Is One Share/One Vote Optimal? *

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Abstract

In a tender offer by a value increasing raider, voting shareholders face a free-rider problem. However, when they are not atomistic, they do not completely free-ride. In contrast, non-voting shareholders, who are never pivotal for the success of the offer, are absolute free-riders. Hence in this case there is a gain from departing from one share/one vote. This departure also has a cost; there is an increased vulnerability to value decreasing raiders, and the optimal governance structure balances the cost and the gain.

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Introduction

Should firms be allowed to issue several kinds of equity which differ in their voting rights? Should instead a one share/one vote rule prevail? That there is fierce controversy over the dual class share system is illustrated by the variety of regulations across countries which either allow, regulate or simply ban shares with restricted voting rights. For instance, it is remarkable that while the New York Stock Exchange removed its one share/one vote requirement in 1986, the EC is planning to restrict the use of dual class shares. ¹ The issue is also contentious amongst investors. ²

Analysing the one share/one vote rule in the context of takeovers on widely held firms, Grossman and Hart (1988) write: ‘One share-one vote maximises the importance of security benefits to security holders relative to benefits to the controlling party and hence encourages the selection of an efficient management team. However, one share-one vote does not always maximize the reward to securityholders in a corporate control contest.’ ³ Their point relies on the assumption that each shareholder is atomistic, in the sense that he believes that his votes can never affect the success of a tender offer.

Our paper shows that, without this assumption, departing from one share/one vote can be optimal even when there is no contest. The starting point is the coordination problem faced by shareholders in a tender offer. Suppose that all shares have voting rights and that a raider who would improve the value of the firm makes a take-it-or-leave-it bid. If the bid price reflects only partially the potential improvement, each shareholder would prefer to hold on to his shares and free-ride on the full improvement (Grossman and Hart 1980). However, if every shareholder does so, the bid fails. Realising that his vote may be pivotal for the offer’s success, a voting shareholder does not completely free-ride and so the raider is able to set a bid price that does not fully reflect the improvement (Bagnoli and Lipman 1988, Bebchuk 1989, Holmström and Nalebuff 1992). In contrast, non-voting shareholders are absolute free-riders. Other things being equal, this provides an incentive to issue some non-voting shares since they allow better surplus extraction than voting ones. In doing so, one would depart from one share/one vote. ⁴

Another way to look at the problem is to focus on the majority rule. As stated above, voting shareholders face a coordination problem when the raider would increase the security benefits post-takeover: who is going to sell, and who is going to free-ride? This can result in coordination failures when too few shareholders tender for the bid to succeed,
or too many tender at a price that does not fully reflect the improvement. Suppose that the majority rule is unanimity, i.e. the raider has to buy all of the outstanding voting shares in order to take control. Then the coordination problem collapses: each voting shareholder is completely pivotal and knows he cannot free-ride on the improvement. However, while the unanimity rule solves the coordination problem, it makes voting shareholders unable to extract any surplus when the raider makes a take-it-or-leave-it offer. Hence, given the unanimity rule, minimising the return rights attached to voting shares is optimal. 

This last example shows that there is potentially a trade-off between the value of voting and non-voting shares. Consider the effects of an increased likelihood of a voting shareholder being pivotal. The tender offer is more likely to succeed, so that the expected social surplus is higher. Non-voting shareholders completely free-ride on a higher expected surplus and are thus better off. Voting shareholders extract a smaller fraction of a larger surplus. The net effect on their wealth is ambiguous.

However, quite intuitively, one is better off completely sacrificing all of a small fraction of the surplus so as to get all of a large one. This idea drives our main result that minimising the number of voting shares is desirable so that in the extreme, a ‘one share/all votes’ governance structure is optimal.

Our finding that there is a gain from departing from one share/one vote (without a corporate control contest) contrasts with Grossman and Hart (1988)’s conclusion that the allocation of votes is irrelevant when dealing with one value increasing raider. An atomistic shareholder completely free-rides on any improvement the raider would implement (Grossman and Hart 1980): if he anticipates that the raider will succeed in taking control, he does not tender his shares at a price below their post-takeover value, thus fully incorporating the improvement. Hence, whatever the allocation of votes, the shareholders extract all the surplus (on security benefits).

In Grossman and Hart (1988), without contests for control, the optimal allocation of votes is a purely defensive measure: one share/one vote aims at raising the cost of acquiring control for parties who would reduce security benefits (but enjoy private gains). Such a party would take advantage of a dual class shares system by acquiring control through shares with a high voting power only, and by letting the other shareholders fully bear the depreciation of their shares. These costs are captured in our setting as well.
But they are only part of the picture, so that when both types of raiders can appear, the optimal governance structure balances the costs and benefits of departing from one share/one vote. It is noteworthy that both approaches do not tend to coincide as the number of shareholders increases.

The paper is organised as follows. The model is presented in section 1. In section 2, we analyse the tender offer game with a value improving raider. Section 3 presents the central result of the paper, namely the optimality of minimising the number of voting shares when only value increasing raiders can appear. In section 4, we allow for value decreasing raiders. Extensions are discussed in section 5. Section 6 concludes.

1 The model

An entrepreneur has a project that needs management and investment. In order to finance it, he founds a firm and issues voting equity (A-shares) and non-voting equity (B-shares). Each A-share has one voting right while B-shares have none. The total number of shares is fixed and denoted \( n \). The same return right is attached to all shares: one \( n^{th} \) of all profit streams.

For simplicity, we assume that the entrepreneur runs the firm at the date it is created. However, he retains no shares and derives no private control gains. His objective is to maximise the gains from the initial public offering. The timing of events is as follows.

At date 0, the entrepreneur designs the governance structure of the firm, which amounts to choosing a couple of integers \( (a, k) \), with \( 1 \leq k \leq a \leq n \), where \( a \) is the number of A-shares and \( k \) is the minimum number of votes required to control the firm. We restrict attention to the case \( k \geq \frac{a}{2} \). The majority rule is called unanimity when \( k = a \), and simple majority when \( k \in \{ \frac{a}{2}, \frac{a+1}{2} \} \). The number of B-shares is \( n - a \).

**Assumption 1** The \( n \) shares are sold to \( n \) different risk neutral investors.  

At date 1, a risk neutral raider appears. His type, drawn from a known distribution, is revealed to the shareholders. A raider of type \( \theta \) is characterised by \( z_\theta \), the private gain he would derive, and \( v_\theta \), the return per share he would implement, once in control.  

At date 2, the raider attempts to take control of the firm by means of a costless, take-it-or-leave-it tender offer. He offers a price per share for each class, \( p_A \) and \( p_B \). We
allow for conditional and restricted offers: the raider can specify that his offer holds only if more than \( C_A \) A-shares and \( C_B \) B-shares are tendered and that he will buy no more than \( R_A \) A-share and \( R_B \) B-shares. If more than \( R_X \) shares of class \( X \) are tendered, the raider picks \( R_X \) of them at random. (Obviously, we can restrict to \( R_A \geq k \).) Thus, an offer is summarised by \((p_A, R_A, C_A, p_B, R_B, C_B)\).

At date 3, the shareholders *simultaneously and independently* choose to tender or not. If the conditions of the offer are met and the raider acquires more than \( k \) voting shares, he manages the firm forever, gets the private gains \( z_\theta \) and the per share value is \( v_\theta \). Otherwise, the entrepreneur runs the firm forever.

In the latter case, the per share value is normalised to zero so that \( v_\theta \) is also the per share value improvement a raider of type \( \theta \) would implement. In what follows, we will distinguish between 'good' raiders, who would increase the security benefits (i.e. with \( v_\theta > 0 \)) and 'bad' ones who would decrease them (i.e. with \( v_\theta < 0 \)).

## 2 The bidding game with a good raider

In this section, we study the outcome of the bidding game (dates 2 and 3) in the case where the raider who appeared at date 1 is a good one, i.e. is such that \( v_\theta > 0 \). The governance structure \((a, k)\) has already been chosen at date 0 and is fixed.

Given an offer, we first analyse the tendering game among the shareholders (date 3). Then we derive the raider's optimal offer (date 2). This section borrows extensively from Holmström and Nalebuff (1992). The coordination game between voting shareholders is analysed in an explicit model. Unlike Grossman and Hart (1980), the coordination problem is less likely to mean that the bid fails as the bid price for voting shares increases. Hence there is a trade-off for the raider between a low bid price on the one hand and a high probability of the bid succeeding on the other.

### 2.1 The bottom line

We place ourselves at date 3. Given an offer, the shareholders play a simultaneous move game in which each of them chooses to tender or not, or uses a mixed strategy. To deal with multiple equilibria, we adopt a selection criterion. 9

**Criterion 1** We select only perfect equilibria.
This implies, in particular, that players do not use weakly dominated strategies. We obtain Bagnoli and Lipman's result on conditional offers. 10

**Lemma 1**  The optimal offer is unrestricted and conditional on all shares being tendered, and the prices are arbitrarily close to zero.

**Proof:** Take any $\varepsilon > 0$. Consider the tender offer with prices $p_A = p_B = \varepsilon$, unrestricted and conditional on all shares being tendered, i.e. $C_A = R_A = a$ and $C_B = R_B = n - a$. By tendering, a shareholder gets $\varepsilon$ if the bid succeeds (which happens if all other shareholders tender) and 0 if it fails. If he does not tender, the bid fails and he gets 0. Hence, not tendering is weakly dominated. In equilibrium, all shareholders tender, the offer succeeds with probability one and the raider's payoff is $n(v_\theta - \varepsilon) + z_\theta$. By taking $\varepsilon$ arbitrarily close to 0, the raider extracts all the surplus, $nv_\theta + z_\theta$. \hfill \Box

The "trick" used by the raider is to make each shareholder absolutely pivotal for the success of the offer, and hence to put him in the weakest position in a take-it-or-leave-it offer. Coordination failures are avoided so that the social surplus is maximised and the raider extracts it all. 11 This leads to the following irrelevance result.

