debt instruments (e.g., repos over bonds).

The collateral rat race in our model is reminiscent of the "maturity rat race" in Brunnermeier and Oehmke (2013). In that paper, short maturity, like collateral in our model, serves to establish priority. However, Brunnermeier and Oehmke (2013) do not study the effects of cash flow pledgeability. Further, our other main results are independent of the rat race (see Subsection 6.6).

More broadly, our paper also relates to the literature on non-exclusive contracts in finance.[7] Our contribution here is to study how collateral can work to mitigate—but, in equilibrium, amplify—the effects of non-exclusivity. We show that collateral is no panacea, because exclusive contracts have a dark side in sequential-borrowing environments.[8] Since collateral allows contracting parties to enter into exclusive relationships selectively it can undermine the claims of other parties—exclusive contracts might not be better than non-exclusive contracts if other non-exclusive contracts are already in place. This suggests a caveat to papers emphasizing how non-exclusive contracts can undermine credit markets, such as Bolton and Scharfstein (1990), Petersen and Rajan (1995), and Donaldson, Piacentino, and Thakor (2016). Also, we study the interaction of limited pledgeability and non-exclusive contracts, which these papers do not.

We also relate to the literature on collateral, covenants, and property rights in law and

[7]See Acharya and Bisin (2014), Attar, Casamatta, Chassagnon, and Décamps (2015), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Leitner (2012), and Parlour and Rajan (2001).

[8]The literature on large shareholder trading vs. monitoring provides another setting in which sequential trade with a third party can decrease efficiency. See, e.g., DeMarzo and Urošević (2006), Faure-Grimaud and Gromb (2004), and Kihlstrom and Matthews (1990).

corporate finance, such as Ayotte and Bolton (2011), Bebchuk and Fried (1996), Kronman and Jackson (1979), Schwarcz (1997), Schwartz (1984), and Stulz and Johnson (1985). The idea of investing in a multi-lateral commitment by ring-fencing, i.e. "collateralizing," a project builds on Kiyotaki and Moore (2000, 2001), who focus on the macroeconomic effects of such multi-lateral commitments.

Our paper is related to the literature on a possible shortage of collateral in funding markets, such as Caballero (2006) and Di Maggio and Tahbaz-Salehi (2015). We offer a new perspective by studying the role of collateral in mitigating non-exclusive contracting.

Layout. The paper proceeds as follows. Section 2 presents the model and includes a discussion of contracting environment. Section 3 analyzes two benchmarks: the first-best outcome and the outcome with exclusive contracting. Section 4 solves the model. Section 5 discusses welfare and policy. Section 6 analyzes a number of extensions and robustness issues. Section 7 concludes. Appendix A contains all proofs.

2 Model

2.1 Players and Projects

There is one good called cash, which is the input of production, the output of production, and the consumption good. A risk-neutral borrower B lives for three dates, $t \in \{0, 1, 2\}$, and consumes at Date 2. B has no cash, but has access to two investment projects, Project 0 at Date 0 and Project 1 at Date 1. Both projects are riskless and payoff at Date 2, but the payoff of Project 1 is revealed only at Date 1. Specifically, Project 0 costs I_0 at Date 0 and pays off X_0 at Date 2 and Project 1 costs I_1 at Date 1 and pays off X_1 at Date 2, where $X_1 \in \{X_1^L, X_1^H\}$ is a random variable realized at Date 1 with $X_1^L < X_1^H$ and $p := \mathbb{P}\left[X_1 = X_1^H\right]$.

B can fund his projects by borrowing I_0 at Date 0 and I_1 at Date 1 from competitive credit markets: we assume that B makes a take-it-or-leave-it offer to borrow I_t from a risk-neutral creditor C_t at Date $t \in \{0, 1\}$. There is no discounting.

2.2 Pledgeability and Collateralizability

B must promise to repay his creditors out of his projects' cash flows under two frictions. First, the pledgeability of cash flows is limited in that B may divert a fraction $(1 - \theta)$ of cash flows, leaving only a fraction θ for his creditors. We refer to θ as the *pledgeability* of cash flows. Second, contracts are non-exclusive in that if B borrows from one creditor at Date 0, he cannot commit not to borrow from another creditor at Date 1, potentially diluting the initial creditor's claim. In other words, when B borrows from C_0 at Date 0, B cannot commit not to borrow from C_1 at Date 1.

The role of collateral in our model is to mitigate the effects of non-exclusive contracting: if a creditor's claim is collateralized (or "secured") by a project, then the creditor has the exclusive right to the project's pledgeable cash flow if the borrower defaults, i.e. he has absolute priority over the project's cash flow.[9] To collateralize a project, B must "fence it off" from the claims of competing creditors. There is a deadweight-cost $(1-\mu)X$ of collateralizing the project with cash flow X.[10] We refer to μ as the *collateralizability* of projects. "Ring-fencing" is the legal analog of physical fence-building: a borrower's ring-fenced assets are legally insulated from its other obligations. The idea that costly ring-fencing is necessary to protect claims from a third party follows Kiyotaki and Moore (2001).[11]

2.3 Borrowing Instruments

B can choose to borrow via unsecured or secured (or "collateralized") debt. At Date t, B borrows I_t from C_t against the promise to repay the fixed face value F_t at Date 2.[12] To borrow secured, B must collateralize his project. We assume that courts respect the absolute priority rule, by which secured creditors are senior to unsecured creditors. Thus, if B collateralizes a project with cash flows X to borrow secured from a creditor, then this creditor has priority over X and X cannot be collateralized and used to borrow secured from another creditor, since collateralization entails ring-fencing to protect the collateral as discussed above.

For simplicity, we assume that if B borrows unsecured from multiple creditors then the creditor that lent first is senior. Hence, C_0's unsecured debt is senior to C_1's unsecured debt. It could also be reasonable to assume that B's unsecured debt is all treated equally, and we discuss this case of pari passu debt in Subsection 6.6. However, in keeping with the non-exclusivity assumption, we rule out the possibility that seniority is a contracting variable.

[9] Note that we assume for simplicity that collateralization is a binary decision—B either collateralizes a project or does not, he cannot collateralize only a fraction of a project. This does not affect the results.

[10] In Subsection 6.3, we show that it is equivalent to assume that to borrow via secured debt B must post a haircut. Further, there are other interpretations of the cost of ring-fencing, e.g., the cost of paying a custodian or warehouse to hold securities, lawyer's fees, ex post monitoring costs (to ensure that collateral stays with the borrower), ex ante auditing (to ensure that collateral is unencumbered), or registering the security in public records. Further, "issuing security is itself costly because the parties would have to negotiate a security agreement, give public notice, and so forth" (Schwartz (1981), p. 9).

[11] They say that a borrower "ring-fences his project in a way that limits the potential for asset-stripping" to a third party (p. 24).

[12] Our restriction to debt contracts maturing at Date 2 is for simplicity. In Subsection 6.5 and Subsection 6.4, we expand the analysis to consider contingent contracts and short-term contracts, respectively, and the main results are unchanged.

2.4 Payoffs

We now give the players' terminal payoffs. First, define the variable μ_t as follows:

$$\mu_t := \begin{cases} \mu & \text{if Project } t \text{ is collateralized,} \\ 1 & \text{if Project } t \text{ is not collateralized.} \end{cases} \tag{1}$$

Thus, the total payoff W is given by

$$W := \begin{cases} 0 & \text{if neither project is undertaken,} \\ \mu_0 X_0 & \text{if only Project 0 is undertaken,} \\ \mu_1 X_1 & \text{if only Project 1 is undertaken,} \\ \mu_0 X_0 + \mu_1 X_1 & \text{if both projects are undertaken.} \end{cases} \tag{2}$$

If B has debt F_0 to C_0 and F_1 to C_1, his payoff is the sum of the non-pledgeable part of the payoff and whatever is left of the pledgeable part of the payoff after repaying the debt to C_0 and C_1: $(1-\theta)W + \max\{\theta W - F_0 - F_1, 0\}$. If B does not default—i.e. $F_0 + F_1 \leq \theta W$—then each creditor C_t gets F_t. If B does default—i.e. $F_0 + F_1 > \theta W$—then C_0 and C_1 divide θW according to priority.

2.5 Assumptions

We impose several restrictions on parameters. These restrict attention to cases of interest, i.e. in which non-exclusivity alone causes the outcome to be inefficient. In our model, decreasing pledgeability increases efficiency because it mitigates the non-exclusive-contracting friction. In general, however, decreasing pledgeability has the direct effect of decreasing efficiency by inhibiting borrowing. We restrict parameters in such a way that this countervailing force is effectively "switched off." This is because we wish to focus on the interaction between pledgeability and non-exclusive contracting (which, to the best of our knowledge, has not been studied before), rather than on the direct effect of pledgeability on borrowing and efficiency (which has been well-studied; see, e.g., Holmstrom and Tirole (1997, 1998) or Kiyotaki (1998)).

Assumption 1. Net of the cost $(1-\mu)$ of collateralization, Project 0 has positive NPV and Project 1 has positive NPV if and only if $X_1 = X_1^H$:

$$0 < I_0 < \mu X_0 \quad \text{and} \quad 0 < X_1^L < I_1 < \mu X_1^H. \tag{3}$$

Assumption 2. The pledgeable cash flow from Project 0 exceeds its cost of investment net of the cost of collateralization, but the pledgeable cash flow from Project 1 does not:

$$I_0 \leq \theta\mu X_0 \quad \text{and} \quad \theta X_1^H < I_1. \tag{4}$$

Assumption 3. The combined pledgeable cash flow from Project 0 and 1 exceeds the combined investment cost if and only if $X_1 = X_1^H$:

$$\theta(X_0 + X_1^L) < I_0 + I_1 < \theta(X_0 + X_1^H). \tag{5}$$

The two parameter restrictions below are less important. They rule out cases that complicate the analysis but do not enrich it.[13]

Assumption 4.

$$X_1^L > \frac{\big(1 - \mu(1-\theta)\big)X_0 - I_0}{\mu(1-\theta)}. \tag{6}$$

This technical restriction ensures that the payoff of Project 1 is always large enough that B has the incentive to undertake it. Specifically, it ensures that if B can fund Project 1 by taking on new debt which dilutes existing debt, he will always do so.[14]

Assumption 5.

$$I_1 < \theta\mu\big(X_0 + X_1^H\big). \tag{7}$$

This is technical restriction simplifies the analysis by ensuring that the cost of Project 1 is not so large that B can never borrow from C_1 to invest in it.