**Proposition 0** When only good raiders can appear, the governance structure is irrelevant to all agents. 12

**Proof:** By Lemma 1, whatever the governance structure, the raider and the shareholders' gains are arbitrarily close to $nv_\theta + z_\theta$ and 0 respectively. \hfill \Box

There may be reasons, however, why the use of conditional offers may be restricted. Some or all conditional offers may be illegal. In addition, the raider may be unable to commit to honour a conditional offer. Suppose that in the optimal offer described above, all A-shares and no B-shares were tendered. Respecting the conditionality would yield zero profit to the raider while accepting the tendered shares would yield a profit of $av_\theta > 0$. We make the following assumption.

**Assumption 2** Offers have to be such that $C_A \leq k$ and $C_B = 0$.

Note that this assumption holds when bids have to be conditional on the raider taking control (as it is the case under the U.K. legislation), i.e. $C_A = k$ and $C_B = 0$. It also holds when conditional bids are simply forbidden.

**Lemma 2** As soon as $C_B = 0$, we can assume without loss of generality that the raider does not bid for non-voting shares.
Proof: If \( C_B = 0 \), a non-voting shareholder's decision has no impact on the success of the tender offer. Thus, he and the raider attribute the same value to a B-share. No profitable trade is then possible and we can assume that no trade takes place. \( \Box \)

In the rest of section 2, we analyse the bidding game. We show that in general, given an offer, several Nash equilibria coexist in the tendering game. However, we can always select a unique symmetric perfect equilibrium: given an offer (possibly restricted and conditional), all voting shareholders play the same strategy, i.e. they all tender their share with the same probability. Then we characterise the raider's optimal offer in Result 1. Since the whole section is an extension of Holmström and Nâlebuff's analysis to conditional and restricted offers, and consists in explaining and proving Result 1, the reader familiar with the exposition might care to skip to section 3 where our main result is developed.

Notations. Suppose all shareholders play the same mixed strategy. Their common probability of tendering is denoted \( t \). We denote by \( P_T^x[t = y] \) the probability that \( x \) shareholders, each tendering with probability \( t \), will tender \( y \) shares. Hence,

\[
P_T^x[t = y] = \binom{x}{y} t^y (1-t)^{x-y}
\]

Result 1 Let \( (a, k) \) be the firm's governance structure. In a good raider's optimal offer, each voting shareholder tenders with probability \( t^* = \frac{k}{a} \).

A voting share is worth: \( v_0.P_t^{k-1}[T \geq k] \);

A non-voting share is worth: \( v_0.P_t^k[T \geq k] \);

The raider's surplus is: \( av_0.t^*P_t^{k-1}[T = k-1] \).

2.2 Equilibrium of the tendering game

We can describe the equilibria of the game, assuming that only A-shareholders are active players (Lemma 2).

- If \( p_A < 0 \), tendering one's A-share is a weakly dominated move. Hence, in a perfect equilibrium no A-share is tendered.

- Conversely, if \( p_A > v_0 \), not tendering one's A-share is a weakly dominated move. Hence, in a perfect equilibrium all A-shares are tendered.

- If \( p_A \in (0, v_0) \), there are no weakly dominated moves. All voting shareholders face the same problem: "Suppose I have not tendered my share yet. (1) If the bid succeeds,
I should keep it, free ride and get \( v_b \). (2) If it fails but more than \( C_A - 1 \) shares are tendered, I should tender and get more than 0. (3) Otherwise, whatever I do I get 0”. Of course, each of them prefers situation (1). But for any shareholder to be able to free ride, at least \( k \) others have to tender. Hence, there is a coordination problem.

First, there are Nash equilibria in which strictly less than \( C_A - 1 \) shareholders tender with a positive probability. In these, the shareholders are trapped in situation (3) and the bid fails with probability one. However, as proved in the appendix, these Nash equilibria are not perfect equilibria.

**Lemma 3** If \( p_A > 0 \), in all perfect equilibria, the probability of success is strictly positive.

There are also equilibria in pure strategies, in which exactly \( k \) shareholders tender while \( a - k \) free ride. In these, the bid succeeds with probability one. Because they realise that their vote is crucial for the offer to succeed, none of the tenderers wants to keep his share. (These equilibria are perfect.) Under unanimity, i.e. if \( k = a \), there is a unique Nash equilibrium (hence a perfect equilibrium since one always exists): all shareholders tender. If \( k < a \), there are other equilibria: strictly less than \( a - k \) A-shareholders retain their share, some others tender for sure and the rest of them randomise.

We have to select among several equilibria. Coordinating on one of them is all the more difficult for the shareholders if they act in a decentralised market. Or is it? If they cannot decide who is to play which role, why not coordinate on an equilibrium in which they all play the same role? If such an equilibrium was unique, it would constitute a compelling focal point (see Schelling 1960).

**Lemma 4** For all \( p_A \in (0, v_b] \), there exists a unique symmetric perfect equilibrium, i.e. in which all A-shareholders play the same strategy.

The proof is in the appendix. However, it is quite simple in the case of unrestricted and unconditional offers.

Take \( p_A \in (0, v_b) \) and consider a symmetric equilibrium in which each A-shareholder tenders his share with probability \( t \). We have already seen that under unanimity, the unique perfect equilibrium is symmetric with \( t = 1 \). If \( k < a \), then \( t \neq 1 \), otherwise the bid would still succeed with probability one even if one shareholder did not tender. He would thus have an incentive to do so. It must also be the case that \( t \neq 0 \) since otherwise each shareholder prefers to tender and get \( p_A \) instead of 0. Thus we must look
for \( t \in (0, 1) \). As a consequence, each shareholder has to be exactly indifferent between tendering and retaining his share. By tendering, he gets \( p_A \). By holding back his share, in case more than \( k \) of the other \( a-1 \) shareholders tender he gets \( v_\theta \), and 0 otherwise. Then, if it exists, \( t \) is a solution to
\[
 p_A = v_\theta \cdot P_i^{a-1}[T \geq k]
\]  
(1)
Since the RHS is obviously strictly increasing in \( t \in (0, 1) \) from 0 to \( v_\theta \), the equation admits a unique solution \( t(p_A) \) and the game a unique symmetric equilibrium which is perfect, since totally mixed.

Take now \( p_A = v_\theta \). The bid succeeds with a strictly positive probability (Lemma 3). If it were to succeed with a probability less than one, it would be a strictly best response for each shareholder to tender. Hence, in the only symmetric perfect equilibrium, all shareholders tender, i.e. \( t = 1 \).

- Suppose \( p_A = 0 \). It is easy to see that, if \( k < a \), in the only symmetric perfect equilibrium no shareholder tenders, i.e. \( t = 0 \). Under unanimity, all strategies are equivalent: they yield a payoff of 0 in any case. Hence, to every \( t \in [0,1] \) corresponds a symmetric perfect equilibrium. However, we will select \( t = 1 \). The idea is that under unanimity the raider can obtain \( t = 1 \) with any strictly positive price. Hence, \( p_A = 0 \) will stand for 'a strictly positive price arbitrarily close to 0.' We can thus select a unique Nash equilibrium.

Criterion 2 We consider only symmetric perfect equilibria.

2.3 The optimal offer

We now tackle the raider’s problem (date 2). We have shown that, for each offer, there corresponds a unique symmetric equilibrium in which all A-shareholders tender with the same probability \( t \). In the case of unrestricted and unconditional offers, \( t \) solves equation (1). Unless \( p_A = v_\theta \), we have \( t < 1 \) and the bid fails with a positive probability. However, \( t \) is increasing in \( p_A \) so that the risk of coordination failure is decreasing in the bid price. The raider thus faces a trade-off between a low price and a high probability of success. We generalise this to all offers (Lemma 5) and derive the optimal ones (Lemma 8).

We can restrict \( p_A \), without loss of generality, to \( p_A \in [0, v_\theta] \): with \( p_A = 0 \) the raider does as well as with \( p_A < 0 \) in general and better under unanimity, and with \( p_A = v_\theta \), he always does better than with \( p_A > v_\theta \).
Lemma 5 If \( k < a \), for all \( C_A \) and \( R_A \), the equilibrium probability to tender is strictly increasing in \( p_A \in [0, v_0] \) from 0 to 1.

The fact that in every equilibrium each A-shareholder is indifferent between tendering or not, together with the assumption that \( C_A \leq k \), have an important implication.

Lemma 6 Each player's profit is a function of \( t \) only.

Proof: The B-shareholders are passive. Hence, a B-share is worth \( v_0 \) if the bid succeeds and 0 otherwise, and so the value of a B-share is \( v_0 \cdot P_t^{a}[T \geq k] \). Each A-shareholder is indifferent between tendering or not. Hence, his expected gain equals his expected gain given that he does not tender. When he does not tender, his share is worth \( v_0 \) if the \( a - 1 \) other A-shareholders tender more than \( k \) shares (this is true only if \( C_A \leq k \)) and 0 otherwise, so that the value of an A-share is \( v_0 \cdot P_t^{a-1}[T \geq k] \). The raider's expected profit is the difference between the expected social surplus and the shareholders' expected gains. The social surplus is \( nv_0 + z_0 \) if the bid succeeds and 0 otherwise, and so is \( (nv_0 + z_0) \cdot P_t^{a}[T \geq k] \). The raider's expected gain is then

\[
(nv_0 + z_0) \cdot P_t^{a}[T \geq k] - (n-a)v_0 \cdot P_t^{a-1}[T \geq k] \quad - av_0 \cdot P_t^{a-1}[T \geq k]
\]

None of \( p_A \), \( C_A \) and \( R_A \) appears in the expressions for the three payoffs. \( \square \)

We obtain a result of irrelevance of the offer's type (which extends Holmström and Nalebuff 1992's Proposition 2).