2.6 Discussion of Contracting Environment

The novel contracting assumptions in our environment are (i) courts treat secured debt as super-senior; (ii) borrowers cannot commit not to use collateral; and, for the collateral-overhang result, (iii) collateralization is costly. As discussed above, (i) is typically satisfied, given the absolute priority rule. In contrast, (ii) and (iii) are more likely to be satisfied for some borrowers than for others.

(ii) is satisfied when so-called negative pledge covenants, which restrict future collateralization, are difficult to write or enforce. As we discuss in Subsection 6.1, this the case

[13] Both restrictions matter only for the proof of Proposition 3.

[14] Note that it might also be reasonable to assume that B gets private benefits from empire building and, therefore, always has the incentive to undertake Project 1, regardless of its NPV (cf. footnote 24). In that case this assumption is unnecessary.

for borrowers with many short-term creditors (such as banks), borrowers in financial distress, and borrowers with assets exempt from bankruptcy stays. Thus, our analysis suggests that collateral use should be increasing in borrowers' number of creditors, liability duration, distress probability or asset volatility, and proportion of repo and derivatives liabilities.

(iii) is satisfied when claims on collateral are difficult to verify or haircuts are high. In the corporate setting, this suggests that the collateral overhang is likely for receivables collateral, which requires auditing, monitoring, and registration, as compared to tangible collateral, which may not. In the financial setting, this suggests that the collateral overhang is more likely for illiquid collateral, which demands a high haircut, than for liquid collateral, which does not.

Two types of borrowers that are likely to satisfy all these assumptions are borrowers that rely on leased capital and borrowers that rely on repo financing. Leasing provides a way for new secured creditors to leapfrog existing creditors. A lease is effectively a super-senior secured loan: leased assets are not stayed in bankruptcy, so a lessor can repossess leased assets even before other secured creditors in the event of a borrower's default. A borrower can dilute his existing creditors by taking on new debt in the form of a lease. For leases, the collateralization cost may correspond to the inefficiencies arising from the separation of ownership and control, as in Eisfeldt and Rampini (2009).

Repos also provide a way for new secured creditors to leapfrog existing creditors, since a repo is formally a sale and repurchase of securities: a borrower sells securities to a creditor and other creditors have no recourse to the securities if the borrower defaults—indeed, like leased assets, these securities are exempt from the automatic stay in bankruptcy. In repo markets, the collateralization cost $(1 - \mu)$ corresponds to the repo haircut (as formalized in Subsection 6.3).

3 Benchmarks

In this section, we present two benchmarks: the first-best outcome and the outcome under exclusive contracting. We show both outcomes coincide.

3.1 First Best

In the first-best outcome, all positive NPV projects are undertaken. It follows immediately from Assumption 1 that the first-best outcome is to undertake Project 0 at Date 0 and Project 1 at Date 1 if and only if $X_1 = X_1^H$. The next proposition gives the associated first-best expected surplus.

Proposition 1. *In the first-best outcome, B undertakes Project 0 and undertakes Project 1 if and only if $X_1 = X_1^H$. The expected surplus is*

$$X_0 - I_0 + p\left(X_1^H - I_1\right). \tag{8}$$

3.2 Exclusive Contracts

Assuming that exclusive contracts are feasible amounts to assuming that B can borrow exclusively from a single creditor, i.e. $C_1 = C_0$.

Proposition 2. *With exclusive contracts the first-best outcome obtains.*

The intuition is that with exclusive contracts B borrows at the fair price to fund each project he undertakes. This is because when B takes on debt at Date 1, he does so from C_0, and, thus, the interest rate that C_0 charges on the new debt reflects its effect on the value of existing debt. As a result, B chooses to undertake only positive NPV projects, which leads to the first-best outcome.[15]

4 Model Solution

To characterize the equilibrium, we first solve two Date-0 subgames differing in whether B borrows via unsecured or secured debt at Date 0. Then we compare B's payoffs across subgames to find B's equilibrium choice of debt at Date 0.

4.1 Unsecured Debt to C_0

Suppose B borrows from C_0 at Date 0 via unsecured debt with face value F_0. We focus on the case in which $F_0 \geq I_0$ without loss of generality, since C_0 must recoup I_0 in expectation. We ask when B can borrow from C_1 via unsecured or secured debt at Date 1.

Unsecured debt to C_1. In that case, the new debt to C_1 is junior to the existing debt to C_0. Thus, C_1 will lend to B via unsecured debt only if the projects' pledgeable cash flow $\theta(X_0 + X_1)$ suffices to repay both I_1 to C_1 and F_0 to C_0, or if

$$I_1 \leq \theta(X_0 + X_1) - F_0. \tag{9}$$

[15] This intuition that with exclusive contracts B wants to undertake all and only positive NPV projects is a general feature of our environment, but the fact that the first-best outcome is achieved is not. In general, limited pledgeability alone could constrain B's borrowing, as we discuss further in Subsection 6.2. However, the assumptions in Subsection 2.5 rule this out, allowing us to focus on the inefficiencies induced by the non-exclusivity of contracts.

Given Assumption 1, this implies that B cannot borrow from C_1 via unsecured debt when $X_1 = X_1^L$.

LEMMA 1. *If B has unsecured debt to C_0, then B cannot borrow unsecured from C_1 if $X_1 = X_1^L$.*

The result follows from Assumption 3 and the fact that $F_0 \geq I_0$: if $X_1 = X_1^L$, the pledgeable cash flow that B has left after repaying C_0 is less than I_1.

Secured debt to C_1. In that case, this new debt to C_1 is effectively senior to the existing debt to C_0. This is because by collateralizing his projects, B protects C_1's claim to its cash flow. Thus, C_1 will lend to B via secured debt as long as B has sufficient pledgeable cash flow to repay I_1 (independently of B's unsecured debt F_0 to C_0), or if

$$I_1 \leq \theta\mu(X_0 + X_1), \tag{10}$$

where the right-hand side is the pledgeable fraction θ of the total cash flows $X_0 + X_1$ net of the collateralization cost $(1 - \mu)(X_0 + X_1)$.

By borrowing from C_1 via secured debt at Date 1, B can dilute his existing debt to C_0. This gives B the incentive to borrow and invest in Project 1 even when it has negative NPV.[16] Thus, B borrows at Date 1 whenever C_1 is willing to lend to him, i.e. whenever his pledgeable cash flow is sufficiently high.

LEMMA 2. *If B borrows unsecured from C_0 and $X_1 = X_1^L$, then B can borrow secured from C_1 if and only if pledgeability is above a threshold*

$$\theta^* := \frac{I_1}{\mu\left(X_0 + X_1^L\right)}. \tag{11}$$

This corollary implies that higher cash flow pledgeability loosens B's borrowing constraint at Date 1.

Subgame equilibrium. If pledgeability θ is low, then B cannot borrow from C_1 via secured debt if $X_1 = X_1^L$ (Lemma 2). Without the risk of being diluted, C_0 lends to B at the risk-free rate and B undertakes Project 1 only when it is efficient; he finances it by borrowing from C_1 via unsecured debt.

If pledgeability θ is high, then B can borrow from C_1 via secured debt (Lemma 2). B dilutes C_0's debt whenever $X_1 = X_1^L$, which occurs with probability p. Whether C_0 is willing to lend unsecured depends on p. If p is high, C_0 is unlikely to be diluted, so it is willing to

[16] Assumption 4 ensures that the payoff X_1^L is large enough that B always wishes to dilute C_0 to undertake Project 1. See the proof of Lemma 2 for the formal argument.

lend via unsecured debt—C_0 charges a high interest rate to compensate for dilution when $X_1 = X_1^L$. If p is low, however, C_0 is so likely to be diluted that it never lends via unsecured debt—C_0 cannot charge an interest rate high enough to compensate for dilution.

The next proposition summarizes B's equilibrium borrowing behavior, given that he borrows from C_0 via unsecured debt.

LEMMA 3. *Assume B can only borrow unsecured from C_0 and define*

$$\theta^{**} := \frac{I_1}{\mu X_0}, \tag{12}$$

$$p^* := \frac{I_0 + I_1 - \theta\mu\left(X_0 + X_1^L\right)}{\theta\left(X_0 + X_1^H\right) - \theta\mu\left(X_0 + X_1^L\right)} \in (0,1), \tag{13}$$

$$p^{**} := \frac{I_0 + I_1 - \theta\left(\mu X_0 + X_1^L\right)}{\theta\left(X_0 + X_1^H\right) - \theta\left(\mu X_0 + X_1^L\right)} \in (0,p^*). \tag{14}$$

- *If $\theta \le \theta^*$, B borrows unsecured from C_0; B borrows unsecured from C_1 if $X_1 = X_1^H$ and does not borrow if $X_1 = X_1^L$.*
- *If either $\theta > \theta^*$ and $p \ge p^*$ or $\theta \ge \theta^{**}$ and $p > p^{**}$, B borrows unsecured from C_0; B borrows unsecured from C_1 if $X_1 = X_1^H$ and secured if $X_1 = X_1^L$.*
- *Otherwise, B does not borrow from C_0 or C_1.*

We can now write B's expected payoff at Date 0. Since C_0 and C_1 break even in expectation, B captures the values of the projects he undertakes. Given B borrows unsecured from C_0, his payoff Π_B^u is given by:

$$\Pi_B^u = \begin{cases} X_0 - I_0 + p\left(X_1^H - I_1\right) & \text{if } \theta \le \theta^*, \\ p\left(X_0 + X_1^H\right) + (1-p)\left(\mu(X_0 + X_1^L)\right) - I_0 - I_1 & \text{if } \theta^* < \theta < \theta^{**} \text{ and } p \ge p^*, \\ p\left(X_0 + X_1^H\right) + (1-p)\left(\mu X_0 + X_1^L\right) - I_0 - I_1 & \text{if } \theta \ge \theta^{**} \text{ and } p \ge p^{**}, \\ 0 & \text{otherwise.} \end{cases} \tag{15}$$

4.2 Secured Debt to C_0

Suppose B borrows from C_0 via secured debt with face value F_0, again focusing on the case in which $F_0 \ge I_0$. We maintain the assumption that $F_0 \le \mu X_0$, and we verify that it holds in equilibrium later. We ask when B can borrow from C_1 via unsecured or secured debt.

Unsecured debt to C_1. In that case, the new debt to C_1 is junior to the existing debt to C_0. Thus, C_1 will lend to B via unsecured debt only if the projects' pledgeable cash flow net of the collateralization cost $\theta(\mu X_0 + X_1)$ suffices to repay I_1 to C_1 after having repaid F_0 to C_0, or if

$$I_1 \leq \theta(\mu X_0 + X_1) - F_0. \tag{16}$$

From Assumption 3 and the fact that $F_0 \geq I_0$, we have that if $X_1 = X_1^L$, the pledgeable cash flow that B has left after collateralizing Project 0 and repaying C_0 is less than I_1. Hence we get the following.

LEMMA 4. *If B has secured debt to C_0, then B cannot borrow unsecured from C_1 if $X_1 = X_1^L$.*

Secured debt to C_1. B's ability to borrow from C_1 via secured debt at Date 1 is limited, because B has already collateralized Project 0 to C_0, protecting C_0's claim to its cash flows. Thus, C_1 will lend to B via secured debt only if the pledge able cash flow $\mu(X_0 + X_1)$ generated by the collateralized projects is sufficient pledgeable to repay both I_1 to C_1 and F_0 to C_0, or

$$I_1 \leq \mu\theta(X_0 + X_1) - F_0. \tag{17}$$

Note that this condition is more restrictive than equation (16), the condition for B to borrow from C_1 via unsecured debt.

LEMMA 5. *If B has secured debt to C_0, B will not borrow secured from C_1.*

This is a result of the fact that if B borrows secured from C_0, then all new debt, secured or unsecured, is effectively junior to C_0's debt. As a result, B is better off borrowing unsecured from C_1 than paying the cost $(1 - \mu)X_1$ of collateralizing Project 1.

Subgame equilibrium. If B borrows secured from C_0, that debt is riskless because it has priority over Project 0's pledgeable cash flow. Thus, B can always set $F_0 = I_0$ and, as a result, B can borrow unsecured from C_1 if

$$I_1 \leq \theta\left(\mu X_0 + X_1\right) - I_0 \tag{18}$$

(i.e., if condition (16) holds for $F_0 = I_0$). We can rewrite this condition as follows:

$$\mu \geq 1 - \frac{\theta\left(X_0 + X_1\right) - I_0 - I_1}{\theta X_0}. \tag{19}$$

Given that B never borrows from C_1 via secured debt (Lemma 5) and never borrows from C_1 if the payoff of Project 1 is low (Lemma 4), we can fully characterize B's Date-1 borrowing.

LEMMA 6. *If B has secured debt to C_0 with face value I_0, B borrows from C_1 if and only if $X_1 = X_1^H$ and collateralizability is above a threshold μ^*, given by*

$$\mu^* := 1 - \frac{\theta\left(X_0 + X_1^H\right) - I_0 - I_1}{\theta X_0}. \tag{20}$$

We can now characterize the subgame's equilibrium.

LEMMA 7. *Assume B can only borrow secured from C_0.*

- *If $\mu \geq \mu^*$, B borrows secured from C_0; B borrows unsecured from C_1 if $X_1 = X_1^H$ and does not borrow from C_1 if $X_1 = X_1^L$.*

- *If $\mu < \mu^*$, B borrows secured from C_0 and B does not borrow from C_1.*

We can now write B's expected payoff at Date 0. Given C_0 and C_1's zero-profit condition, B captures the value of the projects he undertakes and his payoff is

$$\Pi_{\mathrm{B}}^s = \begin{cases} \mu X_0 - I_0 + p\left(X_1^H - I_1\right) & \text{if } \mu \geq \mu^*, \\ \mu X_0 - I_0 & \text{otherwise.} \end{cases} \tag{21}$$

4.3 Equilibrium Borrowing

In equilibrium, B borrows from C_0 unsecured if $\Pi_{\mathrm{B}}^u \geq \Pi_{\mathrm{B}}^s$ and secured otherwise. B's equilibrium choice of debt instrument follows from comparing the expression for Π_{B}^u in equation (15) with that for Π_{B}^s in equation (21).

PROPOSITION 3.

- *If $\theta \leq \theta^*$, B borrows unsecured from C_0.*

- *If either $\theta^* < \theta < \theta^{**}$ and $p < p^*$ or $\theta \geq \theta^{**}$ and $p < p^{**}$, B borrows secured from C_0.*

- *Otherwise, B's equilibrium choice of debt depends on the relative inefficiencies of unsecured and secured debt: B borrows unsecured from C_0 if*

$$p(1-\mu)X_0 + pX_1^H + (1-p)\left[1-(1-\mu)\mathbb{1}_{\{\theta^*<\theta<\theta^{**}\}}\right]X_1^L - I_1 \geq \mathbb{1}_{\{\mu\geq\mu^*\}}p\left(X_1^H - I_1\right) \tag{22}$$

and secured otherwise.

This implies that unsecured and secured debt may coexist in equilibrium.

Corollary 1. *Suppose that either $\theta^* < \theta < \theta^{**}$ and $p < p^*$ or $\theta \geq \theta^{**}$ and $p < p^{**}$.*

If condition (22) holds, secured debt and unsecured debt coexist in equilibrium: B borrows unsecured from C_0 and secured from C_1 when $X_1 = X_1^L$.

If condition (22) is violated and $\mu \geq \mu^*$, secured debt and unsecured debt coexist in equilibrium: B borrows secured from C_0 and unsecured from C_1 when $X_1 = X_1^H$.

5 Welfare and Policy

In this section, we first show that the first-best outcome obtains in equilibrium if and only if pledgeability is sufficiently *low*—there is a "paradox of pledgeability." We then show that borrowing via unsecured debt leads to over-investment and borrowing via secured debt leads to under-investment—there is a "collateral overhang" problem. Finally, we suggest that expanding the supply of collateral may have adverse effects, because it can induce a "collateral rat race."

5.1 The Paradox of Pledgeability

Since creditors C_0 and C_1 are competitive, B's equilibrium payoff $\Pi_\text{B} = \max\{\Pi_\text{B}^u, \Pi_\text{B}^s\}$ coincides with the equilibrium surplus. We can now compare the equilibrium surplus with the first-best surplus.

Proposition 4. (Paradox of pledgeability.) *The first-best level of surplus is attained if and only if pledgeability is low enough, i.e., if $\theta \leq \theta^*$.*

The intuition is as follows. An increase in pledgeability θ allows B to pledge more of his cash flows to C_1, making C_1 more willing to lend. This makes it easier for B to take on new debt to C_1. However, this new debt may dilute B's existing debt to C_0. Thus, C_0 becomes less willing to lend. In other words, increasing pledgeability makes it easier to borrow at Date 1 and, hence, paradoxically, makes it harder to borrow at Date 0.

This result follows from the friction of non-exclusive contracts: when B borrows from C_0, he cannot commit not to borrow from C_1. When pledgeablity is low, this friction does not induce an inefficiency because B is too constrained to borrow from C_1 when $X_1 = X_1^L$—low pledgeability makes B's contract with C_0 effectively exclusive, by allowing B to commit not to borrow from C_1 to dilute C_0's debt. Not so when pledgeablity is high.

5.2 Collateral Rat Race

We now turn to the inefficiency of borrowing via unsecured debt, which arises for high pledgeability. If B borrows unsecured from C_0 and pledgeability is high, B can dilute C_0's debt by borrowing secured from C_1 (Lemma 2). As a result, B's investment in Project 1 is subsidized, since B funds it via secured debt to a new creditor, C_1, at the expense of his old creditor, C_0. In other words, undertaking Project 1 is a way for B to syphon off cash flows from C_0. This subsidy distorts B's incentives, inducing B to undertake Project 1 when $X_1 = X_1^L$, even though it has negative NPV.

Proposition 5. *Suppose $\theta > \theta^*$. If B borrows unsecured from C_0, B over-invests in Project 1 when $X_1 = X_1^L$.*

The resulting inefficiency may be so severe that C_0 is unwilling to lend to B unsecured, even though Project 0's pledgeable cash flow exceeds its investment cost—$\theta X_0 > I_0$ (by Assumption 2).

Proposition 6. (Collateral rat race.) *Suppose either $\theta^* < \theta < \theta^{**}$ and $p < p^*$ or $\theta \geq \theta^{**}$ and $p < p^{**}$. C_0 will not lend unsecured to B. This is due to a "collateral rat race," whereby collateralization is required to protect against future collateralization.*

The intuition is as follows. When pledgeability is high, B would fund the low-return Project 1 by borrowing secured from C_1 to dilute his unsecured debt to C_0. B repays C_1 in full, but defaults on his debt to C_0. Hence, C_0 requires collateral to protect against this. In other words, collateralization is required at Date 0 to protect against collateralization at Date 1: there is a collateral rat race.