Lemma 7 All agents are indifferent to any further constraints on \( C_A \) and \( R_A \) (on top of Assumption 2).

Proof: By Lemma 6, only \( t \) matters. For all values of \( R_A \) and \( C_A \), \( t \) is controlled through \( p_A \): if \( k=a \), \( p_A^* = 0 \) and \( t^* = 1 \); by Lemma 5, the result holds for \( k < a \). \( \square \)

The intuition behind this result is as follows. All that matters for all agents is the individual probability to tender. Tendering is less attractive for a shareholder when the offer is conditional because there are cases in which the bid fails and his share is not taken (when less than \( C_A \) shares are tendered). Hence, to induce a given probability of tendering, the bid price has to be higher when the offer is conditional than when it is not. On the other hand, a conditional offer has the advantage for the raider that he does not always have to buy (overpriced) shares when he does not take control. Hence,
increasing the price is less costly for him when the offer is conditional. In the end, an unconditional offer is equivalent to a conditional one with a higher bid price: the price is higher but paid less often.

The reasoning is the same for restricted offers. A restricted offer makes tendering more attractive for a shareholder because in some cases, the bid succeeds and his share is not taken so that he ends up free-riding on the improvement. On the other hand, a restricted offer has the disadvantage for the raider that he cannot keep all the shares tendered below their post-takeover value. An unrestricted offer is equivalent to a restricted one with a lower price: the price is lower but the raider benefits from this less often.

From now on, without loss of generality, we will only consider unrestricted and unconditional offers. Lemma 7 means that we could have chosen any other specification of these parameters (under Assumption 2). For instance, the analysis also applies when the raider can only make offers conditional on his getting control, i.e. $C_A = k$ and $R_A = a$.

In order to concentrate on the question of how the security benefit improvement is shared between the raider and the shareholders, we assume that a good raider has no private control gain, i.e. $z_\theta = 0$.

**Assumption 3** Good raiders derive no private control gains.

We now extend Holmström and Nalebuff’s characterisation of the optimal offer.

**Lemma 8** In an optimal bid, each A-shareholder tenders with probability $t^* = \frac{k}{a}$.

**Proof:** By Lemma 7, we can restrict attention to unrestricted and unconditional offers. Under unanimity, $p_A = 0$ and $t^* = 1$. If $k < a$, the raider affects $t$ through $p_A$ (Lemma 5). We thus optimise directly with respect to $t$. The raider’s profit is (see Lemma 6):

\[
\Pi_r = n v_\theta.P^*_t[T \geq k] - (n-a)v_\theta.P^*_t[T \geq k] - av_\theta.P^{a-1}_t[T \geq k]
\]

\[
= av_\theta.(P^*_t[T \geq k] - P^{a-1}_t[T \geq k])
\]

\[
= av_\theta.t.P^{a-1}_t[T = k-1]
\]

\[
= av_\theta.(\frac{a-1}{a-1})t^k(1-t)^{a-k}
\]

which is maximised at $t^* = \frac{k}{a}$. 

\[\square\]
3 The optimality of one share/all votes

We now turn to the central concern of the paper: the choice of an optimal governance structure \((a^*, k^*)\) by the entrepreneur (at date 0). In this section, we solve the problem in the polar case in which only good raiders (i.e. with \(v_0 > 0\)) can appear at date 1. We obtain an immediate and striking result.

**Lemma 9** When only good raiders can appear, a non-voting share is worth more than a voting share.

**Proof:** When a good raider appears, an A-share's value is \(v_0 \cdot P^a_{\frac{k}{a}}[T \geq k]\) while a B-share is worth \(v_0 \cdot P^b_{\frac{k}{a}}[T \geq k]\). \(\square\)

The premium for B-shares is \(v_0 \cdot \frac{k}{a} \cdot P^a_{\frac{k}{a}}[T = k-1]\), that is \(v_0\) times the probability for a voting shareholder that his vote is pivotal. The B-shareholders are absolute free riders on the expected improvement. On the other hand, because they can be pivotal, A-shareholders have second thoughts and do not free ride completely. Other things being equal, this provides an incentive to issue non-voting shares. We have found that there is a gain from departing from one share/one vote.

This contrasts with Grossman and Hart (1988)'s result. In their model all shareholders are absolute free-riders and capture all the value improvement. As a consequence, a share's value is independent of its voting power so that the governance structure is irrelevant.

However, in our model, a change in the governance structure does not leave 'other things equal'. We now study the impact of the governance structure on the value of A and B-shares. For both classes of shares, a change in the governance structure has antagonistic effects.

Consider the value of a B-share: it is proportional to the probability that the bid succeeds. This probability, \(P^b_{\frac{k}{a}}[T \geq k]\), is increasing in \(a\) and \(t\) and decreasing in \(k\). However, \(t\) is not independent of \(a\) and \(k\): in response to the optimal offer, the probability of tendering is \(t = \frac{k}{a}\). Hence, an increase in \(a\) means that more shareholders tender but each of them tenders with a smaller probability, since he is less likely to be pivotal. So it is not clear whether the bid is more likely to succeed or not. In other words, the effect on the value of B-shares is ambiguous. (The same reasoning applies to the impact on the value of A-shares.)
The balance between these two effects turns out to differ according to the share’s class. For example, the value of A-shares is minimal when \( a = k \) (i.e. when \( a \) is minimal given \( k \)) while that of B-shares is maximal. This is illustrated in the following lemma which is proved in the appendix.

**Lemma 10** Suppose only good raiders can appear. When \( a \) increases as \( k \) is fixed
(i) the value of non-voting shares decreases and (ii) that of voting shares increases.

This means that the dominant effect of an increase in \( a \) is the lowered willingness to tender of a voting shareholder. In response to the increased optimal bid price, each A-shareholder tenders with a lower probability. This decrease in the probability is such that despite the increased number of A-shareholders, it is less likely that at least \( k \) shares are tendered.

Similarly, the impact of an increase in \( k \) is ambiguous: the voting shareholders tender with a higher probability but the success of the takeover requires that more shares are tendered. There again, the balance between the two effects differs according to the share’s class.

**Lemma 11** Suppose only good raiders can appear. When \( k \) increases as \( a \) is fixed
(i) the value of non-voting shares increases and (ii) that of voting shares decreases.

The entrepreneur’s objective is to maximise the value of the firm, that is the expected value of all shares:

\[
E_{\theta}\{av_{\theta}.P_{k}^{a}[T \geq k] + (n - a)v_{\theta}.P\}_{k}^{a}[T \geq k]\}
\]

Since \( v_{\theta} \) is the only random variable, the entrepreneur’s objective is proportional to the expected improvement \( E_{\theta}\{v_{\theta}\} \), which is positive by assumption. Hence, the optimal governance structure \( (a^*, k^*) \) maximises

\[
g(a, k) = a.\frac{P_{k}^{a-1}[T \geq k]}{a.\frac{P_{k}^{a-1}[T \geq k]}{a}} + (n - a).\frac{P\}_{k}^{a}[T \geq k]}
\]

We prove in the appendix that one can get the best of both worlds, i.e. increase the value of voting and non-voting shares at the same time.

**Lemma 12** Suppose \( a > k \). When only good raiders can appear, the value of both voting and non-voting shares is strictly higher under \( (a, k) \) than under \( (a+1, k+1) \).
Corollary 1 If only good raiders can appear, \((a, k)\) strictly dominates \((a+1, k+1)\).

**Proof:** Suppose \(a = k\). Then \(a+1 = k+1\). Thus, under both structures, A and B-shares are worth 0 and \(v_s\) respectively. Hence, \((a, a)\) dominates \((a+1, a+1)\). The preceding lemma extends this result when \(a > k\).

Corollary 2 The optimal majority rule is either unanimity or simple majority.

**Proof:** Consider a governance structure \((a, k)\) such that \(2k > a > k\). Take \(r = 2k - a\). We have \(r > 0\) and \(a - r = 2(k - r)\). Also \(k - r = a - k\) so that \(k > r\). Thus \((a - r, k - r)\) is a governance structure with the simple majority rule that strictly dominates \((a, k)\).

It is relatively straightforward to compare governance structures with the same majority rule which can be either unanimity or the simple majority rule. A consequence of Corollary 1 is that, given the unanimity rule, one wants to minimise the number of voting shares. This is because under unanimity, in our setting, voting shareholders do not extract any surplus while non-voting ones extract it all. It is proved in the appendix that a similar result holds for the simple majority rule. The idea is that by ‘scaling down’ the set of voting shares but keeping the simple majority rule, the value of voting shares is basically unchanged while the probability of success increases, and so does the value of B-shares.

Lemma 13 Suppose only good raiders can appear. Under the unanimity and the simple majority rule, it is optimal to minimise the number of voting shares.

We then get to our main result.

Proposition 1 If only good raiders can appear, the optimal governance structure is \(a^* = k^* = 1\) (‘one share/all votes’).

**Proof:** From the preceding results, we know that the optimal governance structure is either \([1, 1]\) or \([2, 1]\). The firm’s optimal value is then \((n - 1)v_g\) or \(\frac{2}{3}(n - 2) + 1)v_g\). The difference is \(\frac{n - 2}{4}\) so that \([1, 1]\) strictly dominates \([2, 1]\) (unless \(n = 2\) in which case both governance structures are equivalent).

The idea here is quite simple. Suppose that there are a lot of shares. One share/all votes maximises the number of complete free-riders and the surplus on which they free-ride. It is intuitive that this is optimal as soon as the number of shares is large enough:
even if the voting shareholders get nothing, they hold a very small fraction of the security benefits of the firm. This amounts to completely sacrificing a small fraction of the surplus so as to maximise the surplus of which one gets a large fraction (actually all of it). In that sense, the proposition is simply saying that, in our model, this is still true for small values of \( n \).

The result is robust and still holds under less extreme versions. It can be proved that minimising the number of voting shares, or maximising that of complete free-riders is optimal, even when one share/all votes is ruled out.

**Proposition 2** Suppose that the choice of \( a \) is constrained by \( a \geq a_m \). If only good raiders can appear, it is optimal to minimise the number of voting shares, i.e. \( a^* = a_m \).

**Proof:** Suppose \( a^* > a_m \). If \( a^* > 2k^* \) then \( a^* - 1 \geq 2(k^* - 1) \). Hence a contradiction (Corollary 1). If \( a^* = 2k^* \) then the governance structure \( (a^* - 1, k^*) \) has the simple majority rule. This contradicts Lemma 13. \( \square \)

## 4 The case of 'bad' raiders and the general case

In this section, we complete the study of the optimal governance structure. First, we turn to the opposite polar case, in which only bad raiders (i.e. with \( v_\theta < 0 \)) can appear at date 1. In that case, our framework is similar to Grossman and Hart (1988) and we reach the same conclusion: a one share/one vote structure is optimal, and the closer to this, the better. We then derive the optimal structure in the more general case in which raiders of both types, good and bad, can appear. The optimal governance structure trades off the gains associated with issuing non-voting shares when raiders are good with the costs of doing so when they are bad. Generally, the optimal governance structure will depart from one share/one vote.

### 4.1 Bad raiders (\( v_\theta < 0 \))

What a bad raider is really interested in is control. Hence, he wants to buy the minimum amount of return rights giving him control (at the minimum price), since once he has acquired them, he will bear their depreciation. The lack of coordination among voting shareholders can work in his favour and enable him to let part of the depreciation be
borne by the shareholders from whom he buys the shares (Lemma 15). Again, the coordination failure is resolved by the unanimity rule. It is however still the case that the shareholders whose shares are not acquired, in particular non-voting shares, bear the full amount of the decrease in security benefits. This leads to the optimality of not issuing voting shares (Proposition 3) and, when unanimity is ruled out, of issuing a single class of shares (Proposition 4).

We first quickly study the bidding game (dates 2 and 3). As before, we allow for restricted and conditional offers. As we will see, these are not irrelevant in the case of a bad raider (which, by Lemma 7, contrasts with the case of a good raider).

- If $p_X > v_θ$, not tendering one’s X-share is weakly dominated. Hence, in a perfect equilibrium, all shares of class X are tendered. This is also true for $p_A = 0$ except under the unanimity rule, in which case all strategies are equivalent and yield a payoff of zero. As before, we extend the outcome to that case.

- If $p_X \leq v_θ$, tendering one’s X-share is weakly dominated. Hence, in a perfect equilibrium, no shares of class X are tendered.

Without loss of generality we can restrict attention to the case $p_X \in [v_θ, 0]$, $X = A, B$, and directly get the following lemma.

**Lemma 14** (i) A bad raider’s offer with $p_A \geq 0$ succeeds with probability one;

(ii) In the optimal offer, no shareholder makes a strictly positive profit.

We now turn to the tendering game following a bid with $p_A \in (v_θ, 0)$. Voting shareholders are all better off if the bid fails. When the majority rule is unanimity, tendering one’s A-share is a dominated strategy so that in a perfect equilibrium the bid fails with probability one. Suppose now that the majority rule is not unanimity. Again, all voting shareholders face a coordination problem. However, while with a good raider, they all wanted to do the opposite of what the others did, with a bad raider, they all want to do the same as the others. If a shareholder believes that the bid is going to fail, he does not tender his share; however, if he ‘fears’ that the bid will succeed, he tenders it. There are thus two possible types of outcomes: when ‘confidence’ dominates among voting shareholders, the bid fails with probability one; when ‘suspicion’ dominates, it succeeds with a strictly positive probability. Actually, both types of outcomes can arise in equilibrium.

**Lemma 15** Suppose the majority rule is not unanimity and a bad raider appears. If $p_A \in$
\((v_\theta, 0)\) and either \(p_B > v_\theta\) or \(C_B = 0\) then the tendering game admits both ‘confidence’ and ‘suspicion’ perfect equilibria.

The existence of suspicion equilibria is straightforward to show. The symmetric equilibrium in fully mixed strategies we put forward in the case of good raiders (Lemma 4) remains valid. In this equilibrium, the tender offer succeeds with a strictly positive probability. In the appendix, we prove that there are two other symmetric perfect equilibria: one in which no A-share is tendered and one in which all A-shares are tendered.

Abstracting, for the moment, from the problem of selection among the different equilibria, we are able to derive the optimal governance structure.

**Proposition 3** When only bad raiders can appear, the (weakly) optimal governance structure requires that all shares are voting shares and the majority rule is unanimity, i.e. \(a^* = k^* = n\).

**Proof:** Given the unanimity rule, the optimal offer is unrestricted with \(p_A = 0\) and succeeds with probability one (Lemma 14.i). The shareholders’ surplus is then zero which is the best they can obtain (Lemma 14.ii). \(\square\)

There are two benefits to the unanimity rule. Suppose the number of voting shares is fixed to some \(a\), and that we only optimise over the majority rule, i.e. over \(k\).

First, suppose that we always selected confidence equilibria when \(p_A \in (v_\theta, 0)\). Then an optimal bid price would be \(p_A = 0\). Given that any such offer succeeds with probability one and that the raider makes a loss on each share he buys, the optimal offer would be restricted to \(R_A = k\) voting shares (and no bid would be made for non-voting shares). The shareholders would bear the full depreciation in security benefits on all but \(k\) shares, and not make gains on the remaining \(k\) shares. Hence, given \(a\), it is optimal to maximise \(k\), which implies unanimity as a majority rule.

Second, as mentioned before, when the majority rule is unanimity, tendering one’s share is a dominated move whenever \(p_A < 0\). The unanimity rule prevents suspicion equilibria which exist under all other majority rules (Lemma 15).

Hence, the unanimity rule prevents restricted offers and suspicion equilibria both of which allow a bad raider to let voting shareholders bear part of the decrease in security benefits he implements. Given the unanimity rule, the shareholders’ surplus is \((n - a)v_\theta\) so that it is optimal to have only voting shareholders.
The optimality is weak in the following sense. Suppose that only bad raiders can appear. Take any governance structure with unanimity. To take control, the raider would have to buy at least \( k \) shares at a price \( p_A \geq 0 \) and fully bear the depreciation on these shares, making a profit of \( z_\theta + kv_\theta \). Hence, any such governance structure with
\[
k > \max_\theta \frac{z_\theta}{v_\theta}
\]  
(3)
deters all raiders and yields a zero profit to shareholders. However, as soon as the RHS in (3) is greater than \( n \), \( a^* = k^* = n \) is strictly optimal.

It should be emphasised that Proposition 3 does not really constitute a one share/one vote result. If more than one class of voting shares were introduced in our model, it is easily seen that the distribution of shares among these would be irrelevant: because of the unanimity rule, the raider would have to buy all shares anyway. Hence, the fact that all shares have the same voting power does no harm but is not necessary.

Nevertheless, once unanimity is ruled out as a majority rule, we can reproduce Grossman and Hart’s result. To be able to compare governance structures without the unanimity rule when \( p_A \in (v_\theta, 0) \), we make the following assumption in the rest of the paper.

**Assumption 4** We restrict to \( p_A \geq 0 \).

This can be interpreted in several ways. First, since bids below the pre-tender offer price can reduce shareholders’ welfare in suspicion equilibria, they can simply be ruled out by legal dispositions. Second, as in Bagnoli and Lipman (1988), we could simply assume away suspicion equilibria. Some support is given to this view by noticing that the confidence equilibrium in which no share is tendered is an ideal candidate for selection since, as it is symmetric and strictly Pareto dominates all other equilibria, it constitutes a very convincing focal point. Third, Grossman and Hart (1988) endogenise this restriction by assuming that some arbitrageurs are active in the market. These would make a successful counter-offer to a bid with \( p_A < 0 \).

We obtain a result à la Grossman and Hart (1988).

**Proposition 4** Suppose unanimity is ruled out as a majority rule. When only bad raiders can appear, the one share/one vote rule is optimal.
The reader is referred to Grossman and Hart (1988) for a proof of this proposition. The idea is as follows. Given Lemma 14.ii and Assumption 4, whatever his offer, a bad raider makes a loss of $v_\theta$ on each share he acquires. He thus simply bids for the minimum amount of shares giving him the required fraction of the votes, which hurts the shareholders since they fully bear the depreciation on non-acquired shares. The one share/one vote rule maximises this minimum amount: the fraction of return rights equates that of votes required for control; any other setting would allow the raider to acquire the shares with a high voting power preferentially; unless all shares are voting shares and the majority rule is unanimity, he would not have to buy all shares; hence, the fraction of return rights he would end up buying would be lower than that of voting rights required for control.\footnote{The same reasoning shows that the closer to this vote allocation, the higher the value of the firm.}

\textbf{Proposition 5} Suppose the choice of $a$ is constrained by $a \leq a_M$ and that unanimity is ruled out. When only bad raiders can appear, it is optimal to maximise the number of voting shares, i.e. $a^* = a_M$.