This suggests that the ability to use collateral can create a friction when it allows a borrower to *selectively* enter into an exclusive contract. This rat race can lead to inefficient underinvestment, as we discuss in the next subsection.

Investment Efficiency (for $\theta^{**} > 1$)

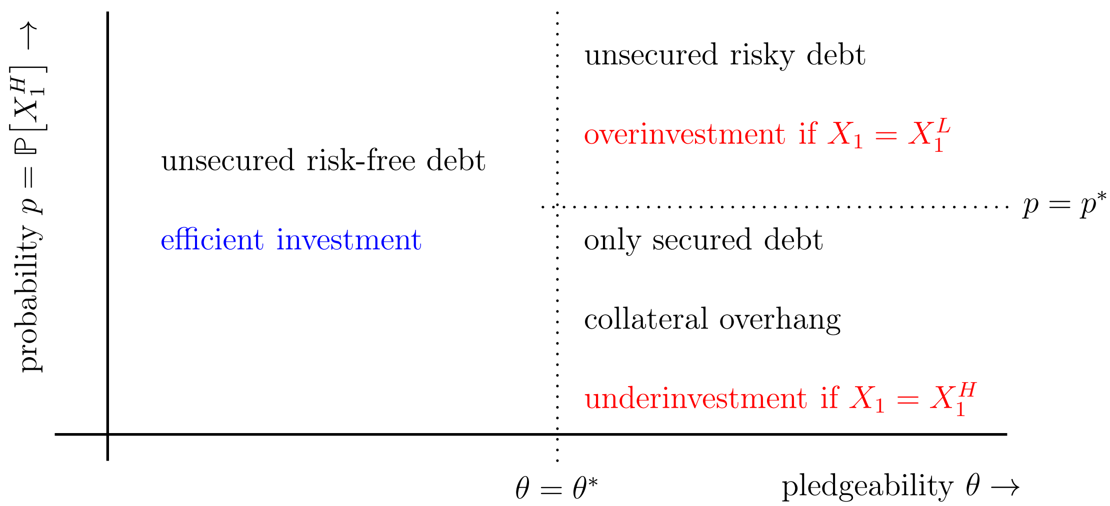

Figure 1: The figure above illustrates B's investment decisions as a function of θ and p. For illustrative purposes, we restrict attention to the case in which $\theta^{**} > 1$. For $\theta < \theta^*$, B takes the efficient action. For $\theta \geq \theta^*$, B over-invests in Project 1 if $p \geq p^*$ and underinvests in Project 1 if $p < p^*$ (cf. Proposition 5 and Proposition 7).

5.3 Collateral Overhang

We now turn to the inefficiency of borrowing via secured debt, which arises for high pledgeability. If B borrows secured from C_0, B pays the collateralization cost $(1 - \mu)X_0$. This cost decreases the surplus to a level below the first-best and it can be amplified in equilibrium because, by collateralizing his project to C_0, B uses up his pledgeable cash flow and thus makes it more difficult to borrow from C_1. In other words, there is a *collateral overhang*, by which collateralizing his project at Date 0 prevents B from borrowing at Date 1. As a result, B may not undertake Project 1, even when it is efficient to do so. Figure 1 depicts which inefficiency arises for different values of the parameters θ and p.

Proposition 7. *(Collateral overhang.) If B borrows from C_0 via secured debt, he can undertake Project 1 when $X_1 = X_1^H$ only if $\mu \geq \mu^*$. Otherwise, collateralizing Project 0 can prevent B from undertaking an efficient Project 1.*

Observe that this collateral overhang kicks in only when collateralizability is below the threshold μ^*. This may seem to suggest that a policy maker should increase collateralizability to prevent this distortion. However, we show next that in fact decreasing collateralizability can increase the surplus.

5.4 Collateral Shortage or Collateral Glut?

We now turn to the effects of varying the collateralizability μ on the surplus.

Proposition 8. *If collateralization is banned, i.e. $\mu = 0$, the first-best surplus is attained in equilibrium.*

The intuition is as follows. For μ sufficiently low, B cannot collateralize his projects to borrow secured from C_1. As a result, B cannot undercut C_0's debt and B's contract with C_0 is effectively exclusive. This leads to the first-best outcome (as in Proposition 2).

This result may cast light on some aspects of the policy debate about which financial assets may be used as collateral in interbank markets as well as how such collateral should be treated in bankruptcy. Within our model, an increase in μ corresponds to an increase in the ease with which assets can be collateralized or as an increase in the total supply of assets that can be used as collateral.[17]

Notably, the special bankruptcy treatment of repo collateral, which makes it effectively super-senior in bankruptcy, corresponds to an increase in μ, since it makes collateralized assets more valuable to creditors. The set of assets eligible for special treatment was expanded in 2005, effectively increasing the supply of repo collateral. Despite this effective increase in the supply of collateral, markets perceived a shortage of collateral. As Caballero (2006) puts it, "The world has a shortage of financial assets. Asset supply is having a hard time keeping up with the global demand for...collateral" (p. 272). Within our model, an increase in μ can also lead to a high dependence on collateral. It makes it easier for B to borrow secured at Date 1, which triggers the collateral rat race, so he must borrow collateralized at Date 0.

6 Extensions and Robustness

In this section, we analyze a number of extensions of our model and confirm the robustness of our main results. First, we include a discussion of covenants arguing that covenants restricting borrowing from third parties—i.e. attempting to circumvent the non-exclusivity friction—may be ineffective, especially for banks.

Second, we analyze a model in which collateral mitigates enforcement problems both between borrowers and creditors and among creditors. Third, we show that the cost $(1-\mu)$ of ring-fencing has an equivalent interpretation as an exogenous haircut on secured debt. Fourth, we relax the assumption that B borrows from C_0 via two-period debt. Fifth, we

[17] We view the supply of collateral in the model as the total cash flow that can used to borrow secured. This is $\mu\theta(X_0 + X_1)$. This is increasing in μ, suggesting an increase in μ corresponds to an increase in the supply of collateral.

study how security design might affect our results, allowing for contingent contracts as well as simple debt. Sixth, we relax the assumption that existing unsecured debt is senior to new unsecured debt.

6.1 Covenants

In this subsection, we discuss the potential use of covenants in our model. We suggest that even though covenants may be effective to mitigate the friction of non-exclusive contracting in some circumstances, their ability to prevent a borrower from taking on new *secured* debt is limited.

The inefficiencies in our model come from the fact that the borrower cannot commit not to dilute its existing debt with new debt, i.e. that contracts are non-exclusive. In reality, debt contracts have so-called "negative pledge covenants," by which a borrower promises its creditor not to borrow from other creditors via secured debt. If such commitments were binding, they could restore efficiency in our model. However, the effectiveness of such covenants is limited. This is because an unsecured creditor holds a claim against only the borrower, not against other creditors. Thus, an unsecured creditor cannot recover collateral that has been seized by a secured creditor. Bjerre (1999) describes these legal restrictions as follows:

> the negative pledge covenant [is a covenant] by which a borrower promises its lender that it will not grant security interests to other lenders. These covenants are common in unsecured loan agreements because they address one of the most fundamental concerns of the unsecured lender: that the borrower's assets will become unavailable to repay the loan, because the borrower will have both granted a security interest in those assets to a second lender and dissipated the proceeds of the second loan. Unfortunately, negative pledge covenants' prohibition of such conduct may be of little practical comfort, because as a general matter they are enforceable only against the borrower, and not against third parties who take security interests in violation of the covenant. Hence, when a borrower breaches a negative pledge covenant, the negative pledgee generally has only a cause of action against a party whose assets are, by hypothesis, already encumbered (pp. 306–307).

The effectiveness of these negative pledge covenants in bankruptcy is especially limited for repo and derivatives liabilities, since these contracts are exempt from automatic stays in bankruptcy—i.e. creditors can liquidate collateral without the approval of the bankruptcy court, making it difficult or impossible for any third party to enforce a claim to the collateral.

Negative pledge covenants may still be useful outside bankruptcy. This is because their violation constitutes a default, and a borrower may adhere to the terms of covenants to avoid a default. However, this may be insufficient to prevent a borrower from taking on debt in general. For example, a borrower in financial distress is likely to default anyway and is therefore willing to violate such covenants to gamble for resurrection by taking on new debt. More generally, it can be difficult to verify that a solvent firm has violated a covenant, especially for complex firms like banks, which may have thousands of counterparties. Indeed, banks effectively do not have to disclose their short-term borrowing:

> There are no specific MD&A requirements to disclose intra-period short-term borrowing amounts, except for [some] bank holding companies [that must] disclose on an annual basis the average, maximum month-end and period-end amounts of short-term borrowings (Ernst & Young (2010)).

There is a another reason that banks in particular may not be able to promise not to dilute existing debt with new debt: the very business of banking constitutes maturity and size transformation, which requires frequent short-term borrowing from many creditors. If a bank agrees to covenants that restrict its ability to borrow in the future, it could undermine its ability to engage in these banking activities. As Bolton and Oehmke (2015) put it:

> debt covenants prohibiting the collateralization...are likely to be...costly to enforce...for financial institutions.... By the very nature of their business, financial institutions cannot assign...collateral to all depositors and creditors, because this would, in effect, erase their value added as financial intermediaries (p. 2356).

This reinforces the idea that non-exclusive contracting is an especially important friction for banks and, therefore, it may add credibility to our thesis that non-exclusive contracting is the reason that interbank markets are heavily reliant on collateral.

6.2 The Two Roles of Collateral

In reality, collateral serves to mitigate enforcement problems both between borrowers and creditors by providing creditors the "right to use" collateral and among creditors by providing some creditor the "right to exclude" others from using collateral. Whereas much of the finance literature has focused on the first role of collateral, we focus on the second. In this subsection, we briefly discuss a model in which both roles of collateral are present. We show that the "right to use" collateral dominates for low pledgeability, whereas the "right to exclude" others from using collateral dominates for high pledgeability.

Consider the following extension of the baseline model. The proportion of pledgeable cash flows is $\theta^s := s\theta$ if B borrows secured and $\theta^u := u\theta$ if B borrows unsecured. We assume not only that collateralization establishes exclusivity, as in the baseline model, but also that collateralization increases pledgeability, i.e. that $\mu\theta^s > \theta^u$ or $\mu s > u$.

We focus on the case in which B always has sufficient pledgeable cash flow to fund Project 0 via secured debt, i.e. $\mu\theta^s X_0 > I_0$. Further, for simplicity, we assume that $p = 0$, so $X_1 = X_1^L$ for sure, but B wants to undertake Project 1 anyway, to benefit from diluting C_0.[18]

PROPOSITION 9. *B collateralizes Project 0 whenever θ is sufficiently small or sufficiently large, i.e.*

$$\theta < \frac{I_0}{uX_0} \quad \text{or} \quad \theta \geq \frac{I_1}{\mu s(X_0 + X_1^L)}. \tag{23}$$

For low θ, B borrows with collateral to increase his pledgeable cash flow—otherwise he could not borrow from C_0 to get Project 0 off the ground. For high θ, B borrows with collateral to offer protection against the claims of other creditors—otherwise he could borrow from C_1 with collateral, diluting C_0's debt, as in the baseline model.

6.3 Collateralization Cost as a Haircut

So far, we have interpreted the cost of collateralization as the cost of ring-fencing assets to protect them from a third party (Subsection 2.2). This cost is important for our collateral overhang result (Proposition 7): because B must pay the cost $(1 - \mu)X$ to collateralize X, collateralization uses up B's pledgeable cash flow. This inhibits his ability to borrow in the future. However, this mechanism is not specific to our interpretation of collateralization as costly ring-fencing. One equivalent interpretation is that B must post a haircut on collateralized debt. To see this, suppose that, in order to borrow I, B must post collateral worth $(1 + m)I > I$. Here, m corresponds to the "margin" and mI corresponds to the haircut. Thus, B can borrow I against a project with cash flow X if its collateral value θX exceeds I plus the haircut mI, i.e. if $\theta X \geq (1 + m)I$ or

$$I \leq \frac{\theta X}{1 + m}. \tag{24}$$

This implies that having to post a haircut mI is equivalent to having to pay the cost of ring-fencing $1 - \mu$. In fact, if the margin $m = (1 - \mu)/\mu$, then the constraint becomes

$$I \leq \frac{\theta X}{1 + m} = \mu\theta X, \tag{25}$$

[18] We take B's incentive to undertake Project 1 as an assumption here (cf. footnote 24). This is just for simplicity, however. An assumption analogous to Assumption 4 would generate this endogenously, as in the baseline model (Proposition 3).

which is just B's constraint to borrow via secured debt in the baseline model.

This analysis implies that posting a haircut leads to the collateral overhang problem just as costly ring-fencing does. Even though B does not pay a deadweight cost to post a haircut like he does to "build" a costly ring-fence, B uses up pledgeable cash flow to post the haircut mI, which tightens his borrowing constraints in the future, potentially leading to underinvestment.

6.4 Short-term Debt

Another possibility we have not considered so far is short-term debt: B could borrow from C_0 via one-period debt and roll over. In this subsection, we show that if debt is required to be renegotiation-proof, then short-term debt cannot improve upon the outcome of the baseline model.

Here we augment the model and suppose that C_0 can lend to B via short-term debt maturing at the end of Date 1, i.e. after B has (potentially) borrowed from C_1 and invested in Project 1. Further, we assume that the debt is subject to renegotiation at Date 1. Specifically, after the debt matures, B can either repay C_0 or offer C_0 an alternative repayment, e.g. he can offer a rescheduling of the debt, so that he repays at Date 2 instead of Date 1.[19] If C_0 accepts B's offer to renegotiate the debt, then B continues his projects. If C_0 reject's B's offer, then C_0 has the right to liquidate. However, since B's projects generate cash flows only at Date 2, we assume that their liquidation value is zero.

PROPOSITION 10. *Suppose B borrows from C_0 via short-term debt. If B borrows from C_1 via secured debt and invests in Project 1 at Date 1, then C_0 prefers to accept a rescheduling of his debt than to liquidate B's assets. I.e. renegotiation-proof short-term debt does not improve on the implementation of long-term contracts.*

6.5 Contingent Debt

So far we have restricted attention to debt contracts, viz. contracts in which the promised repayment is non-contingent. In this subsection, we show that our main results also hold for contingent contracts.[20,21] The inefficiencies in our model result from the fact that contracts

[19] If B and C_0 can also renegotiate *before* B borrows from C_1, then C_0 can write down B's debt to disincentivize dilution when $X_1 = X_1^L$. This can implement the outcome of contingent debt, as discussed in footnote 20.

[20] The results in this subsection imply not only that our results are robust to contingent contracts, but also that they are robust to renegotiable debt: any outcome of renegotiation between B and C_0 at Date 1 can be implemented via contracting contingent on Date-1 information (viz. on the realization of X_1).

[21] It may be worth noting that we analyze only contingent repayments here, not contingent collateralization. However, our results are also robust to contingent collateralization—i.e. B would collateralize his

are non-exclusive, not that they are incomplete (cf. the exclusive contracting benchmark in Subsection 3.2).[22] We focus on debt contracts for simplicity and realism.

Now suppose that B borrows from C_0 via unsecured contingent debt, i.e. B borrows I_0 from C_0 in exchange for the contingent repayments F_0^H when $X_1 = X_1^H$ and F_0^L when $X_1 = X_1^L$. If the payoff of Project 1 is low, i.e. $X_1 = X_1^L$, then B has the incentive to borrow from C_1 via secured debt, diluting C_0's debt.[23] In this event, C_0 is not repaid in full (Assumption 2). In the baseline analysis, C_0 must require collateral to protect against being diluted. Now, with contingent debt, C_0 can protect against being diluted in another way: C_0 can lower the repayment F_0^L when $X_1 = X_1^L$. In particular, if F_0^L is sufficiently low, then the benefits of diluting C_0—and thus avoiding repaying F_0^L—may not compensate for the costs of doing a negative-NPV investment. In other words, if B's promised repayment to C_0 is sufficiently low, then it may be incentive compatible for B not to borrow from C_1.[24] Formally, B's payoff from not borrowing from C_1 and repaying F_0^L to C_0 must exceed his payoff from borrowing from C_1, diverting the fraction $1-\theta$ of his cash flows, and defaulting, i.e. the following incentive constraint must be satisfied[25]

$$X_0 - F_0^L \geq (1-\theta)\mu\left(X_0 + X_1^L\right). \tag{26}$$

This constraint imposes an upper bound on the repayment F_0^L:

$$F_0^L \leq X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right). \tag{27}$$

This expression is less than the cost I_0 of Project 0. This implies that for any incentive-compatible contract, C_0 is not repaid as much as it lent when $X_1 = X_1^L$ (given that θ is high

project to C_0 only if pledgeability is sufficiently high—but with the caveat that B collateralizes only when $X_1 = X_1^L$, not when $X_1 = X_1^H$. This is because C_0 is effectively never diluted when $X_1 = X_1^H$ and thus does not require collateral to protect against dilution.

[22] This finding that the "collateral overhang" of secured credit cannot be resolved by contingent contracting/renegotiation complements Bhattacharya and Faure-Grimaud's (2001) finding that when a firm's investments are non-contractible, renegotiation between borrowers and creditors may not resolve the debt-overhang problem.

[23] This will be feasible whenever pledgeability is high, $\theta \geq \theta^*$, as in Lemma 2, which holds independently of whether debt is contingent or not.

[24] Observe that, with the current setup, contingent contracting can only help insofar as it decreases B's incentive to undertake Project 1 when $X_1 = X_1^L$. However, it might also be reasonable to assume that B always wants to undertake new projects, e.g. because he gets private benefits from new investments. This is the setup in Hart and Moore (1995), in which "management's empire-building tendencies are sufficiently strong that it will always undertake the new investment if it can, even if the investment has negative net present value" (p. 568). Under this alternative assumption, allowing for contingent debt does not change the baseline analysis at all.

[25] We restrict attention to the case in which $I_1 < \theta\mu X_0$, so that B must collateralize both projects to borrow from C_1. We do this just to keep the analysis streamlined and not consider two separate cases.

enough that B can borrow from C_1). So C_0 is repaid in full only with probability p, i.e. in the event that $X_1 = X_1^H$. Hence, if p is sufficiently low, C_0 will not lend to B via unsecured contingent debt, but rather will require collateral. This implies that our main results are robust to allowing for contingent contracts, as the next proposition summarizes.

PROPOSITION 11. *Suppose that p is relatively small,*[26]

$$p < p^{\text{c.d.}} := \frac{I_0 - \Big(X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big)}{\theta\big(X_0 + X_1^H\big) + (1-\theta)\mu\big(X_0 + X_1^L\big) - X_0 - I_0}. \tag{28}$$

- *If $\theta \leq \theta^*$ as defined in equation (11), then B borrows from C_0 via unsecured risk-free debt with face value $F_0^u = I_0$; B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$, and does not borrow from C_1 if $X_1 = X_1^L$.*

  The first-best surplus is attained in equilibrium.