\subsection{The general case}

We now address a more general case in which both good and bad raiders can appear at date 1. It is clear from the previous analysis that the entrepreneur faces a trade-off when deciding on the governance structure since the optimal governance structure differs according to the type of raider. The characteristics of the set of all potential raiders determine the optimal governance structure.

Intuitively, when good raiders dominate, either because they are more likely to appear and/or because they would improve security benefits by a large amount, the number of voting shares will tend to be small. Conversely, when bad raiders dominate, the governance structure will tend to one share/one vote.

For simplicity, we will concentrate on the case in which the set of potential raiders (i.e. those who will actually make an offer at date 2) is independent of the governance structure chosen. In our context, this means that all bad raiders have sufficient private gains that no governance structure deters them, i.e. they are ready to buy all the shares if necessary.\footnote{Also for simplicity we will rule out restricted offers. Note that the possibility that a...}
raider makes restricted offers is detrimental to the shareholders: it is irrelevant for good raiders (Lemma 7) and may be used by bad ones as soon as the governance structure departs from that prescribed in Proposition 3.

**Assumption 5** Restricted offers are ruled out.

We get the very simple following result and its important corollary.

**Lemma 16** When only bad raiders can appear, the majority rule is irrelevant.

**Proof:** The raider will bid for, and only for, all voting shares at price \( p_A = 0 \) and will acquire them all. \( \square \)

**Corollary 3** Under the one share/one vote rule, the simple majority rule is optimal.

**Proof:** Directly follows from Lemma 11 and Lemma 16. \( \square \)

On the one hand, given the mandatory bid requirement (Assumption 5), the majority rule is irrelevant when raiders are bad. On the other hand, the simple majority rule makes each shareholder less likely to be pivotal so that the bid price of a good raider increases.

Under our assumptions, given the number of voting shares, the optimality of the majority rule is only driven by its impact on good raiders. The intuition stated above that there is a trade-off in the setting of the number of voting shares has been formalised in Proposition 2 and Proposition 5.

Let \( q_\theta \) denote the probability that a raider of type \( \theta \) appears at date 1. Define \( V_g \) and \( V_b \), the importance of good and bad raiders respectively, by

\[
V_g = \sum_{\theta} q_\theta \max\{v_\theta, 0\} \quad \text{and} \quad V_b = -\sum_{\theta} q_\theta \min\{v_\theta, 0\}
\]

The ratio \( V_g/V_b \) is the relative importance of good raiders.

**Proposition 6** As the relative importance of good raiders increases from 0 to infinity, the optimal number of voting shares decreases from one share/one vote to one share/all votes.

**Proof:** Fix \( a \) and choose \( k \) optimally. The expected value of the firm is \( V_g \cdot g(a) + V_b \cdot (n - a) \) where \( g(a) = \max_k g(a, k) \) with \( g(a, k) \) as defined in (2). Since \( g(\cdot) \) is decreasing (Proposition 2), the optimal \( a \) is decreasing in \( V_g/V_b \). \( \square \)
Proposition 7 When the relative importance of good raiders is large (small) enough, the optimal majority rule is unanimity (simple majority).

PROOF: When $V_g/V_5$ is large enough, $a^*$ is small compared to $(n-a^*)$. Hence, the trade-off is solved when $k$ is maximised. (In an extreme case, optimality requires one share/all votes, hence unanimity.) The opposite holds when $V_g/V_5$ is small enough. (In the extreme, optimality requires one share/one vote, hence, by Lemma 16 the simple majority rule.)

5 Extensions

5.1 Moral hazard

One important cost of departing from one share/one vote is due to moral hazard on the part of the controlling party. We have assumed that the change in security benefits a raider would implement is fixed. If, instead, it is a choice variable for the raider, it may be important to make sure that he holds a sufficient amount of return rights were he to take control, so that his interest is aligned with that of other shareholders, i.e. security benefits have a high enough weight in his objective function. In other words, the same raider may be good or bad depending on whether he acquires enough shares or not. As shown above, a one share/one vote structure maximises the share of return rights a raider has to acquire to take control. Hence, it maximises the alignment effect. This suggests an additional cost to reducing the number of shares required for control: good raiders will put less emphasis on security benefits so that the total surplus to be shared is lower; moreover, some good raiders may become bad ones.

Though this argument puts a bound on the optimal minimisation of the number of voting shares, it only mitigates our result that, when raiders are good, the entrepreneur wants to minimise the number of voting shares.

Also, it is not obvious that the incentive problems that will arise after the takeover have to be solved through the takeover process. One could imagine to that the governance structure solves the problem of who is to be in control under what conditions, while the interest alignment question is addressed once someone is in control. For instance, the successful raider could be attributed shares, possibly non-voting ones, by the corporate charter.
5.2 Risk-aversion

There correspond different degrees of uncertainty about the outcome of the tender offer in stage 2 to different governance structures. Compare the two extreme cases of one share/one vote and one share/all votes under the unanimity rule. The former involves no risk. Whatever the raider’s type, the wealth of each shareholder is not affected: the shareholders make no profits nor losses. The latter is, however, very risky in that the shareholders surplus depends dramatically on the type of the raider: this structure is better at extracting surplus from a good raider but makes (non-voting) shareholders very vulnerable to bad raiders. Hence, risk aversion tends to make a governance structure with more voting shares attractive.  

However, the more voting shareholders, the more stringent the coordination problem among these. In our framework and unless the majority rule is unanimity, this affects the uncertainty about the outcome of each particular tender offer (not necessarily monotonically though). Hence, potentially there are also costs in terms of the effect on risk of raising the number of voting shares.

5.3 Private gains and takeover costs

Suppose first that for some bad raiders, \(nv_0 + z_0 < 0\). These are not ready to buy the whole firm. Hence, a one share/one vote governance rule may not be necessary to deter them.

Consider now a good raider with private gains. The larger these gains, the more weight he puts on the success of his tender offer, rather than on the sharing of the improvement in security benefits. For a given governance structure, this will result in his bidding a higher price (Lemma 5). Since this reduces the gap between the value of A and B-shares, minimising \(a\) is made less attractive.

A variation in private gains affecting the optimal governance structure in the same direction in the case of good and bad raiders, we have the following result.

**Proposition 8** As private gains decrease (but remain positive), the optimal governance structure tends to one share/all votes.

**Example.** Consider a 1000-share firm. Suppose that there are two types of potential raiders. A good type with \(v_1 = 5\) and \(z_1 = 0\) and a bad type with \(v_2 = -1\) and \(z_2 = 20\). Suppose that the bad type is by far the most likely to appear at date 1 (say \(q_1 = 1/100\)). Then, the optimal governance structure is \(a^* = k^* = 20\).
Suppose now that a raider bears some takeover costs. Note that since the private gains are net of these private costs, the analysis for bad raiders is unchanged. An increase in takeover costs is equivalent to a decrease in private benefits. Hence, it would lead to less voting shares.

With one share/all votes, a good raider gets the surplus on only one share. The takeover costs may not be balanced by this small gain, and thus deter the good raider. It may thus be optimal to depart from one share/all votes, i.e. to raise the number of voting shares, even when only good raiders can appear.

**Example.** Consider a 1000-share firm. Suppose that there is only one type of raider with \( v = 1 \) and \( z = -20 \). Then the optimal governance structure is \( a^* = k^* = 20 \).

### 5.4 Splitting shares

Suppose only good raiders can appear at date 1. We have shown that in the optimal governance rule there is a small number of voting shares and a large number of non-voting ones. However, each voting shareholder is likely to be pivotal so that the bid price is low. It may be possible to reduce the probability that a voting shareholder is pivotal without changing the share that voting shares globally represent in the firm. This amounts to splitting the shares and distributing each of them to a different shareholder so as to spread them more. (Note that the fraction of return rights attached to voting shares is kept constant.) One related intuition is that by splitting shares, one would restore the free rider behavior even for voting shareholders so that one share/one vote would do (almost) as well as one share/all votes when raiders are good, and hence would dominate in the general case as soon as some raiders are bad. Both these intuitions are wrong in general, as proved in the appendix.

**Proposition 9** Under the simple majority rule, splitting the shares is detrimental to the value of the firm. \(^{17}\)

The idea is the following. It is true that as (voting) shares are split and distributed to a larger number of shareholders, voting shareholders free-ride more and more. Hence, as shown by Holmström and Nalebuff, the raider’s surplus goes to zero. \(^{18}\) However, they free-ride on the expected improvement, and the improvement is not certain. It is proved in the appendix that as the shareholders get more of the surplus, the surplus shrinks. In other words, as shares are splitted, the probability of success of the optimal
offer decreases. More precisely, it turns out that the surplus of voting shareholders is not affected, but that of non-voting ones obviously decreases. Hence, splitting shares is detrimental to the value of the firm in two respects. First, the value of shareholders’ surplus in a tender offer is lower. Second, since a good raider’s surplus decreases, in the presence of takeover costs, more good raiders are going to be deterred. (Note that a bad raider’s surplus is not affected as long as the fraction of return rights attached to voting shares is unchanged.)

5.5 Banning partial offers

In section 4, we have shown that a one share/one vote rule was the best response to the threat of bad raiders. As in Grossman and Hart (1988), the point is that such a setting maximises the return rights a bad raider has to acquire in order to get control. There are two ways the raider can buy only part of the return rights. First, he can make a restricted offer within each class of shares. Second, and even if the first type of offer is impossible (as under Assumption 5), he can restrict his offer to certain classes of shares (in our model, to voting shares only).

Hence, ruling out restricted offers within a class is addressing only part of the problem of partial offers. One could argue that a raider should have to bid for all shares in all classes. But such a requirement would be meaningless without a requirement on prices: a price low enough (below $v_0$ for a bad raider) for a certain class of shares amounts to not bidding for this class. As proved by Grossman and Hart, arbitrage is not enough to ensure that $p_B \geq 0$ when $p_A \geq 0$. This leads to the optimality of one share/one vote. Suppose that instead it is possible to enforce the following rule:

'The bid should be for any and all shares at prices above the pre-takeover ones.'

In our model, this would amount to $R_A = R_B = 0$ and $p_A \geq 0$ and $p_B \geq 0$. A bad raider would have to buy all the shares. In Grossman and Hart’s framework, such a rule would lead to the irrelevance of the allocation of votes. In ours, it leads to such an irrelevance in the case of bad raiders but to the optimality of one share/all votes in the general case.

Note that such a rule would prevent bad raiders only from taking control without buying the whole firm. Good raiders instead would be able to de facto bid for a subset of all shares. In our framework, a good raider bidding $p_B = 0$ will acquire no B-shares. This
can be of some importance in situations where allowing partial acquisition is efficient, say because raiders lack sufficient funds to purchase all shares. \(^{19}\)

6 Conclusion

Our analysis suggests that there may be gains to departing from one share/one vote. Issuing non-voting shares is a way to restore the complete free-riding behavior of part of the shareholders. More generally, we make the point that the governance structure, including the allocation of voting rights among shares, can influence the surplus sharing in a tender offer and the probability of success of the takeover.

Unlike previous studies, the gains are not linked to potential corporate control contests. In these, rationales for dual class shares are based on the incumbent controlling party’s attempts to deter hostile takeovers threatening its private gains (surplus protection), or on the shareholders’ attempts to extract part of the raider’s private gains by allowing some rival to compete on a smaller fraction of return rights (private gains extraction). \(^{20}\)

As emphasised in previous studies, there are also costs from departing from one share/one vote. Our model captures the greater vulnerability to value decreasing raiders who can ‘divide and rule’ by acquiring a minimum fraction of the return rights. One share/one vote is a way to ensure equal treatment of all shareholders. The optimal governance structure is then the result of a trade-off.
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Endnotes

1. See Rydqvist (1992). The one share/one vote rule prevails in Australia, Belgium, Norway and the U.K. while no restriction is imposed in Canada, Israel and the Netherlands. Other countries regulate the use of dual class shares. The vote ratio between the two classes is bounded by $1/10$ in Denmark and Sweden, and by $1/20$ in Finland. The fraction of shares with restricted voting rights is limited to 50% in France, Germany and Italy, and to 67% in Japan. In the US, different markets have adopted different rules. See Curtin and Davies (1992) and Lando (1993) on the EC plans.

2. Rupert Murdoch, chairman of News Corporation, has recently asked the Australian Stock Exchange to waive its one share/one vote rule. However, his plan is being fought by institutional investors. See The Economist, 23-29/10/1993; International Herald Tribune, 28/10/1993.

3. See also Harris and Raviv (1988).

4. A very similar point, developed independently, is made by Zingales (1993). Analysing the firm’s optimal ownership structure in the context of takeovers, he writes: ‘A dispersed ownership increases the ability to free ride on any verifiable improvement implemented by the buyer.’

5. In a different perspective, Blair, Gerard and Golbe (1989) analyse some implications of completely unbundling return rights and control rights.

6. However, all our results hold as well with a restriction to $k > \frac{3}{2}$.

7. We do not analyse the case in which a shareholder can hold several shares (though this is done in Holmström and Nalebuff 1992). We are thus unable to properly assess which ownership structure would be reached were the shareholders allowed to trade among themselves. However, we believe that the intuitions behind our main results remain valid.

8. See Barclay and Holderness (1989) for a study of private benefits.


11. Note that this result remains valid when the shareholders hold several shares and
   for more than two classes of shares.

12. This should not be confused with Grossman and Hart (1988)'s irrelevance result,
   which relies on the complete free-riding of all shareholders irrespective of their
   voting rights.


14. In principle, though this is not allowed in our model, a bad raider could blackmail
   the shareholders by conditioning his not acquiring control on his getting a non-
   controlling block of shares (say B-shares) at a low price. A one share/one vote
   governance structure prevents such a strategy.

15. Formally, this corresponds to \( \forall \theta, v_\theta < 0 \) then \( z_\theta + n_v_\theta > 0 \). We discuss departure
   from this assumption in the next section.

16. Thanks are due to Oliver Hart for raising this point.

17. Holmström and Nalebuff (1992) use the term splitting in a different sense. The
   share of each shareholder is split so that he holds more shares but the same return
   rights. We stick to our one share per shareholder assumption.


APPENDIX

A Preliminaries to Appendix B and C

Let the A-shareholders be indexed from 1 to a and let \( \sigma = (t_1, \ldots, t_{i-1}, t_i, \ldots, t_a) \) denote the strategy profile in which shareholder \( i \) tenders with probability \( t_i \). In Appendix B and C, we compare the costs and benefits to shareholder \( i \) of tendering versus retaining his share when the tender price is \( p_{A} > 0 \). In this comparison, the strategy profile of the other shareholders is given, and denoted \( \sigma_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_a) \). Let \( T \) denote the number of shares tendered (which is a random variable) by the \( a-1 \) other A-shareholders. We denote \( P_{\sigma_{-i}}^{a-1}[T = y] \) the probability that \( T = y \).

If \( T < C_A-1 \), no share is purchased: tendering and not tendering are equivalent moves.

If \( C_A-1 \leq T \leq k-1 \), by tendering \( i \) gets \( p_A \) instead of 0. The benefit of tendering is \( p_A \).

If \( k \leq T \leq R_A-1 \), by not tendering \( i \) gets \( v_\theta \) instead of \( p_A \). The cost of tendering is \( v_\theta - p_A \).

If \( T = y \) with \( y \geq R_A \), by not tendering \( i \) gets \( v_\theta \) for sure instead of \( p_A \) when non rationed (which happens with probability \( \frac{R_A}{v_\theta} \)) and \( v_\theta \) otherwise. The cost of tendering is \( (v_\theta - p_A) \frac{R_A}{v_\theta} \).

Hence, the benefits of tendering versus not tendering are

\[
p_A P_{\sigma_{-i}}^{a-1}[T \in [C_A-1, k-1]]
\]

while the costs are

\[
(v_\theta - p_A) P_{\sigma_{-i}}^{a-1}[T \in [k, R_A-1]] + \sum_{y=R_A}^{a-1} \frac{R_A}{y+1} P_{\sigma_{-i}}^{a-1}[T = y]
\]

B Proof of Lemma 3

If \( p_A \geq v_\theta \), in all Nash equilibria, the bid succeeds with probability 1. For \( p_A < v_\theta \), we prove the lemma by contradiction. Given \( p_A \in (0, v_\theta) \), \( C_A \) and \( R_A \), let \( \sigma = (t_1, \ldots, t_i, \ldots, t_a) \) be a perfect equilibrium in which the bid fails with probability 1. Assume without loss of generality that the shareholder are indexed so that \( t_i = 0 \) if and only if \( i > x \). Since
$x$ is the number of shareholders tendering with a positive probability, it must be that $x < k$.

Being a perfect equilibrium, $\sigma$ is the limit of a sequence of totally mixed strategy profiles $\{\sigma^\nu\}_{\nu \in \mathbb{N}}$ such that, for any shareholder $i$, $t_i$ is a best response to $\sigma^\nu_{-i}$, for all $\nu$. In particular, for shareholder $a$, not tendering must be a best response to $\sigma^\nu_{-a} = (t^\nu_1, ..., t^\nu_{a-1})$ for all $\nu$. Hence, the costs of tendering versus not tendering must outweigh the benefits. The idea of the proof is to show that as one lets enough individual probabilities go to 0, (5) converges to 0 faster than (4).

We have

$$P_{\sigma^\nu_{-a}}[T = y] = \sum_{\#J = a - 1 - y} \prod_{j \in J} (1 - t^\nu_j) \cdot \prod_{i \not\in J} t^\nu_i = \prod_{i = 1}^{a-1} t^\nu_i \cdot \sum_{\#J = a - 1 - y} \prod_{j \in J} \frac{1 - t^\nu_j}{t^\nu_j}$$

By (4), the benefits are greater than $p_A \cdot P_{\sigma^\nu_{-a}}[T = k-1]$. By (5), the costs are lower than $(v_{g} - p_A) \sum_{y = k}^a P_{\sigma^\nu_{-a}}[T = y]$. To compare them, we can simplify by $\prod_{i = 1}^{a-1} t^\nu_i$. Hence, we have to compare

$$p_A \cdot \sum_{\#I = a - k} \prod_{i \in I} \frac{1 - t^\nu_i}{t^\nu_i}$$

and

$$(v_{g} - p_A) \cdot (1 + \sum_{y = 1}^{a - k - 1} \sum_{\#J = y} \prod_{j \in J} \frac{1 - t^\nu_j}{t^\nu_j})$$

(4) is a sum of $M = \sum_{y = 0}^{a - k - 1} \binom{a - 1}{y}$ products. Let $J^*$ be the set of indexes corresponding to the largest of these. Hence, (4) is lower than $(v_{g} - p_A) \cdot M \cdot \prod_{j \in J^*} \frac{1 - t^\nu_j}{t^\nu_j}$.