- *If $\theta > \theta^*$ and $p < p^{\text{c.d.}}$, then B cannot borrow from C_0 via unsecured debt (even if the debt is contingent).*

6.6 Pari Passu Debt

We now argue that our result that increasing pledgeability leads to more collateralized borrowing—the paradox of pledgeability—does not depend on the assumption that new unsecured debt is effectively senior to old unsecured debt. Increasing pledgeability can increase the use of collateral even if collateral cannot be used to establish priority over existing debt. The result obtains as long as taking on new debt has some negative effect on old debt.[27] We show this by considering the case of pari passu debt in detail.

Here we focus on the case in which all unsecured debt is treated equally (pari passu). Consider the following twist on the baseline model. At Date 1, B cannot borrow from C_1 via secured debt, for example because it is too late to collateralize assets or because secured debt is not legally prioritized over existing debt.[28] But B can borrow from C_1 via pari passu unsecured debt, i.e. if B defaults with unsecured debt to C_0 with face value F_0 and unsecured debt to C_1 with face value F_1, each creditor is repaid a pro rata fraction of B's pledgeable cash flows. Thus, if B defaults after undertaking both Project 0 and Project 1,

[26] The cutoff $p^{\text{c.d.}}$ in equation (11) is always between zero and one by Assumption 3 and Assumption 4.

[27] Even so, we argue in Subsection 6.1 below, that our results apply most pertinently in the baseline case, in which new secured debt does have priority over old unsecured debt.

[28] In many circumstances, such as the interbank market, this assumption may not be realistic. Secured debt typically has legal priority, as in the baseline model. See, e.g., Bjerre (1999), as well as the other legal literature cited in the Introduction and the discussion of covenants below.

the repayment to $\mathrm{C}_t \in \{\mathrm{C}_0, \mathrm{C}_1\}$ is as follows:

$$\text{repayment to } \mathrm{C}_t \;= \frac{F_t}{F_0 + F_1}\theta(X_0 + X_1). \tag{29}$$

If B undertakes Project 1 when $X_1 = X_1^L$, B's portfolio of projects $X_0 + X_1$ does not generate sufficient pledgeable cash flow to cover the costs of the projects $I_0 + I_1$ (Assumption 3), so B must default. However, B may still be able to borrow from C_1 via unsecured debt by diluting his debt to C_0. Specifically, B can borrow from C_1 whenever the repayment it recieves in the event of default is greater than I_1. Using equation (29) above, this says that, given $X_1 = X_1^L$, B can borrow I_1 from C_1 via debt with face value F_1 as long as

$$\theta \geq \frac{F_0 + F_1}{F_1}\frac{I_1}{X_0 + X_1^L}. \tag{30}$$

Here, B promises C_1 a high face value F_1 to dilute C_0's claim, effectively subsidizing B's investment in Project 1, just as in the baseline case with secured borrowing. This is feasible if B can offer C_1 a sufficiently high face value F_1 to ensure C_1 is repaid in full even in the event of default (even though C_1's debt is not prioritized in bankruptcy). In other words, despite the fact that C_0 is supposedly on equal footing with C_1 in bankruptcy, C_1's debt has diluted C_0's debt so severely that C_1's debt is in fact risk free. Mathematically, B can borrow from C_1 as long as F_1 is sufficiently high to satisfy inequality (30). Since $(F_0 + F_1)/F_1 \to 1$ as $F_1 \to \infty$, inequality (30) is satisfied if and only if pledgeability θ is sufficiently large, or

$$\theta > \theta^{\text{p.p.}} := \frac{I_1}{X_0 + X_1^L}. \tag{31}$$

Thus, if pledgeablity is sufficiently high, B borrows from C_1 via unsecured debt. We can solve for the face value by setting the repayment to C_1 equal to I_1 in equation (29):

$$F_1 = \frac{I_1 F_0}{\theta\left(X_0 + X_1^L\right) - I_1}. \tag{32}$$

Observe that B defaults on his debt to C_1 and repays $I_1 < F_1$. However, the debt is still "risk free" in the sense that C_1 has a deterministic return equal to the risk-free rate (zero).

Now turn to B's debt to C_0. Since C_1 is always repaid I_1, the repayment to C_0 if $X_1 = X_1^L$ is given by the total pledgeable cash flow $\theta(X_0 + X_1^L)$ less the repayment I_1 that is made to C_1 (supposing that it is positive), i.e.

$$\text{repayment to } \mathrm{C}_0 = \theta\left(X_0 + X_1^L\right) - I_1. \tag{33}$$

This is less than I_0 by Assumption 2. Thus, if B borrows from C_0 via unsecured debt, B repays C_0 less than I_0 whenever $X_1 = X_1^L$. Thus, if the probability $1-p$ that $X_1 = X_1^L$ is high, C_0 is rarely repaid. As a result, C_0 will not lend to B via unsecured debt, but only via secured debt. In other words, the paradox of pledgeability also holds with pari passu debt. This is the next proposition.

Proposition 12. *Suppose that p is relatively small,*[29]

$$p < p^{\text{p.p.}} := \frac{I_0 + I_1 - \theta\left(X_0 + X_1^L\right)}{\theta\left(X_1^H - X_1^L\right)}. \tag{34}$$

- *If $\theta \leq \theta^{\text{p.p.}}$ as defined in equation (31), then B borrows from C_0 via unsecured risk-free debt with face value $F_0^u = I_0$; B borrows from C_1 via risk-free unsecured debt if $X_1 = X_1^H$ and does not borrow from C_1 if $X_1 = X_1^L$.*

  The first-best surplus is attained in equilibrium.

- *If $\theta > \theta^{\text{p.p.}}$, then B cannot borrow from C_0 via unsecured debt.*

This result demonstrates that the driving force in our model is not the borrower's ability to use collateral to establish priority over existing debt, but rather the borrower's ability to take on new debt more generally, i.e. the fact that contracts are non-exclusive. However, in reality creditors take contractual measures to approximate exclusive relationships with their borrowers. Notably, they impose covenants in debt contracts that restrict future borrowing. These covenants offer limited protection against future *secured* borrowing, however, for reasons we discuss in the next subsection.

7 Conclusion

We have considered a model in which collateral serves to protect creditors against dilution with new debt. High pledgeability increases the risk of dilution, since it makes it easy to take on new secured debt and thus, paradoxically, makes creditors less willing to lend unsecured. Collateralization is required is to protect against future collateralization—there is a collateral rat race.

This reliance on collateral leads to a collateral overhang problem, whereby collateralized assets are encumbered and cannot be used to raise liquidity. We find that increasing the

[29] The cutoff $p^{\text{p.p.}}$ in equation (34) equals the cutoff p^* in equation (13) if collateralizability $\mu = 1$. This reflects the fact that pari passu debt allows B to effectively prioritize C_1's debt without bearing the cost of collateralization. It follows from Assumption 2 that the cutoff is always between zero and one.

supply of collateral or deviating from the absolute priority rule may mitigate this problem, by preventing the collateral rat race from getting started.

A Proofs

A.1 Proof of Proposition 1

The argument is in the text. □

A.2 Proof of Proposition 2

Suppose B borrows from C_0 at the risk-free rate, $F_0 = I_0$. Since the contract with C_0 is exclusive, B must borrow from C_0 at Date 1. By Assumption 2, B can borrow from $C_1 = C_0$ if and only if $X_1 = X_1^H$, since C_0 lends at Date 1 only if its total surplus from the *two* loans increases. Thus, he can undertake Project 1 if and only if $X_1 = X_1^H$. In summary, B invests in Project 0 and invests in Project 1 when $X_1 = X_1^H$.

TO COMPLETE

□

A.3 Proof of Lemma 1

The argument is in the text. □

A.4 Proof of Lemma 2

Suppose B borrows from C_0 via unsecured debt with face value F_0. If $X_1 = X_1^L$, unsecured borrowing from C_1 is impossible (by Lemma 1), but secured borrowing is possible provided that condition (10) holds, i.e. if $\theta > \theta^*$. To prove that B borrows secured from C_1 when $\theta > \theta^*$, we compare B's payoff from doing so with his payoff from not borrowing from C_1. We assume for now that $F_0 \leq \theta X_0$ and verify this in the proof of Proposition 3 below (cf. equations (42) and (49)).

If B borrows secured from C_1 the pledgeable cash flows are insufficient to repay both C_0 and C_1, so he defaults (by Assumption 3). If B does not borrow from C_1, Project 0's pledgeable cash flow suffices to repay C_0, his only creditor, so he does not default (this follows from $F_0 \leq \theta X_0$). Thus, B prefers to borrow from C_1 via secured debt as long as

$$\mu(1-\theta)\big(X_0 + X_1^L\big) > X_0 - F_0 \tag{35}$$

which is always satisfied given Assumption 4 and the fact that $F_0 \geq I_0$.

A.5 Proof of Proposition 3

Frist, we prove a preliminary result that we employ later.

LEMMA 8. *Suppose B has borrowed from C_0 via unsecured debt and that B can borrow from C_1 via unsecured debt and not default, i.e.*

$$\theta\big(X_0 + X_1\big) \geq F_0 + I_1. \tag{36}$$

B prefers to borrow from C_1 via unsecured debt than via secured debt.

Proof. Here we suppose that B has unsecured debt to C_0 with face value F_0 and we compare B's payoff from borrowing from C_1 via unsecured debt and via secured debt.

If B borrows from C_1 via unsecured debt, he does not default by assumption (equation (36)). Thus, his payoff is

$$\Pi_{\text{B}}^{\text{unsec.}} = X_0 + X_1 - F_0 - I_1. \tag{37}$$

Observe that this is larger than the payoff if B defaults and diverts the fraction $1 - \theta$ of his cash flow:

$$X_0 + X_1 - F_0 - I_1 = \theta\big(X_0 + X_1\big) + (1-\theta)\big(X_0 + X_1\big) - F_0 - I_1 \tag{38}$$

$$\geq (1-\theta)\big(X_0 + X_1\big), \tag{39}$$

since, by the no-default assumption, $\theta\big(X_0 + X_1\big) \geq F_0 + I_1$.