But we have $\#J^* \leq a - k - 1 < a - k \leq a - x - 1$. Since $a - x - 1$ is the number of shareholders such that $\frac{1 - t^\nu_j}{t^\nu_j}$ goes to infinity, at least one term in (3) is of the form

$$p_A \cdot \prod_{i \in I^*} \frac{1 - t^\nu_i}{t^\nu_i} \cdot \prod_{j \in J^*} \frac{1 - t^\nu_j}{t^\nu_j} \quad \text{with} \quad I^* \cap \{x+1, ..., a-1\} \neq \emptyset$$

Hence, at rank $\nu$, the ratio of the benefits on the costs of tendering is greater than

$$\frac{p_A}{(v_{g} - p_A) \cdot M} \cdot \prod_{i \in I^*} \frac{1 - t^\nu_i}{t^\nu_i}$$

Since $I^* \cap \{x+1, ..., a-1\} \neq \emptyset$ at least one term in the product goes to infinity as $\nu$ goes to infinity. Hence, at some point, tendering is shareholder $a$'s strictly best response to $\sigma^\nu_{-a}$, a contradiction. \qed
C Proof of Lemmas 4 and 5

Take \( p_A \in (0, v_\theta) \). In equilibrium, each shareholder is indifferent between tendering and retaining his share. Hence, the benefits of tendering must equal the costs. This equality has the following form

\[
P_A \sum_{i=C_A-1}^{k-1} \beta_i t^i (1-t)^{a-i} = (v_\theta - p_A) \sum_{j=k}^{\sigma-1} \beta_j t^j (1-t)^{a-j}
\]

or

\[
\frac{p_A}{v_\theta - p_A} = \sum_{j=k}^{\sigma-1} \frac{1}{\sum_{i=C_A-1}^{k-1} \beta_i \frac{1-t^i}{t^i}}
\]

Since each term \( \frac{1-t^i}{t^i} \) with \( j > i \) is strictly decreasing in \( t \in (0, 1) \) between \(+\infty\) and 0, the right hand side is strictly increasing in \( t \in (0, 1) \) between 0 and \(+\infty\). Also, \( \frac{p_A}{v_\theta - p_A} \) is strictly increasing in \( p_A \). Hence, the mapping between \( p_A \in (0, v_\theta) \) and \( t \in (0, 1) \) is one-to-one and strictly increasing.

Since there is a unique symmetric perfect equilibrium \( (t=1) \) when \( p_A = v_\theta \), Lemma 4 is proved. Since \( t=0 \) in the equilibrium selected when \( p_A = 0 \), the mapping is strictly increasing from \([0, v_\theta]\) to \([0, 1]\) and Lemma 5 is proved.

\[\square\]

D Preliminary function study

Let us study \( F_{x,y}(t) = P^x_t[T \geq y] \) with \( x \geq y \).

\[
F_{x,y}(t) = \sum_{i=y}^{x} \binom{x}{i} t^i (1-t)^{x-i}
\]

\[
F'_{x,y}(t) = x \binom{x-1}{y-1} t^{y-1}(1-t)^{x-y} = x P^x_t[T = y-1]
\]

\( F_{x,y} \) is strictly increasing on \([0, 1]\). If \( x = y \), \( F'_{x,y}(t) \geq 0 \) on \([0,1]\). If \( x > y \), for all \( t \in [0,1] \),

\[
F'_{x,y}(t) = x(x-1) \binom{x-1}{y-1} [\frac{x-1}{x-1} - t] t^{y-2}(1-t)^{x-y-1}
\]

If \( x > y \), \( F_{x,y} \) is strictly convex on \([0, \frac{x-1}{x-1}]\) and strictly concave on \((\frac{x-1}{x-1}, 1]\), i.e. \( F'_{x,y} \) is single peaked.
E  Proof of Lemma 10

(i) We want to show that the difference $\Delta_1 = P_{\frac{a}{k}}^a[T \geq k] - P_{\frac{a+1}{a+1}}^a[T \geq k]$ is strictly positive. It is easy to check that this is true if $a = k$. Suppose now $a > k$.

$$\Delta_1 = F_{a+1,k}(\frac{k}{a}) - F_{a+1,k}(\frac{k}{a+1}) - \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$$

Since $F_{a+1,k}$ is strictly concave on $(\frac{k-1}{a+1}, 1)$, and $\frac{k-1}{a+1} < \frac{k}{a+1} < \frac{k}{a}$ we have

$$\Delta_1 > \left(\frac{k}{a} - \frac{k}{a+1}\right) F'_{a+1,k}(\frac{k}{a}) - \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$$

$$> \frac{k}{a(a+1)}(a+1)P_{\frac{a}{k}}^a[T = k-1] - \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$$

Hence $\Delta_1 > 0$. \(\Box\)

(ii) We want to show that the difference $\Delta_2 = \frac{a}{k}P_{\frac{a}{k}}^a[T > k] - \frac{a-1}{k}P_{\frac{a-1}{k-1}}^a[T > k]$ is strictly positive. It is easy to check that this is true if $a = k$. Suppose now $a > k$.

$$\Delta_2 = F_{a,k}(\frac{k}{a}) - F_{a,k}(\frac{k}{a+1}) + \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$$

Denote $\tau = \arg max\{F'_{a,k}(t), t \in [\frac{k}{a+1}, \frac{k}{a}]\}$. Since $F_{a,k}$ is strictly concave on $(\frac{k-1}{a+1}, 1)$ we have $\tau \in \{\frac{k-1}{a+1}, \frac{k}{a+1}\}$.

$$\Delta_2 \geq \left(\frac{k}{a+1} - \frac{k}{a}\right)F'_{a,k}(\tau) + \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$$

$$\geq \frac{k}{a+1}P_{\frac{a}{k}}^a[T = k-1] + \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$$

Since $\frac{k}{a+1} \leq \tau$, we have $\Delta_2 \geq -\tau P_{\frac{a}{k}}^a[T = k-1] + \frac{k}{a}P_{\frac{a}{k}}^a[T = k-1]$.

Since $\frac{k}{a} = \arg max\{tP_{\frac{a}{k}}^a[T = k-1], t \in [0, 1]\}$ and $\tau \neq \frac{k}{a}$, $\Delta_2 > 0$. \(\Box\)

F  Proof of Lemma 11

(i) We want to show that the difference $\Delta_3 = P_{\frac{a}{k+1}}^a[T \geq k+1] - P_{\frac{a}{k}}^a[T \geq k]$ is strictly positive. (This makes sense only if $a > 1$ and $a > k$.)

$$\Delta_3 = P_{\frac{a}{k+1}}^a[T \geq k+1] - P_{\frac{a}{k}}^a[T \geq k+1] - P_{\frac{a}{k}}^a[T = k] = F_{a,k+1}(\frac{k+1}{a}) - F_{a,k+1}(\frac{k}{a}) - \frac{1}{a}F'_{a,k+1}(\frac{k}{a})$$

Denote $\tau = \arg min\{F'_{a,k+1}(t), t \in [\frac{k}{a+1}, \frac{k+1}{a+1}]\}$.

$$\Delta_3 > \frac{1}{a}[F'_{a,k+1}(\tau) - F'_{a,k+1}(\frac{k}{a})]$$
Since $F'_{a,b+1}$ is single peaked, $\tau \in \{\frac{k}{a}, \frac{b+1}{a}\}$. Thus, it suffices to show that $\tau = \frac{k}{a}$. This amounts to proving that $\ln\left(\frac{F'_{a,b+1}(\frac{b+1}{a})}{F'_{a,b+1}(\frac{k}{a})}\right) > 0$. We have

$$\ln\left(\frac{F'_{a,b+1}(\frac{b+1}{a})}{F'_{a,b+1}(\frac{k}{a})}\right) = k \ln(1 + \frac{1}{k}) + (a-k-1) \ln(1 - \frac{1}{a-k})$$

Let us study the function $g_a(x) = x \ln(1 + \frac{1}{x}) + (a-x-1) \ln(1 - \frac{1}{a-x})$ on $[\frac{a}{2}, a-1]$.

$$g_a'(x) = \ln(1 + \frac{1}{x}) - \frac{1}{x+1} - \frac{1}{a-x} - \frac{1}{a-x}$$

$$g_a''(x) = \frac{1}{(a-x)^2} - \frac{1}{x+1} + \frac{1}{x} - \frac{1}{a-x}$$

Hence, $g_a''(x)$ is strictly positive if $x > a-x-1$. Hence, $g_a$ is strictly convex on $[\frac{a}{2}, a-1]$. Hence $g_a'(\frac{a}{2}) = \min\{g_a'(x), x \in [\frac{a}{2}, a-1]\}$. $g_a'(\frac{a}{2}) = \ln(1 + \frac{2}{a}) - \frac{1}{1+\frac{1}{a}} - \ln(1 - \frac{2}{a}) - \frac{2}{a}$. We study $h(x) = \ln(1 + \frac{1}{x}) - \frac{1}{1+\frac{1}{x}} - \ln(1 - \frac{1}{x}) - \frac{1}{x}$ on $(1, +\infty)$. We have $h'(x) = \frac{1}{(1+\frac{1}{x})^2} - \frac{1}{x+1} + \frac{1}{x} - \frac{1}{a-x}$. Hence, $h'(x) < 0$. Since $\lim_{x \to +\infty} h(x) = 0$, we have $h(x) > 0$, that is for all $x \in [2, +\infty)$, $g_a'(\frac{a}{2}) > 0$. Hence, for all $x \in [\frac{a}{2}, a-1]$, $g_a'(x) > 0$. It now suffices to show that for all $a \in [2, +\infty)$, $g_a'(\frac{a}{2}) > 0$. We have $g_a'(\frac{a}{2}) = \frac{a}{2} \ln(1 + \frac{2}{a}) + (\frac{a}{2} - 1) \ln(1 - \frac{2}{a})$. Let us study $\ell(x) = \frac{a}{2} \ln(1 + \frac{2}{a}) + (\frac{a}{2} - 1) \ln(1 - \frac{2}{a})$.

$$\ell'(x) = \ln(1 - \frac{1}{x^2}) - \frac{1}{x+1} + \frac{1}{x} \quad \ell''(x) = \frac{(2x+1)(x-2)}{2x^2(x-1)(x+1)^2}$$

Hence $\ell''(x) < 0$ on $(2, +\infty)$. Since $\lim_{x \to +\infty} \ell'(x) = 0$, $\ell$ is strictly increasing on $(2, +\infty)$. Since $\lim_{x \to 2^+} \ell(x) = 0$, $\ell$ is strictly positive on $(2, +\infty)$. 