Now turn to the case in which B borrows from C_1 via secured debt. In this case, he may or may not default with C_0. Denoting the total final payoff by W, as in equation (2), B's payoff is

$$\Pi_{\text{B}}^{\text{sec.}} = \max\left\{W - F_0 - I_1\, ,\, (1-\theta)W\right\}. \tag{40}$$

We can see immediately that this is less than $\Pi_{\text{B}}^{\text{unsec.}}$ above as follows: if B borrows secured, then $W < X_0 + X_1$, since $\mu < 1$. Thus, first term in the max function is less than the expression in equation (37) and the second term in the max function is less than the expression in equation (39). □

We now proceed with the construction of the equilibrium, given that B borrows from C_0 via unsecured debt. We break the proof up for different regions of the parameter space: we analyze first the case in which θ is low, then the case in which θ is high and p is high, and finally the case in which θ is high and p is low.

Low pledgeability: $\theta \leq \theta^*$. For $\theta \leq \theta^*$, we proceed as follows. We assume that B borrows from C_0 via risk-free debt. We show that B borrows from C_1 via risk-free junior debt when $X_1 = X_1^H$ and does not borrow from C_1 when $X_1 = X_1^L$. We confirm that B's initial debt to C_0 is indeed risk free.

Suppose that $\theta \leq \theta^*$ and that B borrows from C_0 via risk-free debt, so that $F_0 = I_0$. If $X_1 = X_1^H$, then B has sufficient pledgeable cash flow to borrow from C_1 via unsecured risk-free debt by Assumption 3 which says $\theta(X_0 + X_1^H) \geq I_0 + I_1$. By Lemma 8 above, B indeed borrows via unsecured debt rather than secured debt.

If $X_1 = X_1^L$, B cannot borrow from C_1 via unsecured debt (by Lemma 1) or via secured debt (by Lemma 2).

We now show that B's debt to C_0 is indeed risk free. First observe that if $X_1 = X_1^H$, then B repays both C_0 and C_1 since $\theta(X_0 + X_1^H) \geq I_0 + I_1 = F_0 + F_1$, having used Assumption 3 and the fact that the risk-free rate is zero. Now observe that when $X_1 = X_1^L$, B repays C_0 since B does not borrow from C_1 (since θ is low) and $\theta X_0 > I_0$ by Assumption 2.

High pledgeability and high probability that $X_1 = X_1^H$. Recall that B borrows secured when $X_1 = X_1^L$ (Lemma 2). There are three ways to borrow secured: (i) collateralize only Project 1, (ii) collateralize only Project 0, and (iii) collateralize both projects. Case (i) is infeasible because $\mu\theta X_1 < I_1$. Case (ii) is preferable to case (iii) because the deadweight loss from collateralization is lower. We now show that case (ii) arises when $\theta \geq \theta^{**}$ and $p \geq p^{**}$ and that case (iii) arises when $\theta^* < \theta < \theta^{**}$ and $p \geq p^*$.

(ii) For $\theta > \theta^{**}$ and $p \geq p^{**}$, we proceed as follows. We assume that B borrows from C_0 via risky debt with face value F_0, where

$$I_0 < F_0 \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{41}$$

We then show that, given this condition, B borrows from C_1 via risk-free junior debt when $X_1 = X_1^H$ and borrows from C_1 via risk-free secured debt when $X_1 = X_1^L$. We confirm that the face value F_0 of B's initial debt to C_0 is indeed in the range specified in equation (41).

Suppose that B borrows from C_0 via risky debt, so $F_0 > I_0$. Suppose also that F_0 is lower than the upper bound in equation (41) above. Note that this implies that $F_0 < \theta X_0$ since, simply rearranging equation (41) implies that

$$F_0 < \theta X_0 + \big(\theta X_1^H - I_1\big) \tag{42}$$

and $\theta X_1^H < I_1$ by Assumption 2. If $X_1 = X_1^H$, then B has sufficient pledgeable cash flow to borrow from C_1 via unsecured risk-free debt by the hypothesis in equation (41). By Lemma 8 above, B indeed borrows via unsecured debt rather than secured debt.

Thus, B is repaid in full if $X_1 = X_1^H$.

If $X_1 = X_1^L$, B borrows secured from C_1 (collateralizing only Project 0) and invests in the negative NPV project (by Lemma 2). Thus, B defaults on his debt to C_0 when $X_1 = X_1^L$. C_0 gets the pledgeable cash flow after B has repaid C_1:

$$\text{repayment to } C_0 \text{ if } X_1^L \; = \theta\big(\mu X_0 + X_1^L\big) - I_1. \tag{43}$$

We now show that the face value F_0 of B's debt to C_0 is in the range given in equation (41). The fact that $F_0 > I_0$ follows from the fact that B defaults when $X_0 = X_1^L$, since $\theta\big(\mu X_0 + X_1^L\big) - I_1 < I_0$ by Assumption 2. We now show that F_0 is less than the upper bound in equation (41). Given the analysis above, C_0's break-even condition reads

$$I_0 = pF_0 + (1-p)\left(\theta\big(\mu X_0 + X_1^L\big) - I_1\right) \tag{44}$$

so

$$F_0 = \frac{I_0 - (1-p)\Big(\theta\big(\mu X_0 + X_1^L\big) - I_1\Big)}{p}. \tag{45}$$

Thus, F_0 is less than the required upper bound whenever

$$\frac{I_0 - (1-p)\Big(\theta\big(\mu X_0 + X_1^L\big) - I_1\Big)}{p} \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{46}$$

We can rewrite this condition as

$$p \geq \frac{I_0 + I_1 - \theta\big(\mu X_0 + X_1^L\big)}{\theta\big(X_0 + X_1^H\big) - \theta\big(\mu X_0 + X_1^L\big)} \equiv p^{**}, \tag{47}$$

which is satisfied by assumption.

(iii) For $\theta^* < \theta < \theta^{**}$ and $p \geq p^*$, we proceed as follows. We assume that B borrows from C_0 via risky debt with face value F_0, where

$$I_0 < F_0 \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{48}$$

We then show that, given this condition, B borrows from C_1 via risk-free junior debt when $X_1 = X_1^H$ and borrows from C_1 via risk-free secured debt when $X_1 = X_1^L$. We confirm that the face value F_0 of B's initial debt to C_0 is indeed in the range specified in equation (48).

Suppose that B borrows from C_0 via risky debt, so $F_0 > I_0$. Suppose also that F_0 is lower than the upper bound in equation (48) above. Note that this implies that $F_0 < \theta X_0$ since, simply rearranging implies that

$$F_0 < \theta X_0 + \big(\theta X_1^H - I_1\big) \tag{49}$$

and $\theta X_1^H < I_1$ by Assumption 2. If $X_1 = X_1^H$, then B has sufficient pledgeable cash flow to borrow from C_1 via unsecured risk-free debt by the hypothesis in equation (48). By Lemma 8 above, B indeed borrows via unsecured debt rather than secured debt. Thus, B is repaid in full if $X_1 = X_1^H$.

If $X_1 = X_1^L$, B borrows secured from C_1 (collateralizing both Project 0 and Project 1) and invests in the negative NPV project (by Lemma 2). Thus, B defaults on his debt to C_0 when $X_1 = X_1^L$. C_0 gets the pledgeable cash flow after B has repaid C_1:

$$\text{repayment to } C_0 \text{ if } X_1^L \ = \theta\mu\big(X_0 + X_1^L\big) - I_1. \tag{50}$$

We now show that the face value F_0 of B's debt to C_0 is in the range given in equation (48). The fact that $F_0 > I_0$ follows from the fact that B defaults when $X_0 = X_1^L$, since $\theta\mu\big(X_0 + X_1^L\big) - I_1 < I_0$ by Assumption 2. We now show that F_0 is less than the upper bound in equation (48). Given the analysis above, C_0's break-even condition reads

$$I_0 = pF_0 + (1-p)\left(\theta\mu\big(X_0 + X_1^L\big) - I_1\right) \tag{51}$$

so

$$F_0 = \frac{I_0 - (1-p)\Big(\theta\mu\big(X_0 + X_1^L\big) - I_1\Big)}{p}. \tag{52}$$

Thus, F_0 is less than the required upper bound whenever

$$\frac{I_0 - (1-p)\Big(\theta\mu\big(X_0 + X_1^L\big) - I_1\Big)}{p} \leq \theta\big(X_0 + X_1^H\big) - I_1. \tag{53}$$

We can rewrite this condition as

$$p \geq \frac{I_0 + I_1 - \theta\mu\big(X_0 + X_1^L\big)}{\theta\big(X_0 + X_1^H\big) - \theta\mu\big(X_0 + X_1^L\big)} \equiv p^*, \tag{54}$$

which is satisfied by assumption.

High pledgeability and low probability that $X_1 = X_1^H$. For high θ and low p, we proceed as follows. We first explain that the analysis above implies that B defaults when $X_1 = X_1^L$ and therefore B must repay $F_0 > \theta\big(X_0 + X_1^H\big) - I_1$ when $X_1 = X_1^H$, given that p is small. We then argue that this repayment is infeasible.

The analysis of cases (ii) and (iii) above implies that B borrows from C_1 when $X_1 = X_1^L$ and defaults on his debt to C_0, making a repayment less than I_0 (given in equations (43) and (50)).