(ii) This point is a consequence of Lemma 9 and 11: the value of A-shares is greater under $[a+1, k+1]$ than under $[a, k+1]$ (Lemma 9), and greater under $[a, k]$ than under $[a+1, k+1]$ (Lemma 11).

G Proof of Lemma 12

A-shares. We want to show that if $a > k$ then the difference $\Delta_4 = P_a^{a-1}[T \geq k] - P_a^{a}[T \geq k+1]$ is strictly positive.

$$\Delta_4 = P_a^{a-1}[T \geq k+1] + P_a^{a-1}[T = k] - P_a^{a}[T \geq k+1]$$

$$= P_a^{a}[T \geq k+1] - P_a^{a}[T = k] + P_a^{a-1}[T = k] - P_a^{a}[T \geq k+1]$$

$$= (1 - \frac{k}{a}) P_a^{a-1}[T = k] + F_{a,k+1}(\frac{k}{a}) - F_{a,k+1}(\frac{b+1}{a})$$

Denote $\tau = \arg\max\{F'_{a,k+1}(t), t \in [\frac{k}{a-1}, 1]\}$. We know that $F_{a,k+1}$ is strictly concave on $(\frac{k}{a-1}, 1)$. It is easy to check that $2k > a$ imply that $\frac{k}{a-1} > \frac{b+1}{a}$. Since $\frac{b+1}{a} > \frac{k}{a}$ we have $V$
\[ \tau = \frac{b_{11}}{a_{11}} \] Hence,

\[ \Delta_4 > (1 - \frac{\theta}{a}) P_{\frac{a}{b}}^A[T = k] - (\frac{b_{11}}{a_{11}} - \frac{k}{a}) \cdot F_{e, k+1}(\frac{b_{11}}{a_{11}}) \]

\[ > (\frac{a-1}{k})(\frac{\theta}{a})^k (1 - \frac{k}{a})^{a-k} - \frac{a-k}{a(a+1)}, a, (\frac{a-1}{k})(\frac{b_{11}}{a_{11}})^k (1 - \frac{b_{11}}{a_{11}})^{a-k-1} \]

\[ > (\frac{a-1}{k})(\frac{\theta}{a})^k (1 - \frac{k}{a})^{a-k} - (\frac{a-1}{k})(\frac{b_{11}}{a_{11}})^k (1 - \frac{b_{11}}{a_{11}})^{a-k} \]

Since \( x^k(1-x)^{a-k} \) is maximal at \( x = \frac{k}{a} \), we have \( \Delta_4 > 0 \). \( \square \)

**B-shares.** Proving \( \Delta_5 = P_{\frac{a}{b}}^A[T \geq k] - P_{\frac{a_{11}}{b_{11}}}^A[T \geq k+1] > 0 \) if \( a > k \) is similar. \( \square \)

### H Proof of Lemma 13

If \( a \) is odd, \( a = 2k-1 \). Then, by Corollary 1, \([2k-1, k]\) is dominated by \([2(k-1), k-1]\) which also corresponds to the simple majority rule. If \( a \) is even, \( a = 2k \). Define the differences \( \Delta_6 \) and \( \Delta_7 \) by

\[ \Delta_6 = P_{\frac{2k-1}{2k}}^A[T \geq k] - P_{\frac{2k}{2k}}^A[T \geq k] \]

\[ \Delta_7 = P_{\frac{2k-1}{2k}}^A[T = k-1] - P_{\frac{2k-2}{2k-1}}^A[T = k-1] \]

We want to prove that \( n\Delta_6 + k\Delta_7 > 0 \).

\[ \Delta_6 = F_{2k-1,k}(\frac{k}{2k-1}) - F_{2k-1,k}(\frac{1}{2}) - \frac{1}{2}P_{\frac{2k-1}{2k}}^A[T = k-1] \]

Since \( F_{2k-1,k}' \) is maximal at \( \frac{1}{2} \), we have

\[ \Delta_6 > (\frac{k}{2k-1} - \frac{1}{2}) F_{2k-1,k}(\frac{k}{2k-1}) - \frac{1}{2}P_{\frac{2k-1}{2k}}^A[T = k-1] \]

\[ > \frac{1}{2(2k-1)}(2k-2)(\frac{k}{2k-1})^{k-1}(1 - \frac{k}{2k-1})^{k-1} - \frac{1}{2}(\frac{2k-1}{k-1})(\frac{1}{2})^{k-1} \]

\[ > \frac{1}{2}(\frac{2k-1}{k-1})(\frac{k}{2k-1})^{k-1}(1 - \frac{k}{2k-1})^{k-1} - \frac{1}{2}(\frac{2k-1}{k-1})(\frac{1}{2})^{k-1} \]

Since \( x^k(1-x)^{k-1} \) is maximal at \( \frac{k}{2k-1} \), \( \Delta_6 > 0 \).

\[ \Delta_7 = (\frac{2k-1}{k-1})^{2k-1} - (\frac{2k-1}{k-1})(\frac{k}{2k-1})^{k-1}(1 - \frac{k}{2k-1})^{k-1} < 0 \]

We remark that \( \Delta_6 > -\frac{1}{2}\Delta_7 \). Since \( n \geq 2k \) we have \( n\Delta_6 + k\Delta_7 > 0 \). \( \square \)

### I Proof of Lemma 15

If \( p_B \leq v_B \) no B-shareholder tenders. Then if \( C_B > 0 \) the bid fails with probability one, i.e. there are no 'suspicions' equilibria. Suppose that \( C_B = 0 \). Consider a sequence of
symmetric fully mixed strategy profiles $\sigma'' = (t_\nu, \ldots, t_\nu)$, i.e. with $t_\nu \in (0, 1)$. Taking into account that $p_A$ and $(v_\beta - p_A)$ are strictly negative, comparing the costs and benefits of tendering for one shareholder amounts to comparing (see Appendix C):

$$\frac{p_A}{v_\beta - p_A} \quad \text{and} \quad \sum_{j=k}^{\sigma-1} \frac{1}{\sum_{i=0}^{k-1} \frac{\beta_j}{\beta_i} \left(1+ \frac{1-t}{t}\right)^{j-i}}$$

The LHS is strictly positive.

Existence of ‘confidence’ equilibria: Take $t_\nu = \frac{1}{\nu}$. The RHS goes to zero as $\nu$ goes to infinity. Hence, for $\nu$ large enough, $t_\nu = 0$ is a best response to $\sigma''$. $\sigma = (0, \ldots, 0)$ being the limit of the sequence, it is a perfect equilibrium.

Existence of ‘suspicion’ equilibria: Take $t_\nu = 1 - \frac{1}{\nu}$. The RHS goes to infinity as $\nu$ goes to infinity. Hence, for $\nu$ large enough, $t_\nu = 1$ is a best response to $\sigma''$. $\sigma = (1, \ldots, 1)$ being the limit of the sequence, it is a perfect equilibrium.

Note that, as in the case of a ‘good’ raider, equation (8) defines a symmetric equilibrium in totally mixed strategies, hence a perfect one. In this, the bid succeeds with a strictly positive probability.

The reasoning is the same when $C_B > 0$: $C_B$ plays the role of $k$ for B-shareholders. □

J Proof of Proposition 9

For simplicity, we restrict to even values of $r$, so that $k = \frac{r}{2}$. The proof is in two steps. The value of the set of voting shares is shown to be unchanged as shares are split, while that of voting shares decrease. Suppose the voting shares represent a fixed fraction $\alpha$ of the return rights. The value of all voting shares is

$$A = 2k \cdot \frac{\alpha v_\beta}{2k} \cdot P_{\frac{1}{2}}^{2k-1}[T \geq k]$$

By symmetry, with $t = \frac{1}{2}$, the complementary events $T \in [0, k-1]$ and $T \in [k, 2k-1]$ are equiprobable. Hence $A = \frac{\alpha v_\beta}{2}$, independently of $a$.

The value of the set of B-shares is

$$B = (1-\alpha)v_\beta \cdot P_{\frac{1}{2}}^{2k}[T \geq k]$$

Again, by symmetry, the events $T \in [0, k]$ and $T \in [k, 2k]$ are equiprobable but this time not complementary. Hence

$$B = (1-\alpha) (\frac{1}{2} + \frac{1}{2} P_{\frac{1}{2}}^{2k}[T = k])$$

$$= (1-\alpha) (\frac{1}{2} + (\frac{1}{2})^{2k+1} \binom{2k}{k})$$

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Hence, we only have to prove that \( h(k) = \left( \frac{1}{2} \right)^{2k+1} \binom{2k}{k} \) is decreasing in \( k \).

\[
\frac{h(k)}{h(k+1)} = \frac{2^{2k+3}}{2^{2k+1}} \frac{(2k)!}{k!k!} \frac{(k+1)!(k+1)!}{(k+2)!} = \frac{2(k+1)}{2k+1} > 1
\]