C_0's break-even condition implies that F_0 must be larger than $\theta\big(X_0 + X_1^H\big) - I_1$ (this is implied by equation (47) and (54) and the analysis that precedes them). Thus, F_0 must be so high that B cannot borrow from C_1 via unsecured debt if $X_1 = X_1^H$. If B borrows via unsecured debt, B defaults on his debt to C_0 and C_0's break-even condition is violated. □

A.6 Proof of Lemma 4

The argument is in the text. □

A.7 Proof of Lemma 5

The argument is in the text. □

A.8 Proof of Lemma 6

The argument is in the text. □

A.9 Proof of Lemma 7

The proof proceeds as follows. We first recall that if B borrows from C_0 via secured debt, then (i) B's debt to C_0 is risk free and (ii) B does not borrow from C_1 when $X_1 = X_1^L$. We then analyze what happens when $X_1 = X_1^H$, which depends on the collateralizability μ.

If B borrows from C_0 via secured debt, C_0 is effectively always a senior clamant on the pledgeable collateralized cash flows from Project 0, $\theta\mu X_0$. Since this is greater than the cost I_0 of Project 0 (by Assumption 2), B can borrow from C_0 risk free.

Recall also that Lemma 4 says that if B borrows from C_0 via risk-free secured debt, then B does not borrow from C_1 when $X_1 = X_1^L$.

We now analyze what happens when $X_1 = X_1^H$. Given that B has borrowed from C_0 via secured debt, he never borrows from C_1 via secured debt (by Lemma 4). If B borrows from C_1 via unsecured debt, C_1 is effectively junior to C_0. Thus, C_0 lends only if B's pledgeable cash flow *net repayment to* C_0 exceeds the cost of Project 1, or

$$I_1 \leq \theta\big(\mu X_0 + X_1^H\big) - I_0 \tag{55}$$

or

$$\mu \geq 1 - \frac{\theta\big(X_0 + X_1^H\big) - I_0 - I_1}{\theta X_0} \equiv \mu^*. \tag{56}$$

Thus, when $\mu \geq \mu^*$, B borrows from C_1 and invests in Project 1 when $X_1 = X_1^H$, but when $\mu < \mu^*$, B is constrained and does not invest in Project 1 when $X_1 = X_1^H$. □

A.10 Proof of Lemma 3

The result follows immediately from comparing the expression for Π_B^u in equation (15) with the expression for Π_B^s in equation (21).

A.11 Proof of Corollary 1

The statement follows immediately from Lemma 3.

A.12 Proof of Proposition 4

The argument is in the text. □

A.13 Proof of Proposition 5

The argument is in the text. □

A.14 Proof of Proposition 6

The argument is in the text. □

A.15 Proof of Proposition 7

The argument is in the text. □

A.16 Proof of Proposition 8

The argument is in the text. □

A.17 Proof of Proposition 12

Much of the argument is already in the text. However, there are a few gaps to fill in. Most importantly, we argued that C_0 does not lend unsecured if the probability p that Project 1 has the high payoff is sufficiently small. It remains to show that any $p < p^{\text{p.p.}}$ is indeed "sufficiently small." Below we complete the proof. We first summarize the case in which $\theta < \theta^{\text{p.p.}}$ (as defined in equation (31)) and then proceed to analyze the case in which $\theta \geq \theta^{\text{p.p.}}$.

Low pledgeability: $\theta < \theta^{\text{p.p.}}$. When $\theta < \theta^{\text{p.p.}}$, B cannot borrow when $X_1 = X_1^L$, as shown in analysis leading up to equation (31). In contrast, when $X_1 = X_1^H$, B borrows via risk-free unsecured debt. To see this, note that B prefers to borrow via unsecured debt than via secured debt (by Lemma 8) and that B has sufficient pledgeable cash flow to borrow (by Assumption 2). Thus, C_0 and C_1 both lend via risk-free unsecured debt, as stated in the proposition.

High pledgeability: $\theta \geq \theta^{\text{p.p.}}$. For $\theta \geq \theta^{\text{p.p.}}$, we proceed as follows. We first analyze B's repayments to C_0 and C_1 when $X_1 = X_1^L$. We show that B does not repay C_0 in full. We then ask under what circumstances B can promise C_0 a high enough repayment when $X_1 = X_1^H$ to offset this loss when $X_1 = X_1^L$. This analysis gives the threshold $p^{\text{p.p.}}$ given in the proposition.

When $X_1 = X_1^L$, B can borrow from C_1 via secured debt, as shown in analysis leading up to equation (31). Further, recall that B cannot borrow via unsecured debt (by Assumption 2) and, further, that B prefers to borrow than not to borrow (by Assumption 4). Given that C_1 breaks even, B's repayment to C_0 when $X_1 = X_1^L$ is given by B's total pledgeable cash flow minus the repayment I_1 to C_1:

$$\text{repayment to } C_0 \text{ if } X_1^L \; = \theta\left(X_0 + X_1^L\right) - I_1 \tag{57}$$

as shown in the text (equation (33)). This is less than I_0 by Assumption 2. Thus, it constitutes a default on C_0's debt. We now ask whether B can promise to repay C_0 enough when $X_1 = X_1^H$ to compensate C_0 for this loss when $X_1 = X_1^L$.

When $X_1 = X_1^H$, B makes the repayment F_0 to C_0. F_0 must satisfy two conditions (i) C_0's break-even condition and (ii) B's limited liability constraint if $X_1 = X_1^H$ (where by "limited liability constraint" we mean that B's total repayment to all his creditors cannot exceed his pledgeable cash flow). C_0's break-even condition reads:

$$I_0 = pF_0 + (1-p)\Big(\theta\left(X_0 + X_1^L\right) - I_1\Big), \tag{58}$$

having substituted in from equation (57) above. B's limited liability constraint if $X_1 = X_1^H$ reads:[30]

$$\theta\big(X_0 + X_1^H\big) \geq F_0 + F_1 = F_0 + I_1. \tag{59}$$

Substituting the expression for F_0 implied by the break-even condition in equation (58) into this the limited liability constraint above implies that we must have

$$\theta\big(X_0 + X_1^H\big) \geq \frac{I_0 - (1-p)\Big(\theta\big(X_0 + X_1^L\big) - I_1\Big)}{p} + I_1 \tag{60}$$

which can be re-written as

$$p \geq \frac{I_0 + I_1 - \theta\big(X_0 + X_1^L\big)}{\theta\big(X_1^H - X_1^L\big)} \equiv p^{\text{p.p.}}, \tag{61}$$

where $p^{\text{p.p.}}$ is defined in equation (34). Thus, for $\theta \geq \theta^{\text{p.p.}}$ and $p < p^{\text{p.p.}}$, B cannot borrow from C_0, as stated in the proposition. □

A.18 Proof of Proposition 11

In this proof we argue that for high θ the incentive constraint puts an upper bound on B's repayment to C_0 when $X_1 = X_1^L$. This upper bound is less than the size of C_0's loan I_0, so if C_0 lends unsecured, it must take a loss when $X_1 = X_1^L$. If the probability $1-p$ that $X_1 = X_1^L$ is sufficiently large than C_0 will not lend to B via unsecured debt.

First observe that this incentive constraint can bind only if pledgeability is high. If $\theta \leq \theta^*$ as in Proposition 3, then B cannot borrow from C_1, so C_0 does not risk dilution.

For high pledgeability, $\theta > \theta^*$, in contrast, the incentive constraint in equation (26) puts an upper bounds on B's repayment if $X_1 = X_1^L$.

$$F_0^L \leq X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right). \tag{62}$$

Thus, for any feasible repayment $F_0^H \geq I_0$,[31] we can substitute this upper bound into C_0's

[30] Here we have tacitly assumed that B undertakes Project 1 when $X_1 = X_1^H$. This is implied by Assumption 5. See the the proof of Proposition 3 for further explanation.

[31] It is without loss of generality to restrict attention to the case in which $F_0^H \geq I_0$. Otherwise, C_0 is repaid less than I_0 not only when $X_1 = X_1^L$ but also when $X_1 = X_1^H$ and C_0 will not lend unsecured.

break-even condition to find a necessary condition for C_0 to lend to B via unsecured debt:

$$I_0 = pF_0^H + (1-p)F_0^L \tag{63}$$
$$\leq pF_0^H + (1-p)\Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big]. \tag{64}$$

Observe that Assumption 4 says that the term in square brackets above is less than I_0,

$$I_0 - \Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big] > 0. \tag{65}$$

Thus, we can rewrite the necessary condition as

$$p \geq \frac{I_0 - \Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big]}{F_0^H - \Big[X_0 - (1-\theta)\mu\left(X_0 + X_1^L\right)\Big]}. \tag{66}$$

Observe that the right-hand side above is positive. Thus, p must be sufficiently large in order for C_0 to lend to B via unsecured debt. In other words, given that $\theta > \theta^*$, for small p C_0 lends only via secured debt, as desired.

The expression for the cutoff $p^{\text{c.d.}}$ in equation (28) comes from considering the loosest lower bound in equation (66) above. This follows by considering the largest feasible repayment $F_0^H = \theta(X_0 + X_1^H) - I_1$. □

A.19 Proof of Proposition 10

The result follows immediately from the fact that B has no cash flows at Date 0, so C_0 has zero recovery value in the event of liquidation. Thus, C_0 always prefers to accept a rescheduling to Date 2 than to liquidate at Date 1. Hence, renegotiation-proof one-period contracts do not improve on the two-period contracts we focus on in the baseline model.[32]

A.20 Proof of Proposition 9

B can finance Project 0 only if his pledgeable cash flow exceeds I_0. B borrows from C_0 via unsecured debt if (i) Project 0's unsecured pledgeable cash flows are sufficient to cover the investment and (ii) C_0 is not at risk of dilution by the new debt to C_1. Condition (i) says that

$$\theta^u X_0 > I_0 \tag{67}$$

[32] This result is subject to the caveats about the timing of renegotiation in footnote 20 and about contingent debt in Subsection 6.5.

and condition (ii) says that

$$\mu\theta^s(X_0 + X_1^L) \leq I_1. \tag{68}$$

Substituting $\theta^u = u\theta$ and $\theta^s = s\theta$ gives the conditions in the proposition.

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